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Weyl Invariance of the Green-Schwarz Heterotic Sigma Model

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Abstract

Weyl invariance at one loop is proven for the heterotic non-linear sigma model coupled to curved superspace with non-trivial fermionic and bosonic background fields. A general formula is given relating the Weyl anomaly to the sigma model β-functions at arbitrary sigma-model loop order. The Weyl anomaly is also considered in the presence of Yang-Mills backgrounds.

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1. Introduction

The intriguing connection between the non-linear sigma model and string theory has been investigated extensively. In the bosonic sigma-model it is well-known that the requirement of Weyl invariance at one loop, a necessary requirement for a consistent string theory coupled to a spacetime background, leads to the equations of motion for the spacetime background fields [1,2]. At higher loops, this requirement leads to string corrections to the equations of motion of the spacetime background fields, that is, corrections to the low-energy effective theory [3]. This procedure has also been followed for the Neveu-Schwarz-Ramond (NSR) string [4]. However, it is not known at this time how to couple fermionic backgrounds to the NSR sigma-model, so the string corrections obtained in this way are not supersymmetric. Supersymmetric string corrections can be obtained from superspace considerations: the Bianchi identities (i.e., closure of the supersymmetry algebra) constrain the form of the fermionic contributions to the string corrections [5]. It is not known whether the corrections obtained in this way are consistent with the propagation of the string in a background. The Green-Schwarz (GS) formulation allows coupling to fermionic backgrounds, and hence a possible way to resolve these questions. Some results for GS sigma-models in bosonic backgrounds have been obtained [6]. Until recently, however, difficulties in the covariant quantization of GS strings have hindered such investigations. The GS string has now been quantized in "semi-covariant" [7-9] and "covariant" [10] gauges, however the "covariant" gauges suffer from problems related to the infinite tower of ghosts in the theory [11]. The "semi-covariant" gauge restricts the spacetime curvature to be flat in the direction determined by the gauge-fixing. Here we will use the "semi-covariant" method to investigate the Weyl anomaly of the theory to one loop.

Quantization of the GS sigma-model in the semi-light-cone gauge [7] has been shown to lead to an effective action[3] which is renormalizable in the sense of Friedan [1] when the spacetime background satisfies its equations of motion [8]. It was shown in [8] that there exists a renormalization scheme in which the β-functions are zero. However, from the bosonic case it is known that the vanishing of the β-functions is not equivalent to the condition of Weyl invariance [12]. In fact, if the background spacetime satisfies $R_{mn} = \nabla_{(m}v_{n)}$ for an arbitrary vector v_m, then there is a scheme in which the β-functions are zero, however, the Weyl anomaly is non-zero. If the vector v_m is the derivative of the dilaton, and the Fradkin-Tseytlin term [2] is included in

[3] We will use the term "effective action" to refer to the two-dimensional quantum one-loop effective action. This should not be confused with the effective action for the background spacetime fields, which is obtained by integrating Einstein's equations.

the action, then the sigma model is Weyl invariant at one loop provided the background spacetime satisfies Einstein's equations. The condition of vanishing β-functions is not a coordinate-invariant condition, and depends on the renormalization scheme chosen. The Weyl anomaly, on the other hand, is a physical quantity that must be the same in any renormalization scheme. Thus we are led to ask for the GS string whether the Weyl anomaly vanishes. We will show that it does indeed vanish, if the Fradkin-Tseytlin term is included in the action. The Fradkin-Tseytlin term is not κ-invariant, however. We will discuss the possibility of a κ-anomaly and ways it might be avoided.

In section 2, quantization of the GS sigma model in the semi-light-cone gauge is reviewed. The κ-noncovariance of the normal coordinate expansion requires that one impose the same gauge-fixing condition on the background fields as is imposed on the quantum fields. Section 3 presents the calculation of the divergences of the effective action, expressing them in terms of a supercoordinate transformation together with a gauge transformation of the antisymmetric tensor. The calculation is done without imposing conformal gauge on the world-sheet metric, keeping terms to linear order in the two-metric perturbation. It is found that both the bosonic and the fermionic parts of the supercoordinate transformation are required. Section 4 gives the extension to the GS case of the expression for the Weyl anomaly [13-15]. In section 5, diffeomorphisms are considered and the proof of Weyl invariance is given for the case when there are no Yang-Mills backgrounds. Section 6 considers the coupling to Yang-Mills backgrounds, and it is shown that the ambiguity in our regularization procedure implies that Weyl anomaly cancellation is also possible in this case.

2. Normal coordinates and gauge fixing

We start with the action for the GS sigma-model in a spacetime background of supergravity coupled to a tensor multiplet:

$$S = \int d^2\sigma \, [\frac{1}{2}\gamma^{ij} V_i^a V_{ja} + \epsilon^{ij} V_i^C V_j^D B_{CD}] + S_{het} \quad ; \tag{2.1}$$

$$V_i^A \equiv \partial_i Z^M E_M{}^A \quad , \quad Z^M = \{X^m, \theta^\mu\} \ ,$$

$$\gamma^{ij} \equiv \sqrt{g} g^{ij} \ ,$$

and S_{het} is the action for the left-moving heterotic fermions.

The heterotic fermions in (2.1) do not couple to the background superspace when the gauge fields are set to zero. Thus they only contribute to the flat background Weyl anomaly, which vanishes in ten spacetime dimensions [7], as long as boundary effects are ignored [16]. The inclusion of fermionic backgrounds here will not disturb the cancellation of the flat-spacetime Weyl anomaly. We will consider the contribution of the heterotic fermions in section 6, when we turn the Yang-Mills background back on.

We do a background-quantum splitting of the coordinates: $Z^M \to Z^M + \Pi^M(y)$, where the perturbation y^A is treated simultaneously as a quantum perturbation and a normal coordinate. Normal coordinate expansion in superspace [17] leads to the quadratic action given in [8]:

$$
\begin{aligned}
L^{(2)} &= L_{BB} + L_{FF} + L_{BF};\\
L_{BB} &= \frac{1}{2}\gamma^{ij} D_i y^a D_j y_a + (\gamma^{ij} + \epsilon^{ij}) V_i^a D_j y^b y^c H_{cba}\\
&\quad + \frac{1}{2}\gamma^{ij} V_i^a y^c V_j^B y^d R_{dBca}\\
&\quad + \epsilon^{ij} y^\alpha V_i^b y^c (T_{cb}\Gamma_a V_j) + \frac{1}{2}\epsilon^{ij} V_i^d V_j^c y^b y^a \nabla_a H_{bcd},\\
L_{FF} &= (\gamma^{ij} + \epsilon^{ij})[V_i^a (D_j y \Gamma_a y)\\
&\quad + V_i^\alpha V_j^\beta y^\alpha T_{\alpha\beta}{}^\gamma (\Gamma_a y)_\gamma + 2(V_i \Gamma^a y)(V_j \Gamma_a y)],\\
L_{BF} &= 2(\gamma^{ij} + \epsilon^{ij})[D_i y^a (V_j \Gamma_a y)\\
&\quad - 2V_i^c y^b H_{bcd}(V_j \Gamma^d y) + V_i^a V_j^b y^c (T_{cb}\Gamma_a y)].
\end{aligned} \tag{2.2}
$$

Here we have used the superspace constraints of [8]. The relevant constraints for our purpose are

$$
\begin{aligned}
T_{\alpha\beta}{}^c &= 2\Gamma^c{}_{\alpha\beta} = 2H_{\alpha\beta}{}^c,\\
T_{abc} &= -2H_{abc},\\
T_{ab}{}^D &= H_{\alpha bc} = H_{\alpha\beta\gamma} = 0.
\end{aligned} \tag{2.3}
$$

In ten dimensions, imposing the constraints automatically requires that the spacetime fields satisfy their equations of motion. We have the following field equations [8]:

$$
\begin{aligned}
R_{(ab)} &= 2\nabla_{(a}\nabla_{b)}\Phi\\
R_{[ab]} &= -\nabla^c T_{abc} = 2T_{ab}{}^\alpha \nabla_\alpha \Phi + 4H_{ab}{}^c \nabla_c \Phi\\
R_{\alpha b} &= -2(T_{bc}\Gamma^c)_\alpha = 2\nabla_b \nabla_\alpha \Phi.
\end{aligned} \tag{2.4}
$$

The world-sheet covariant derivative in (2.2) contains both world-sheet and spacetime connection terms. For example, acting on a field U_i^A with both types of indices:

$$
D_i U_j^A = \partial_i U_j^A + U_j^D V_i^B \omega_{BD}{}^A - \omega_{ji}^k U_k^A. \tag{2.5}
$$

In order to carry out quantum sigma model calculations it is necessary to deal with the κ symmetry of the action (2.1). The quantum κ-transformations to lowest order are given (using the convention $(\eta^{ij}+\epsilon^{ij})A_iB_j = A_-B_+$) by [8]

$$\begin{aligned}\delta y^M &= V_-^b(\Gamma_b\kappa_+)^\alpha E_\alpha{}^M,\\ \delta\gamma_{++} &= -4V_+^\alpha\kappa_{+\alpha}.\end{aligned} \tag{2.6}$$

Instead of attempting to deal with the infinite towers of ghosts that appear in "covariant" quantization schemes, we use the semi-light-cone gauge for calculations [7-9]. In the approach of Kallosh, this involves choosing two light-cone vectors, M^a, N^a, such that $M^2 = N^2 = 0$ and $M^aN_a = \frac{1}{2}$. Defining spacetime lightcone indices by $M^a(\Gamma_a)_{\alpha\beta} \equiv (\Gamma^\ominus)_{\alpha\beta}$, $N^a(\Gamma_a)_{\alpha\beta} \equiv (\Gamma^\oplus)_{\alpha\beta}$, we impose the ghost truncation condition on the κ-ghosts: $(\Gamma^\ominus\Gamma^\oplus)_\alpha{}^\beta C_\beta = C_\alpha$. For the choice of the gauge fixing fermion as in [8], this imposes the light-cone gauge condition on the fermionic normal coordinate:

$$\Gamma^\oplus_{\alpha\beta}y^\beta = 0. \tag{2.7}$$

Additionally, the truncation condition restricts the spacetime curvature: $R_{abcd}M^cN^d = 0$. We assume that this restriction does not obscure the anomaly structure.

A curious aspect of the normal coordinate split performed here is that, while it preserves manifest supersymmetry covariance (which however is lost when we impose the gauge-fixing), it is not covariant with respect to kappa symmetry. A great advantage of the background field method for Yang-Mills and supergravity theories is that it can be done in such a way as to preserve covariance under the gauge group, and therefore the gauge choice for the external legs of a Feynman diagram is independent of the choice for the propagating internal lines. In other words, the gauge for the background fields can be chosen independently of the gauge for the quantum fields [18]. In our case this is not true! Since we have not maintained kappa covariance in the background-quantum split, we must impose the *same* gauge condition on the background fields that is imposed on the quantum fields, just as if we were calculating in theory with no background-quantum split. That is, we impose

$$\Gamma^\oplus_{\alpha\beta}\theta^\beta = 0. \tag{2.8}$$

As we will see below, there are divergent one-loop diagrams at first order in the two-metric perturbation which appear to make the theory non-renormalizable even in flat space, in contradiction to the results of [7]. These terms vanish when the gauge condition is applied to the background fields, as they must. This observation allows us to settle some questions that have been raised about the consistency of semi-light-cone gauge quantization [19,20]. The Lorentz non-invariance reported in [19] for the type II string is proportional to the gauge-fixing condition, and thus vanishes when this condition is taken into account. The same is true of the Weyl anomaly reported in [20], when the terms involving ϵ^{ij} are carefully regulated, as we will show in section 5. Part of the results and observations that allow us to recover a consistent sigma model quantized by ghost truncation appeared previously in [21], where it was shown that making a light-cone gauge choice for the background fields $(\Gamma^{\oplus}V_+)_\alpha = 0$ and using the two-dimensional field equations produces the vanishing of *both* the Weyl anomaly, *and* a one-loop counterterm that would make the sigma model non-renormalizable, *and* a Lorentz non-covariant gravitational anomaly. In [21] we proved the need of including the Fradkin-Tseytlin term in the effective action, in order to have no conformal anomaly, when the choice for the background $(\Gamma^{\oplus}V_+)_\alpha = 0$ is made. The normal coordinate splitting is useful in spite of its κ-noninvariance, since it allows a normal coordinate expansion which is manifestly covariant in the superspace background fields, as we have seen.

3. The divergent effective action

Our goal is to show that the divergent part of the effective action can be removed by a renormalization of the supercoordinate together with a gauge transformation on the antisymmetric tensor, as long as the spacetime fields satisfy their (ten-dimensional) equations of motion.[4] Thus the situation is very similar to the bosonic NLSM, in the case when the spacetime background of that model satisfies its equations of motion. For our model, this proof has been given *in the conformal gauge* in [8]. However, since the Feynman diagrams for the gauge-fixed theory are no longer covariant with respect to two-dimensional or ten-dimensional general coordinate transformations, it is no longer possible to use invariance arguments to determine the anomaly structure. The importance of using a general reparametrization gauge to investigate the Weyl anomaly has been emphasized in the flat-space case in [16]. Here we will perform the calculation without imposing conformal gauge, keeping terms to first order in the two-metric perturbation. We will find that outside the conformal gauge an additional renormalization of the supercoordinate is required, which will not affect the Weyl anomaly.

We first note that Feynman diagrams which have only bosonic quantum propagators are explicitly covariant in the sense of two-dimensional reparametrizations (as well as being covariant with respect to spacetime reparametrizations and local supersymmetry). This means we can use the same dimensional argument as in the bosonic case [12] to show that the complete divergent contribution from these diagrams is given by the covariantization of the conformal gauge result in [8]. The only

[4] In the bosonic case it is possible to explicity derive the anomaly that appears when the spacetime background does not satisfy the ten-dimensional equations of motion. We cannot do that here, since the constraints we have imposed automatically impose the ten-dimensional equations of motion.

Feynman diagram that needs to be calculated is the bosonic tadpole. We regularize the divergent integral that appears using

$$\frac{1}{(2\pi)^2}\int d^2k\frac{1}{k^2}\rightarrow\frac{1}{(2\pi)^2}\int d^{2+\epsilon}k\frac{1}{(k^2+m^2)}=\frac{1}{4\pi}\Big(\frac{m}{\mu}\Big)^{\epsilon}\Gamma(-\frac{\epsilon}{2}). \tag{3.1}$$

We are assuming that the infrared divergences can be regulated as in [12]. Using the field equations (2.4), the divergent part of the effective action is [8]

$$\begin{aligned}\Gamma^{\infty}_{bos} &= \frac{1}{4\pi\epsilon}\Big(\frac{m}{\mu}\Big)^{\epsilon}\int d^{2+\epsilon}\sigma\Big\{\,2\gamma^{ij}[V_i^bV_j^a\nabla_b\nabla_a\Phi+V_i^\beta V_j^a\nabla_a\nabla_\beta\Phi]\\ &\qquad +\epsilon^{ij}V_i^bV_j^a[-T_{ab}{}^c\nabla_c\Phi+T_{ab}{}^\gamma\nabla_\gamma\Phi]-2\epsilon^{ij}V_i^bV_j^\alpha\nabla_b\nabla_\alpha\Phi\Big\}\\ &=\frac{1}{\epsilon}\Big(\frac{m}{\mu}\Big)^{\epsilon}\int d^{2+\epsilon}\sigma\Big\{\,v^M_{bos}\frac{\delta L}{\delta Z^M}-2\epsilon^{ij}D_i(V_j^\alpha\nabla_\alpha\Phi)\Big\}\\ &=\frac{1}{\epsilon}\Big(\frac{m}{\mu}\Big)^{\epsilon}\int d^{2+\epsilon}\sigma\Big\{\,v^M_{bos}\frac{\delta L}{\delta Z^M}+\epsilon^{ij}\partial_iZ^M\partial_jZ^N\partial_N\Lambda_M\Big\},\end{aligned} \tag{3.2}$$

where

$$v^M_{bos}=\frac{1}{2\pi}E_a{}^M\nabla^a\Phi\quad,\quad\Lambda_M=-\frac{1}{2\pi}E_M{}^\alpha\nabla_\alpha\Phi,$$

and the two-dimensional equations of motion are

$$E_a{}^M\frac{\delta L}{\delta Z^M} = -\gamma^{ij}D_iV_{ja}+\epsilon^{ij}V_i^CV_j^BH_{aBC}=0, \tag{3.3a}$$

$$E_\beta{}^M\frac{\delta L}{\delta Z^M} = (\gamma^{ij}+\epsilon^{ij})V_i^aV_j^\alpha(\Gamma_a)_{\alpha\beta}=0. \tag{3.3b}$$

Thus the divergent effective action arising from the diagrams with only bosonic lines can be written in terms of a supercoordinate transformation and a gauge transformation of the antisymmetric tensor.

We now turn to the diagrams with fermions propagating in the loop. As mentioned already, these diagrams are not manifestly covariant, so we cannot use the usual covariance arguments to fix the form of the effective action. In principle, then, we need to calculate the divergent contributions to all orders in the two-metric perturbation. Here we calculate to first order.

Define the two-metric perturbation by

$$g_{ij}\equiv\eta_{ij}-h_{ij}\quad,\quad\bar h_{ij}\equiv h_{ij}-\frac{1}{2}\eta_{ij}h^k_k\ , \tag{3.4}$$

so that

$$\gamma^{ij}=\eta^{ij}-\bar h^{ij}+H^{ij}\ , \tag{3.5}$$

where H^{ij} is quadratic in the metric perturbation. We carry out our computation of the one-loop effective action to linear order in the metric perturbation h_{ij}. Note that $\bar h^{ij}$ is of order ϵ, using dimensional regularization in $(2+\epsilon)$ dimensions. Using the conventions of [8], we have $(\eta^{ij} + \epsilon^{ij})A_i B_j = A_- B_+$.

The terms of L_{FF} that are needed for the calculation of the divergent part of the effective action to linear order in h_{ij} are:

$$V_-^{\oplus}(D_+ y \Gamma^{\ominus} y) - \bar{h}^{++} V_+^{\oplus}(D_+ y \Gamma^{\ominus} y) - \bar{h}^{--} V_-^{\oplus}(D_- y \Gamma^{\ominus} y). \tag{3.6}$$

The first term gives the propagator for the fermion as:

$$\alpha \;\_\_\_\_\_\_\; \beta \quad = \quad i\frac{p_-}{p^2}(\Gamma^{\oplus})^{\alpha\beta} \;, \tag{3.7}$$

while the propagator for the boson is:

$$a \;-\,-\,-\,-\,-\,-\; b \quad = \quad -\frac{\eta^{ab}}{p^2}. \tag{3.8}$$

In writing the fermion propagator we have performed the rescaling, as in [8]:

$$y^{\alpha} \rightarrow (V_-^{\oplus})^{\frac{1}{2}} y^{\alpha}. \tag{3.9}$$

I has been argued in [16] that this will not contribute to the effective action via the chiral anomaly as long as $V_-^{\oplus}$ satisfies its equation of motion. Every term in the interaction Lagrangian will pick up a factor of $(V_-^{\oplus})^{\frac{1}{2}}$ for each fermion y^{α}. For ease of writing, we omit these factors until the results are given.

First consider one-loop diagrams with one or two external legs. If both propagators are fermions, then to get a divergent integral we need at least one vertex with a D_+ derivative. All diagrams with one derivative vertex and one non-derivative vertex are zero, because of the symmetry of the spinor indices: derivative vertices are symmetric and non-derivative vertices are anti-symmetric in spinor indices. The fermionic tadpole diagram is a purely quadratic divergence, which is zero in dimensional regularization. The diagram with two D_+ derivatives gives the momentum integral

$$\int d^{2+\epsilon}k \; \frac{k_-(k+p)_-(2k+p)_+^2}{k^2(k+p)^2} = 0. \tag{3.10}$$

So there is no contribution to the divergent effective action from this sector.

Considering now diagrams with a boson and a fermion propagator, we need the interaction terms

$$\begin{aligned} &2D_- y^a(V_+\Gamma_a y) - 2\bar{h}^{++} D_+ y^a (V_+\Gamma_a y) - 2\bar{h}^{--} D_- y^a (V_-\Gamma_a y) \\ &- 2V_-^c y^b H_{bcd}(V_+\Gamma^d y) + V_-^a V_+^b y^c (T_{cb}\Gamma_a y). \end{aligned} \tag{3.11}$$

Again we will need at least one D_+ derivative to get a divergent integral. If there is one D_+ and one D_- derivative, then the momentum integral becomes $\int d^d k \frac{k_-}{k^2}$, which is zero by symmetric integration. Thus we only need to calculate the diagram in figure 1a, with D_+ at one vertex and the spacetime background fields at the other vertex. This term is non-zero, however, and we find the resulting contribution to the effective action is, to linear order in h^{ij},

$$\Gamma^{\infty}_{fer} = \frac{2}{\pi\epsilon}\Big(\frac{m}{\mu}\Big)^{\epsilon}\int d^{2+\epsilon}\sigma\,\frac{\bar{h}^{++}}{V^{\oplus}_{-}}\,[2H_{abc}V^{a}_{-}(V_{+}\Gamma^{b}\Gamma^{\oplus}\Gamma^{c}V_{+}) - V^{a}_{-}V^{b}_{+}(V_{+}\Gamma^{c}\Gamma^{\oplus}\Gamma_{a}T_{bc})]. \qquad (3.12)$$

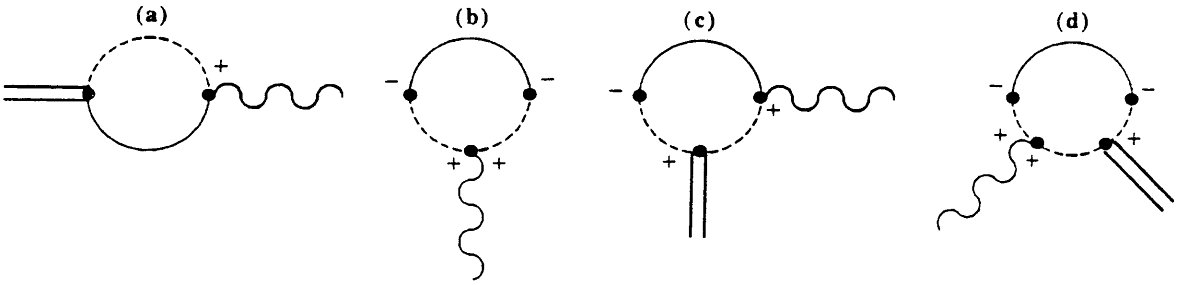

FIG. 1 - Feynman diagrams for the divergent effective action. Wiggly lines represent the metric perturbation $\bar{h}^{ij}$, double lines represent spacetime background fields, + and – represent two-dimensional derivatives D_+ and D_-.

From diagrams with more external legs, counting powers of k_+ and k_- reveals that only the diagrams of figures 1b, 1c, and 1d can contribute. Figure 1d, however, is zero by symmetric integration, and the other two, though they are of the same form as the first term in (3.12), are found to cancel each other, so the complete divergent effective action from this sector is that given in (3.12).

Using the two-dimensional equations of motion (3.3a, b) and the Γ-matrix algebra, we can write (3.12) as

$$\Gamma^{\infty}_{fer} = \frac{1}{\epsilon}\Big(\frac{m}{\mu}\Big)^{\epsilon}\int d^{2+\epsilon}\sigma\Big[v^{M}_{fer}\frac{\delta L}{\delta Z^{M}} + \sqrt{g}(V_{+}\Gamma^{\oplus}\lambda)\Big], \qquad (3.13)$$

where

$$v^{M}_{fer} = \frac{4}{\pi}\frac{\bar{h}^{++}}{V^{\oplus}_{-}}V^{b}_{+}T_{b}{}^{\oplus\alpha}E_{\alpha}{}^{M},$$

and

$$\lambda^{\alpha} = -\frac{2}{\pi}\frac{\bar{h}^{++}}{V^{\oplus}_{-}}V^{a}_{-}[2H_{abc}(\Gamma^{b}\Gamma^{c}V_{+})^{\alpha} + V^{b}_{+}(\Gamma^{c}\Gamma_{a}T_{bc})^{\alpha}].$$

The term involving λ^{α} vanishes when we impose the gauge-fixing condition on the background fields, as discussed above.

Finally, we put the two sectors together and find that the complete divergent effective action to linear order in h_{ij} is

$$\Gamma^{\infty} = \frac{1}{\epsilon}\Big(\frac{m}{\mu}\Big)^{\epsilon}\int d^{2+\epsilon}\sigma\Big[v^{M}\frac{\delta L}{\delta Z^{M}} + \epsilon^{ij}\partial_{i}Z^{M}\partial_{j}Z^{N}\partial_{N}\Lambda_{M}\Big], \qquad (3.14)$$

where

$$v^M = \frac{1}{2\pi}(\nabla^a\Phi)E_a{}^M + \frac{4}{\pi}\frac{\bar{h}^{++}}{V_-^{\oplus}}V_+^b T_b{}^{\oplus\alpha}E_\alpha{}^M \quad , \quad \Lambda_M = -\frac{1}{2\pi}E_M{}^\alpha\nabla_\alpha\Phi. \qquad (3.15)$$

Thus we see that the divergent effective action at one loop can be written as a shift of the supercoordinate plus a gauge transformation on the antisymmetric tensor when the spactime fields satisfy their equations of motion.

The gauge transformation term, in spite of appearances, is *not* a total derivative, as noted by Tseytlin [15]. This is because ϵ^{ij} is not a constant in $(2+\epsilon)$ dimensions. Rather, we have $\epsilon^{ij} \rightarrow \Omega^\epsilon\epsilon^{ij}$. We cannot simply drop this term, but must include it in the β-function for the antisymmetric tensor. As we will see below, this term is crucial for the cancellation of the Weyl anomaly.

4. The Weyl anomaly of the heterotic GS NLSM

We now derive a formula for the Weyl anomaly of the model (2.1), following the bosonic case.

Introduce the generating functional $W[J]$ by

$$e^{-W[J]} \equiv \int [dZ]exp\Big[-S[Z] - \int d^{2+\epsilon}\sigma\sqrt{g}J(Z(\sigma),\sigma) + S_{ghost}\Big], \qquad (4.1)$$

with $J(Z(\sigma),\sigma)$ a (bare) generalized source term (see [14] for details of this procedure). In (2.1) the ghosts have been included implicitly in the measure $[dZ]$. The ghosts do not contribute to the background-dependent part of the anomaly, and will be ignored in the following. For instance, in the semi-light-cone gauge used in section 3, the ghost terms are essentially Lagrange multipliers for the constraints. The calculations we will perform below involve taking derivatives with respect to the renormalization parameter μ. The contributions to these Green functions from the ghosts will be proportional to the constraints and hence will vanish. An alternative derivation of the Weyl anomaly can be given starting from the gauge fixed and normal coordinate expanded action (2.2), following Hull [12]. Here we will follow Shore's approach [14], and take the action (4.2) (see below), before normal coordinate expansion, as our starting point.

We can rewrite the classical action (2.1) (ignoring the heterotic fermions and ghosts) as

$$S = \int d^{2+\epsilon}\sigma\Big\{\nabla_i Z^M\nabla_j Z^N\Big[\frac{1}{2}\gamma^{ij}G_{NM} + \epsilon^{ij}B_{NM}\Big] + \sqrt{g}\Big(\frac{a}{2}\Big)R^{(2)}\Phi(Z)\Big\}, \qquad (4.2)$$

where we have defined $G_{NM} \equiv E_{Na}E_M{}^a$, and we have included a Fradkin-Tseytlin term with a to-be-determined coefficient a. The world-sheet curvature is denoted $R^{(2)}$. Note that G_{NM} is not a "metric for superspace", in particular, it does not have an inverse. It should be pointed out explicitly that, *a priori*, there seems to be no need to identify the Φ field appearing in (4.2) with the dilaton field introduced in (2.4), effectively when solving the constraints (2.3). As it will become clear in the next section, the fields are determined, one in terms of the other, by the choice $a = \frac{1}{2\pi}$ that follows from the requirement of Weyl invariance, at the quantum level.

Consider a Weyl rescaling in $2+\epsilon$ dimensions: $g_{ij} \longrightarrow \Omega^2 g_{ij}$, $\epsilon^{ij} \longrightarrow \Omega^\epsilon \epsilon^{ij}$. Then the Weyl anomaly is

$$\sqrt{g} < T^i_i(\sigma) >= -\Omega(\sigma) \frac{\delta \Gamma}{\delta \Omega(\sigma)}\Big|_{\Omega=1}. \tag{4.3}$$

The anomaly can be related to the renormalization group by the trick of introducing a position-dependent renormalization mass $\mu(\sigma)$ [14]. The renormalized fields (denoted with a superscript (r)) are defined by

$$G_{MN}(Z,\sigma) = \mu^\epsilon(\sigma)\Big[G^{(r)}_{MN}(Z) + \sum_{n=1}^{\infty} \epsilon^{-n}\xi_n(G^{(r)}, B^{(r)}, \Phi^{(r)})\Big], etc. \tag{4.4}$$

With the β-functions (defined for *constant* μ)

$$\mu \frac{d}{d\mu} G^{(r)}_{MN} \equiv \hat{\beta}^G_{MN}, \quad etc., \tag{4.5}$$

we find the renormalization group equation (on-shell, i.e., at $J^{(r)} = 0$)

$$\begin{aligned} -\mu(\sigma)\frac{\delta}{\delta\mu(\sigma)} W[0] = \\ \int dZ \Bigg[\hat{\beta}^G_{MN}(Z,\sigma)\frac{\delta}{\delta G^{(r)}_{MN}(Z,\sigma)} + \hat{\beta}^B_{MN}(Z,\sigma)\frac{\delta}{\delta B^{(r)}_{MN}(Z,\sigma)} \\ + \hat{\beta}^\Phi(Z,\sigma)\frac{\delta}{\delta \Phi^{(r)}(Z,\sigma)}\Bigg] W[0]. \end{aligned} \tag{4.6}$$

The local $\hat{\beta}$-functions are defined by [14]

$$\hat{\beta}^G_{MN}(Z,\sigma) = \hat{\beta}^G_{MN}\big(G^{(r)}(Z,\sigma), B^{(r)}(Z,\sigma), \Phi^{(r)}(Z,\sigma)\big). \tag{4.7}$$

Except for the Fradkin-Tseytlin term, the renormalization mass enters the action in the same way as the Weyl scale factor. Thus the anomaly can be written

$$\sqrt{g} < T^i_i(\sigma) >= -\mu(\sigma)\frac{\delta}{\delta\mu(\sigma)} W[0] + a\sqrt{g} < \Delta\Phi >, \tag{4.8}$$

where $\Delta\Phi \equiv (\frac{1}{\sqrt{g}})\partial_i(\sqrt{g}g^{ij}\partial_j\Phi)$. The renormalization group equation gives the first term on the RHS. The second term can be rewritten using

$$\sqrt{g}\Delta\Phi = \gamma^{ij}V_i^a V_j^B \nabla_B \nabla_a \Phi + \epsilon^{ij} V_i^D V_j^B H^a{}_{BD} \nabla_a \Phi$$
$$+ \frac{a}{2}\sqrt{g}R^{(2)}(\nabla^a\Phi\nabla_a\Phi) - \frac{\delta S}{\delta Z^M} E_M{}^a \nabla_a \Phi$$
$$+ \sqrt{g}D^i(V_i^\alpha \nabla_\alpha \Phi). \tag{4.9}$$

Taking the vacuum expectation value of this expression, and using the fact that the vacuum expectation value of an arbitrary composite operator is given by

$$< F(Z) >_{J=0} = \int dZ\ F(Z) \frac{\delta W[J]}{\delta J(Z,\sigma)}\Big|_{J=0}, \tag{4.10}$$

we get

$$\sqrt{g}\ < \Delta\Phi >=$$
$$\int dZ \Big[\ 2E_N{}^B \nabla_B \nabla_a \Phi E_M{}^a \frac{\delta W[0]}{\delta G_{NM}(Z,\sigma)} + H^a{}_{NM}\nabla_a\Phi \frac{\delta W[0]}{\delta B_{NM}(Z,\sigma)}$$
$$+ \nabla^a\Phi\nabla_a\Phi \frac{\delta W[0]}{\delta\Phi(Z,\sigma)} + \sqrt{g}D^i(V_i^\alpha\nabla_\alpha\Phi)\frac{\delta W[0]}{\delta J(Z,\sigma)}\Big]. \tag{4.11}$$

Combining (4.6), (4.8), and (4.11), we find that the Weyl anomaly is given by

$$\sqrt{g} < T^i{}_i(\sigma) >= (f.p.)\int dZ\ \Big[(\hat{\beta}^G_{NM} + 2aE^{(r)}{}_N{}^B \nabla^{(r)}{}_B \nabla^{(r)}{}_a \Phi^{(r)} E_M{}^a)\frac{\delta W[0]}{\delta G^{(r)}_{NM}(Z,\sigma)}$$
$$+ (\hat{\beta}^B_{NM} + aH^{(r)a}{}_{NM}\nabla^{(r)}_a\Phi^{(r)})\frac{\delta W[0]}{\delta B^{(r)}_{NM}(Z,\sigma)}$$
$$+ (\hat{\beta}^\Phi + a\nabla^{(r)a}\Phi^{(r)}\nabla^{(r)}{}_a\Phi^{(r)})\frac{\delta W[0]}{\delta\Phi^{(r)}(Z,\sigma)}$$
$$+ aD^i(V_i^{(r)\alpha}\nabla^{(r)}{}_\alpha\Phi^{(r)})\frac{\delta W[0]}{\delta J^{(r)}(Z,\sigma)}\Big]. \tag{4.12}$$

Here the notation $(f.p.)$ denotes the finite part, which is necessary when rewriting the bare quantities in terms of renormalized quantities, since the Green functions obtained by taking variations of $W[J]$ with respect to the position-dependent coupling functions $G^{(r)}_{NM}(Z,\sigma)$, etc., are not guaranteed to be finite. The energy-momentum tensor, on the other hand, is guaranteed to be finite, so these poles must cancel against the poles introduced by rewriting everything in terms of the renormalized quantities [14].

We can replace the $\hat{\beta}$-functions in (4.12) by the actual β-functions

$$\beta^G_{MN} = \epsilon G^{(r)}_{MN} + \hat{\beta}^G_{MN} \tag{4.13}$$

as long as the contribution

$$\int dZ\Big(G^{(r)}_{MN}\frac{\delta}{\delta G^{(r)}_{MN}(Z,\sigma)} + B^{(r)}_{MN}\frac{\delta}{\delta B^{(r)}_{MN}(Z,\sigma)} + \Phi^{(r)}\frac{\delta}{\delta\Phi^{(r)}(Z,\sigma)}\Big)W[0] \tag{4.14}$$

is finite *off shell* [14]. In our case this means we cannot drop terms in Γ^∞ proportional to the field equations, but must perform our subtractions such that all divergences are absorbed by the counterterms for G_{MN}, B_{MN}, and Φ. This is the procedure we will follow. The alternative is to use a formalism which allows for renormalization of the supercoordinate Z^M, which involves some subtleties which we wish to avoid [13-15]. In the next section we derive a Ward identity which allows us to show that as long as the divergent effective action takes the form (3.14) it is possible to make the expression (4.14) finite.

The main difference between (4.12) and the corresponding expression in the bosonic case is the last term. Before discussing the fate of this term, we will discuss how the expression (4.12) changes under a diffeomorphism in superspace.

5. A Ward Identity for Diffeomorphisms

It is important to understand the possible sources of ambiguity in the expression (4.12) for the Weyl anomaly. Starting from the classical GS action (4.2) and performing the change of variables $Z^M \to Z^M + \zeta^M$, one finds the identity

$$\zeta^M \frac{\delta S}{\delta Z^M} = \gamma^{ij} V_i^a V_j^D (\nabla_D \zeta_a + \zeta^B T_{BD}{}^a) + \epsilon^{ij} V_i^D V_j^B \zeta^A H_{ABD} + \frac{a}{2}\sqrt{g} R^{(2)} \zeta^A \nabla_A \Phi - \sqrt{g} D^i (V_i^a \zeta_a). \tag{5.1}$$

Performing the same change of variables in the expression (4.1) for $W[J]$ and evaluating at $J = 0$, one obtains the Ward identity

$$(f.p.) \int dZ \Big[2E_N^{(r)D} (\nabla^{(r)}{}_D \zeta^{(r)}{}_a + \zeta^{(r)B} T^{(r)}{}_{BDa}) E^{(r)}{}_M{}^a \frac{\delta W[0]}{\delta G^{(r)}{}_{NM}(Z,\sigma)} + \zeta^P H^{(r)}{}_{PNM} \frac{\delta W[0]}{\delta B^{(r)}{}_{NM}(Z,\sigma)} + \zeta^{(r)A} \nabla^{(r)}{}_A \Phi^{(r)} \frac{\delta W[0]}{\delta \Phi^{(r)}(Z,\sigma)} - D^i (V^{(r)}{}_i{}^a \zeta_a^{(r)}) \frac{\delta W[0]}{\delta J^{(r)}(Z,\sigma)} \Big] = 0. \tag{5.2}$$

This identity can be used in (4.12) to give

$$\begin{aligned} &\sqrt{g} < T_i^i(\sigma) >= \\ &(f.p.) \int dZ \Big\{ [\hat\beta^G_{NM} + 2E_N^{(r)D} (\nabla_D \zeta_a + \zeta^B T_{BDa} + a \nabla_D \nabla_a \Phi)^{(r)} E_M{}^{(r)a}] \frac{\delta W[0]}{\delta G^{(r)}_{NM}(Z,\sigma)} \\ &+ [\hat\beta^B_{NM} + (\zeta^A + a\delta^A_a \nabla^a \Phi) H_{ANM}]^{(r)} \frac{\delta W[0]}{\delta B^{(r)}_{NM}(Z,\sigma)} \\ &+ [\hat\beta^\Phi + \zeta^A \nabla_A \Phi + a \nabla^a \Phi \nabla_a \Phi]^{(r)} \frac{\delta W[0]}{\delta \Phi^{(r)}(Z,\sigma)} \\ &- D^i (V_i^a \zeta_a - a V_i^\alpha \nabla_\alpha \Phi)^{(r)} \frac{\delta W[0]}{\delta J^{(r)}(Z,\sigma)} \Big\}. \end{aligned} \tag{5.3}$$

We have used an obvious notation to denote that all of the quantities appearing within a set of brackets are renormalized.

We now apply this expression to the results of section 3. To one-loop order, $W[0]$ in (5.3) can be replaced by the classical action, and the $\frac{\delta W}{\delta \Phi}$ term can be ignored, as it is of order $(\alpha')^2$. Also, $\frac{\delta W}{\delta J^{(r)}}$ becomes $\sqrt{g}\delta(Z'-Z)$. The β-functions are identified from the divergent effective action by subtracting the pole and taking $\mu\frac{\delta}{\delta\mu}$. The supercoordinate transformation is rewritten using (5.1), integrated over the world-sheet, to identify the coefficients of $\frac{\delta S}{\delta G_{MN}}$, etc. Note that the diffeomorphism current term does not appear, because the effective action is under an integral. This gives us the identity which proves that a divergent effective action of the form (3.14) can always be absorbed by a renormalization of the coupling functions without a renormalization of the supercoordinate Z^M. The result is

$$\begin{aligned}\sqrt{g} < T^i{}_i(\sigma) >_{1-loop} = {} & 2\gamma^{ij} V_i^{(r)^a} V_j^{(r)^B} [\nabla_B(\zeta_a - v_a + a\nabla_a\Phi) + (\zeta^D - v^D)T_{DBa}]^{(r)} \\ & + \epsilon^{ij} V^{(r)}{}_i{}^D V^{(r)}{}_j{}^B (\zeta^A - v^A + a\delta_a{}^A\nabla^a\Phi)^{(r)} H^{(r)}{}_{ABD} \\ & + \frac{1}{2\pi}\epsilon^{ij} D_i(V_j{}^\alpha\nabla_\alpha\Phi)^{(r)} \\ & - \sqrt{g} D^i(V_i{}^a\zeta_a - aV_j{}^\alpha\nabla_\alpha\Phi)^{(r)}. \end{aligned} \tag{5.4}$$

The spinor terms, involving v^α, which came from the Feynman diagrams with fermion propagators, give a contribution proportional to the spinor equation of motion (3.3*b*). Note that there is no diffeomorphism current for the spinors because the derivation of the spinor equation of motion does not involve an integration by parts.

Using the superspace constraints (2.3) and the two-dimensional equations of motion (3.3*a*, *b*), the dependence on the diffeomorphism parameter ζ^A in (5.4) is explicitly seen to cancel, as it must, because of the Ward identity (5.2). Our final expression for the Weyl anomaly is then

$$\sqrt{g} < T_i^i(\sigma) >_{1-loop} = \left(a\gamma^{ij} + \frac{1}{2\pi}\epsilon^{ij}\right) D_i(V_j^\alpha\nabla_\alpha\Phi)^{(r)} + \left(a - \frac{1}{2\pi}\right) D_i(V_j^a\nabla_a\Phi)^{(r)}. \tag{5.5}$$

It is clear that without the Fradkin-Tseytlin term ($a=0$), there is a non-vanishing Weyl anomaly. However, if we take $a = \frac{1}{2\pi}$ then we are left with

$$\sqrt{g} < T_i^i(\sigma) >_{1-loop} = \frac{1}{2\pi}(\gamma^{ij} + \epsilon^{ij}) D_i(V_j^\alpha\nabla_\alpha\Phi)^{(r)} = 0. \tag{5.6}$$

This follows from the equation of motion (3.3*b*) and the gauge condition (2.8) imposed on the background fields. It is crucial that the gauge transformation on B_{MN} and the contribution from the Fradkin-Tseytlin term combine to give $(\gamma^{ij}+\epsilon^{ij})$, which is exactly the combination appearing in the spinor equation of motion.

In concluding our treatment of the potential ambiguities in the anomaly calculation, two remarks are in order. If we set the Fradkin-Tseytlin term to zero ($a=0$), it is possible to shift the anomaly from one sector (a Weyl anomaly) to another (a diffeomorphism current anomaly). In fact, in the $a=0$ case the first two terms in

(5.4) can be eliminated by taking $\zeta^a = v^a$, but this will leave the diffeomorphism current as an anomaly. As a final comment, we note that Shore's approach [14] gives us the correct coefficient of the notoriously tricky ϵ^{ij} term for anomaly cancellation. As was mentioned in section 3, the ϵ^{ij} term is not a surface term, in $(2+\epsilon)$ dimensions. In the treatment we adopted here, this term appears exactly with the appropriate coefficient to ensure the vanishing of the Weyl anomaly.

6. Yang-Mills Backgrounds

The effect of including Yang-Mills backgrounds [22] on the Green-Schwarz beta-functions has been considered by Grisaru et al. [23]. The action for the heterotic fermions is

$$S_{het} = \frac{1}{2}\int d^{2+\epsilon}\sigma\sqrt{g}e_-{}^i\psi^s(\delta^{st}D_i - V_i^A A_A^{st})\psi^t. \tag{6.1}$$

Grisaru, et. al. have shown that the beta functions are absorbed by a superdiffeomorphism with

$$\epsilon^a = \frac{1}{2\pi\epsilon}\nabla^a\Phi + \frac{1}{16(4\pi)^2\epsilon}\frac{V_-^c}{V_-^{\oplus}}(\chi^s\Gamma_{c\ominus}{}^a\chi^s)\,, \tag{6.2a}$$

$$\epsilon^\alpha = -\frac{1}{8(4\pi)^2\epsilon}\chi^{s\alpha}(\nabla_\beta\chi^{s\beta}) \tag{6.2b}$$

(here $\chi^{s\beta}$ is the gaugino), together with a gauge transformation on the gauge potential A_A^{st}. Thus the gauge potential renormalizes in the same way as the other coupling functions, as in (4.4). Our expression (4.8) for the Weyl anomaly becomes

$$\sqrt{g} < T_i^i(\sigma) > = -\mu(\sigma)\frac{\delta}{\delta\mu(\sigma)}W[0] + a\sqrt{g} < \Delta\Phi > -\frac{1}{2}\sqrt{g} < \psi^s e_-{}^i D_i\psi^s > . \tag{6.3}$$

The expression (4.12) for the Weyl anomaly gets an extra contribution

$$(f.p.)\int dZ\ (\hat{\beta}_M^A + \nabla^a\Phi F_{Ma})^{(r)}\frac{\delta W[0]}{\delta A_M^{(r)}}\ . \tag{4.12'}$$

The Φ term is cancelled by the contribution to $\hat{\beta}^A$ coming from the first term on the r.h.s of (6.2a) in the same way that happens for the analogous Φ terms in (4.12). The gauge transformation on A_A^{st} gives no contribution to the Weyl anomaly because of the Yang-Mills Ward identity. (As noted in [23], the anomaly induced by the rotation of the chiral fermions in the Yang-Mills gauge transformation $\psi \to \Lambda\psi$, $\delta A_i^{st} = D_i\Lambda^{st}$ is cancelled by assigning a variation to the antisymmetric tensor field: $\delta B_{MN} = -\frac{1}{16\pi}\Lambda^{st}\partial_{[M}A_{N)}^{st}$. This cancellation is plagued by the ambiguity arising from the definition of the ϵ^{ij} tensor in $2+\epsilon$ dimensions.) The contribution from the fermionic part of the superdiffeomorphism is also zero, as discussed above. This leaves the bosonic part of the superdiffeomorphism and the fermion bilinear $< \psi D_-\psi >$ to be considered. From the results of the previous sections, we know that the bosonic part of the superdiffeomorphism will contribute to the Weyl anomaly through the diffeomorphism current. The first term in the expression (6.2a) for ϵ^a will be cancelled by the Fradkin-Tseytlin term using the gauge condition on the background, as discussed in section 5. The second term on the r.h.s. of (6.2a) is a new contribution, arising from the two-loop diagram in fig. 2 [23], which contributes to the Weyl anomaly as in (5.3):

$$\sqrt{g} < T_i^i > = -\frac{1}{16(4\pi)^2}\gamma^{ij} D_i[\frac{V_j^a V_-^c}{V_-^{\oplus}}(\chi^s \Gamma_{c\ominus a}\chi^s)]. \tag{6.4}$$

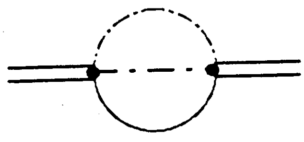

FIG. 2 - Feynman diagram that gives rise to the contribution (6.4). Dash-dot lines represent the heterotic fermions Ψ.

Due to the chiral rotations performed in [23] in order to write the beta-functions in terms of the parameters in (6.3), the fermion bilinear $< \psi D_- \psi >$ has the possibility to contribute to the Weyl anomaly. This could cancel the unwanted anomaly in (6.4). In fact, the regularization ambiguities that arise allow for another solution to the problem of Weyl anomaly cancellation.

This can be seen by considering the contribution to the effective action from the Feynman diagram in fig. 2 which, in the conformal gauge, reads

$$\frac{1}{16(4\pi)^2\epsilon}\frac{V_-^c}{(V_-^{\oplus})^{\frac{1}{2}}}\left[\chi^s\Gamma_c\Gamma_\ominus\Gamma_a\partial_+\left(\frac{V_-^a}{(V_-^{\oplus})^{\frac{1}{2}}}\chi^s\right)\right]. \tag{6.5}$$

There is an ambiguity, however, in how one writes this term in an "invariant" manner. If one picks a pair of world-sheet indices and covariantizes them via e.g. $\partial_+ V_-^a \to (\gamma^{ij} - \epsilon^{ij})\partial_i V_j^a$, then the ambiguity resides in the option of writing the same term as $\partial_+ V_-^a \to \partial_i(\gamma^{ij} - \epsilon^{ij})V_j^a$. The difference between these expressions comes about because the ϵ^{ij} tensor is not a constant in $2+\epsilon$ dimensions, as discussed above. This difference is proportional to

$$\epsilon^{ij} D_i[\frac{V_j^a V_-^c}{V_-^{\oplus}}(\chi^s \Gamma_{c\ominus}{}^a\chi^s)]. \tag{6.6}$$

Using this ambiguity, we can write the contribution to the Weyl anomaly from the Yang-Mills terms in (6.2) by combining (6.4) and (6.6) with appropriate coefficients to give

$$-\frac{1}{16(4\pi)^2\epsilon} D_+[\frac{V_-^a V_-^c}{V_-^{\oplus}}(\chi^s \Gamma_{c\ominus}{}^a\chi^s)] \; = \; 0. \tag{6.7}$$

We suspect that a more careful regularization procedure for the chiral fermion action (6.1) would give the correct coefficient for this cancellation to take place.

7. Discussion

We have derived a general expression for the Weyl anomaly of the GS sigma model in a curved superspace which satisfies its equations of motion. By explicit calculation, we found the one-loop beta functions to first order in the perturbation of the two-dimensional metric, and found that the resulting Weyl anomaly vanished when the semi-light-cone gauge conditions were applied to the background fields, and for a non-zero coefficient of the Fradkin-Tseytlin term. This provides the demonstration of the vanishing of the Weyl anomaly for a string in non-zero fermionic backgrounds. The coupling to Yang-Mills backgrounds does not seem to spoil this cancellation, although in this case an ambiguity in the treatment of the ϵ^{ij} tensor in dimensional regularization plays a role in the cancellation of the Weyl anomaly.

The non-vanishing of the Fradkin-Tseytlin term is a puzzle. This term is not κ-invariant. Therefore our introduction of it to cancel the Weyl anomaly introduces a potential anomaly in the κ-symmetry. In the absence of a covariant quantization scheme for the Green-Schwarz sigma model, it is difficult to check κ-invariance at the quantum level. It is possible, of course, that the κ-variation of the Fradkin-Tseytlin term is precisely what is needed to cancel a κ-anomaly coming from the rest of the action, in the same fashion as occurs for the Weyl anomaly.

The flat-superspace calculations of [16] demonstrate a strange boundary effect: the Weyl anomaly is shown to cancel only up to boundary terms on the world-sheet. As noted there, it is not always possible to ignore boundary terms on the world-sheet. It would be interesting to see how the coupling to superspace backgrounds affects these boundary terms.

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REFERENCES

[1] D. Friedan, Phys. Rev. Lett. 45 (1980) 1057; Ann. of Phys. 163 (1985) 318; L. Alvarez-Gaumé, D.Z. Freedman and S. Mukhi, Ann. Phys. 134 (1981) 85; C.G. Callan, D. Friedan, E.J. Martinec and M.J. Perry, Nucl. Phys. B262 (1985) 593.

[2] E.S. Fradkin and A.A. Tseytlin, Phys. Lett. 158B (1985) 316; Nucl. Phys. B261 (1985) 1.

[3] C.M. Hull and P.K. Townsend, Nucl. Phys. B301 (1988) 197; and references therein.

[4] M.T. Grisaru, A. van de Ven and D. Zanon, Nucl. Phys. B277 (1986) 388, 409.

[5] S. Bellucci, D.A. Depireux and S.J. Gates, Jr., Phys. Lett. 238B (1990) 315; L. Bonora, M. Bregola, K. Lechner, P. Pasti, M. Tonin and Nucl. Phys. B296 (1988) 877; and references therein.

[6] A. Sen, Phys. Rev. D32 (1985) 2162; Phys. Rev. Lett. 55 (1985) 1846.

[7] S. Carlip, Nucl. Phys. B284 (1987) 365; R.E. Kallosh, Phys. Lett. B195 (1987) 369.

[8] M.T. Grisaru, H. Nishino and D. Zanon, Phys. Lett. 206B (1988) 625; M.T. Grisaru and D. Zanon, Nucl. Phys. B310 (1989) 57.

[9] G. Gilbert and D. Johnston, Phys. Lett. 205B (1988) 273.

[10] R.E. Kallosh, Phys. Lett. 195B (1987) 369; ibid. 224B (1989) 273; A.H. Diaz and F. Toppan, Phys. Lett. 211B (1988) 285; S.J. Gates, Jr., M.T. Grisaru, U. Lindström, M. Roček, W. Siegel, P. van Nieuwenhuizen and A.E. van de Ven, Phys. Lett. 225B (1989) 44; M. Green and C.M. Hull, Phys. Lett. 225B (1989) 57; R.E. Kallosh and E. Bergshoeff, Nucl. Phys. B333 (1990) 605.

[11] J.M.L. Fisch and M. Henneaux, Bruxelles preprint ULB TH2/89-04; F. Bastianelli, G. Delius and E. Laenen, Phys. Lett. B229 (1989) 223.

[12] C.M. Hull and P.K. Townsend, Nucl. Phys. B274 (1986) 349; C.M. Hull, lecture notes (unpublished, 1987).

[13] A.A. Tseytlin, Phys. Lett. 178B (1986) 34.

[14] G.M. Shore, Nucl. Phys. B286 (1987) 349.

[15] A.A. Tseytlin, Nucl. Phys. B294 (1987) 383.

[16] U. Kraemmer and A. Rebhan, Phys. Lett. 236B (1990) 255; F. Bastianelli, P. van Nieuwenhuizen and A. Van Proeyen, "Superstring Anomalies in the Semi-Light Cone Gauge", preprint ITP-SB-90-47.

[17] J.J. Atick and A. Dhar, Nucl. Phys. B284 (1987) 131.

[18] L.F. Abbott, Nucl. Phys. B185 (1981) 189 and references therein.

[19] S.J. Gates, Jr., P. Majumdar, B. Radak and S. Vashakidze, Phys. Lett. 226B (1989) 237.

[20] S. Bellucci, Phys. Lett. B227 (1989) 61; P. Majumdar, R.N. Oerter and A. E. van de Ven, Phys. Lett. 233B (1989) 123.

[21] S. Bellucci, "Effective Action Approach to the Green-Schwarz String", INFN-Frascati preprint LNF-90/056(PT), to appear in the proceedings of the "Fourth Regional Conference on Mathematical Physics", Sharif University of Technology, Tehran, 12-17 May 1990; ibid. Mod. Phys. Lett. A5 (1990) 2253.

[22] E. Bergshoeff, B. de Wit, M. de Roo and P. van Nieuwenhuizen, Nucl. Phys. B195 (1982) 97; G.F. Chapline and N.S. Manton, Phys. Lett. 120B (1983) 105; J.J. Atick, A. Dhar and B. Ratra, Phys. Rev. 33D (1986) 2824.

[23] M.T. Grisaru, H. Nishino and D. Zanon, Nucl. Phys. B314 (1989) 363.