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CHIRAL INVARIANCE ON THE LATTICE

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ABSTRACT

A definition of fermions on a lattice is proposed which does not violate chiral invariance. The no-go theorem of Nielsen and Ninomiya is evaded by using a second order formalism for the fermions.

In this note we propose a definition of fermions on a lattice which does not violate chiral invariance. The no-go theorem of Nielsen and Ninomiya[1)] is evaded by using a second order formalism for fermions, which makes their propagator on the lattice similar to that of the bosons as far as the poles are concerned.

A second order equation for spinors equivalent to the Dirac equation was first considered by Feynman and Gell-Mann[2)]. No action was found, however, from which the Feynmam-Gell-Mann equation would follow[3)]. We introduce in fact the Feynmam-Gell-Mann wave operator into the path integral through a change of variables whose result cannot be cast in the form of a path integral for a second order fermionic action.

Let us start from the standard expression of the fermionic path integral in the continuum

$$Z_F = \int \prod_x d\,\overline{\psi}(x)\, d\,\psi(x)\, e^{-i\int d^4x\, \mathcal{L}_F} \tag{1}$$

where

$$\mathcal{L}_F = \bar{\psi} (i \not{\mathcal{D}} + m) \psi + \bar{\eta} \psi + \bar{\psi} \eta . \tag{2}$$

In the above equation ψ is a Dirac spinor, η its source and $\mathcal{D}_\mu$ the covariant derivative describing its coupling to a gauge field that we do not need to specify. We consider massive spinors because the method we are going to present is relevant also to this case since it avoids the unwanted degenerate states. The chiral case can obviously be obtained by putting the mass equal to zero.

Let us perform in (1) the change of variables

$$\psi = (- i \not{\mathcal{D}} + m) \psi' \tag{3}$$

so that

$$Z_F = \int \prod_x d \bar{\psi} (x) d \bar{\psi}' (x) \left[\det (- i \not{\mathcal{D}} + m) \right]^{-1} e^{- i \int d^4 x \mathcal{L}'_F} \tag{4}$$

where

$$\mathcal{L}'_F = \bar{\psi} (\not{\mathcal{D}}^2 + m^2) \psi' + \bar{\psi} \eta + \bar{\eta} (- i \not{\mathcal{D}} + m) \psi'. \tag{5}$$

The states with vanishing eigenvalues of the Dirac wave operator are understood to be projected out in a gauge-invariant way, otherways $Z_F = 0$. Since this projection is not peculiar of the lattice, we will not discuss it here.

$\mathcal{L}'_F$ is a convenient form for a lattice transcription. Let us in fact consider its free part

$$\mathcal{L}_F^{(0)} = \bar{\psi} (\Box + m^2) \psi' + \bar{\psi} \eta + \bar{\eta} (- i \not{\partial} + m) \psi'. \tag{6}$$

On the lattice we define left and right derivatives

$$\Delta_\mu^{\pm} f (x) = \pm \frac{1}{a} \left[f (x \pm \mu) - f (x) \right] , \tag{7}$$

where x is a vector with integral components, μ a vector with components $\mu_\nu = \delta_{\mu\nu}$ and a the lattice spacing. The lattice d'Alambertian is

$$\Box = \Delta_\mu^{(+)} \Delta_\mu^{(-)} . \tag{8}$$

We then see that the fermion propagator is

$$S_F^{-1} = -(-i\gamma^\mu \Delta_\mu^{(+)} + m)\ \frac{1}{\Delta_\mu^{(+)}\ \Delta_\mu^{(-)} + m^2} \tag{9}$$

which has the right pole.

For the lattice transcription of the full fermionic path integral Z_F it is convenient to introduce compensating pseudobosons through the equation

$$\int \prod_x d\,\overline{\varphi}\,(x)\, d\,\varphi\,(x)\ \ e^{-i\int d^4x\, \overline{\varphi}\,(\not{D}^{\,2} + m^2)\,\varphi} = |\det(-i\ \not{D} + m)|^{-1}\,. \tag{10}$$

φ are bosons in the spinor representation subject to the constraint

$$i\,\gamma_5\,\varphi = \varphi \tag{11}$$

in the absence of which the determinant in Eq. (10) would appear squared.

Using Eq. (10) Z_F can be rewritten

$$Z_F = \int \prod_x\ d\,\overline{\psi}\,(x)\, d\,\overline{\psi}'\,(x)\ \ d\,\overline{\varphi}\,(x)\, d\,\varphi\,(x)\,\sigma\ \cdot$$

$$e^{-i\int d^4x\,[\,\overline{\varphi}\,(\not{D}^{\,2} + m^2)\,\varphi + \overline{\psi}\,(\not{D}^2 + m^2)\,\psi' + \psi\,\eta + \overline{\eta}\ (-i\,\not{D} + m\,]\psi'} \tag{12}$$

where

$$\sigma = \operatorname{sgn}\det(-i\ \not{D} + m). \tag{13}$$

In Euclidean space $\sigma = 1$.

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