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QUARK-GLUON ADMIXTURE IN HADRONS IN STRONG COUPLING AND SMALL VOLUME EXPANSIONS*

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ABSTRACT

It is shown that in strong coupling expansions with noncompact lattice regularization as well as in the small volume expansion quarks and gluons are coupled to leading order, at variance with Wilson's strong coupling expansion. These approaches suggest that quarks and gluons contribute in comparable amount to the structure of all hadrons (there should be no glueballs) in agreement with observation about the momentum and the spin of the proton.

There is firm experimental evidence that gluons give a contribution to the momentum(1) and the spin(2) of the proton comparable to that of quarks. Yet the different quark models are based on the assumption of decoupling of quarks from gluons which also leads to the prediction of glueballs(3). Let us in fact stress that this prediction is based on this unproven assumption.

Now it is understood that quark-gluon (q-g) decoupling is a zeroth order approximation. The point we want to raise here is that in view of the above experimental evidence a zeroth order approximation including a substantial q-g admixture might be better.

In principle the extent of q-g decoupling should be derived from QCD, but this is at the moment out of reach. The justification of decoupling which to our understunding is considered more sound lies in the strong coupling expansion of Wilson lattice gauge theories(4). The use of such an expansion in hadron spectroscopy goes back to Wilson's proposal(5) of the lattice regularization, and it has been fully exploited in papers(6) based on the Hamiltonian formalism developed by Kogut and Susskind(7).

An attractive feature of the strong coupling expansion is that it accounts for confinement so that it is intrinsically consistent.

* Presented by F. Palumbo

The obvious weakness of this approach is that if one takes two different actions with the same continuum limit one gets two different strong coupling expansions. For instance addition to Wilson's action of a plaquette term where the link variables belong to the adjoint representation of the gauge group changes completely the phase diagram at strong coupling[(8)]. An even more dramatic change arises from using noncompact lattice regularizations[(9,10)]. The way to avoid such arbitrariness is to obtain the infrared effective action by renormalization group methods from the continuum, i.e. small coupling, but this is again out of reach.

In the framework of strong coupling expansions we are therefore bound for the time being to assume as infrared effective action some regularized action at strong coupling, but we suggest to do it in such a way that the resulting q-g admixture be large already to zeroth order. We will show that this indeed happens with noncompact lattice regularizations. Moreover in one of such regularizations a new mechanism for quark confinement emerges, which makes the strong coupling expansion intrinsically consistent. The mechanism is quite natural, in that, unlike Wilson's one, it holds only for nonabelian gauge theories.

From the point of view of an actual estimate of q-g admixture another approach seems more interesting, namely the small volume expansion[(11)]. The reason is that it does not require any assumption. We will show that to leading order q-g coupling is very similar to that of the strong coupling expansion with noncompact regularizations.

We will illustrate how strong q-g mixing can arise using the unregularized QCD Hamiltonian formalism, and then we will mention how our analysis is affected by the regularization, and what is the connection with the small volume expansion.

We will confine ourselves to the SU(2) color group. The Hamiltonian density in the continuum can be written

$$H = \frac{1}{2} (\mathcal{E}_k^a)^2 + \frac{1}{2} \sum_{h<k} (\mathcal{F}_{hk}^a)^2 + \frac{i}{2} \bar{\psi} \gamma^k \mathcal{D}_k \psi + \text{h.c.} + m \bar{\psi} \psi \qquad (1)$$

where,

$$\mathcal{F}_{hk}^a = \partial_h \mathcal{A}_k^a - \partial_k \mathcal{A}_h^a + g \,\varepsilon_{bc}^a \mathcal{A}_h^b \mathcal{A}_k^c \qquad (2)$$

$$\mathcal{E}_k^a = \mathcal{F}_{ok}^a , \qquad (3)$$

$$\mathcal{D}_k = \partial_k - ig \, A_k^a \frac{1}{2} \tau_a , \qquad (4)$$

ε_{bc}^a being the antisymmetric tensor and τ_a the Pauli matrices.

The electric strength components are canonically conjugate to the gauge field components

$$\left[\mathcal{A}_h^a (x), \mathcal{E}_k^b(y) \right] = i \, \delta_{hk} \, \delta^{ab} \delta (x - y). \qquad (5)$$

The physical states are restricted by the Gauss law

$$\left(\mathcal{D}_{kb}^a \, \mathcal{E}_k^b + g \, j_o^a \right) \Psi = 0, \qquad (6)$$

where

$$j_o^a = \psi^* \frac{1}{2} \tau^a \psi \tag{7}$$

are the color charge densities and Ψ is the state functional.

In order to obtain a sensible limit for $g \to \infty$ we must scale $\mathcal{A}_k^a$ and $\mathcal{E}_k^b$ according to

$$\begin{aligned} \mathcal{A}_k^a &\to g^{-1/3} \mathcal{A}_k^a \\ \mathcal{E}_k^a &\to g^{1/3} \mathcal{E}_k^a. \end{aligned} \tag{8}$$

We get in this way in terms of the scaled variables to leading order in g

$$H = g^{2/3} \left\{ \frac{1}{2} (\mathcal{E}_k^a)^2 + \frac{1}{2} \sum_{h<k} \left(\varepsilon_{bc}^a \mathcal{A}_k^b \mathcal{A}_k^c \right)^2 - \bar{\psi} \gamma^k \frac{1}{2} \tau_a \psi \mathcal{A}_k^a \right\} + m \bar{\psi} \psi \tag{9}$$

with the Gauss constraint

$$(\varepsilon_{bc}^a \mathcal{A}_k^c \mathcal{E}_k^b + j_k^o) \Psi = 0. \tag{10}$$

The strong coupling limit provides a hyperlocal approximation, all spatial derivatives disappearing. As anticipated there remains a term coupling quarks to gluons which can in no way be considered a priori negligible.

The mentioned mechanism for confinement has been proved only for SU(2) but we believe it to have a general validity. It works as follows. We can rewrite the Gauss constraint in the form

$$\left(\mathcal{L}^a + \mathcal{S}^a \right) \Psi = 0, \tag{11}$$

where

$$\mathcal{S}^a = \psi^* \frac{1}{2} \tau^a \psi \tag{12}$$

$$\mathcal{L}^a = \sum_k l_k^a \tag{13}$$

$$l_k^a = \varepsilon_{bc}^a \mathcal{A}_k^c \mathcal{E}_k^b \, . \tag{14}$$

l_k^a are, for fixed k, the components (labeled by the color index a) of an orbital angular momentum. $\mathcal{L}$ is therefore a total orbital angular momentum whose eigenvalues must be integral. The halfintegral ones belong in fact to two-valued eigenfunctions which are excluded from the domain of the Hamiltonian which contains a Laplacian

$$(\mathcal{E}_k^a)^2 = - \frac{\delta^2}{\delta(\mathcal{A}_k^a)^2} \tag{15}$$

Since the eigenvalues of $\mathcal{S}^a$ are half integral the Gauss law can be satisfied only if quarks plus antiquarks occur in even number at each point. A quark cannot be separated from another one even by an infinitesimal distance, what we have called superconfinement. It is worth while noticing that confinement might take a different form according to the scale at which it is observed. The above mechanism might explain confinement when we look at hadrons on a scale much larger than their dimensions, even if the relevant mechanism when we study hadrons on a smaller scale is different.

Let us now consider the effect of the regularization. First of all it is obvious that the above analysis does not apply with Wilson regularization. Instead of the Laplacian (15) there is in that case the square of an angular momentum, so that two-valued eigenfunctions are admissible and the present confining mechanism does not work. Moreover, since the coupling of matter fields to gauge fields is through unitary matrices, there can be no quark term increasing with g, which leads to q-g decoupling.

Two noncompact regularizations have been recently proposed[(9,10)], where the q-g coupling is similar to that of Eq. (9). We believe that this is a general feature of noncompact regularizations. Only one has been studied in some detail. In the Hamiltonian formalism its strong coupling limit[(9)] exactly coincides with (9)

$$H = \frac{g^{3/2}}{a}\left\{-\frac{1}{2}\frac{\partial^2}{\partial A_{ka}^2}+\frac{1}{2}\sum_{h<k}\left(\varepsilon^a_{bc}A^b_h A^c_k\right)^2-\bar{\psi}\gamma^k\frac{1}{2}\tau_a\psi A^a_k\right\}+m\bar{\psi}\psi \tag{16}$$

with the Gauss constraint

$$\left[\varepsilon^a_{bc}A^c_k\left(-i\frac{\partial}{\partial A^b_k}\right)+\ \psi^*\frac{1}{2}\tau_a\psi\right]\Psi=0 \tag{17}$$

In the above equations a is the lattice spacing and all the fields are dimensionless and defined at lattice sites.

A strong coupling expansion shows that at finite but large g the q-q wave function spreads over the lattice but quarks remain confined. The q-q wave function decreases in fact at large separation distance Na like

$$|\Psi|^2 \sim g^{-\frac{4}{3}N} \tag{18}$$

vanishing for $N\rightarrow\infty$.

The drawback of the regularization of Ref. (9), however, is that it makes use of gauge invariant variables which arise from the solution of a nonrenormalizable gauge fixing. The curvilinear nature of these variables poses additional difficulties[(12)]. So although Eqs. (16)-(17) can be used to have an estimate, crude as it may be, of q-g admixture, it may turn out to be difficult to use them as the starting point of a systematic expansion. Moreover, as already emphasized, this approach is based on the assumption that the regularized Hamiltonian at strong coupling approximates the infrared effective Hamiltonian.

In order to estimate q-g admixture may therefore be more convenient to use the small volume expansion which allows to evaluate systematically higher order corrections and does not contain any ad hoc assumption.

In the small volume expansion one considers the gauge field in a cubic box of edge L with periodic boundary conditions[(11)]. The zero momentum modes (the spatially constant modes) of the gauge field and electric strength are scaled according to Eq. (8) while the other modes are not scaled. One then writes an effective Hamiltonian in the Fock space generated by zero momentum modes*

* The notation is slightly different from Ref. (11).

$$H = H_0 + H_1 + H_E, \tag{19}$$

where H_0 is a free field Hamiltonian, H_E takes in to account the effect of nonzero momentum modes and

$$H_1 = \frac{1}{L} g^{2/3} \left\{ \frac{1}{2} e_{ka}^2 + \frac{1}{2} \sum_{h<k} \left(\varepsilon^{abc} c_{hb} c_{kc} \right)^2 \right\} . \tag{20}$$

In the above equation c_{ha} and e_{ha} are spatially constant modes of gauge field and electric strength. If H_E is evaluated perturbatively it is given by an expansion in $g^{1/3}$, the leading term being of order g. The quantization box must in such a case be small because the running coupling costant appears at a value of the momentum equal to 1/L. So H_1 is the leading interaction term, and it is the same as the pure gauge part of Eq.(16).

What is essential with respect to q-g admixture (and so far unnoticed as far as we know), is that there is a q-g term of order $g^{2/3}$

$$\frac{1}{L} g^{2/3} c_{ka} \int d^3x \, \overline{\psi}(x) \frac{1}{2} \tau_a \gamma_k \psi(x) \tag{21}$$

which is very similar (although not exactely equal) to the term appearing in Eq. (16) and that should be added to H_1.

The Gauss low in the present case does not lead to quark confinement, but only states that in the quantization box the number of quarks plus antiquarks must be even (for SU(2)).

The effective Hamiltonian in the space of the constant modes, once the term (21) is included, might serve to estimate q-g admixture. The agreement between small volume expansion and Monte Carlo calculations [13] for the pure gauge Hamiltonian gives some support to the reliability of the results in the presence of the q-g interaction term (21).

In conclusion we have shown that two different approaches give to leading order a term coupling quarks to gluons which can in no way be considered negligible with respect to pure gluons or pure quarks (the mass term) terms. This suggests that strong q-g admixture be a general feature of all hadrons, in agreement with observation about the momentum and the spin of the proton. If this turns out to be the case, a reassessement of the notion of glueball is in order.

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