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AROUND THE Z_0.

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ABSTRACT.

Analitycal expressions are presented for e.m. radiative effects in the reaction $e^-e^+ \longrightarrow \mu^-\mu^+$, in the vicinity of the Z_o mass, in the Weinberg-Salam model. Our formulae contain all finite first order corrections, as well as soft photon effects resummed to all orders, with no restriction on the relative magnitudes of the energy resolution $\Delta\omega$ and the Z_o-width Γ. Hard photon effects, other than the tail effect, are not included. Weak interaction are considered only to lowest order.

Radiative corrections are explicitly shown to change drastically the lowest order results, and must therefore be taken in full account for a realistic description of experiments at LEP energies.

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1. INTRODUCTION

High energy e^-e^+ colliding beams provide a unique opportunity[1] for testing the properties of weak neutral currents, particularly for energies near the intermediate boson mass. Interference with electromagnetic process gives characteristic effects which are extremely sensitive to the weak vector and axial couplings.

Experimental investigation of these effects will provide however a sensitive test of the theory[2] only when radiative corrections have been fully taken into account. More precisely, we can distinguish two types of radiative corrections, namely those associated with higher order pure weak effects and the more standard ones due to electromatic effects.

The radiative corrections of the first kind, which are finite by virtue of the renormalizability[3] of the theory, themselves play an important role as a test of the theory. They have been recently calculated by Passarino and Veltman[4] for $e^-e^+ \rightarrow \mu^- \mu^+$ and by Consoli[5] for $e^-e^+ \rightarrow e^-e^+$.

On the other hand, pure electromagnetic radiative corrections, which add no additional theoretical information, are expected to be rather important, especially in the vicinity of the Z_0-boson, and could change sizably the naive expectations. In particular, one must evaluate to all orders in α the corrections due to soft photon emission, which become increasingly important as the energy increases. Usually, i.e. for non-resonant cross-sections, this leads to the exponentiated form $(\Delta\omega/E)^{(4\alpha/\pi)\ln(2E/m)}$, where $\Delta\omega$ is the resolution and 2E the c.m. energy of the experiment. In the case of resonance production, the above factor is modified and for a very narrow resonance (like J/Ψ) the correction[6,7] becomes $\mathcal{L}(\Gamma/M)^{(4\alpha/\pi)\ln(2E/m)}$. Physically this is understood by saying that the width Γ provides a natural cut-off in damping the energy loss in the initial state. For the case of the Z_0-boson, where neither of the preceding case applies ($\Gamma/M \sim \Delta\omega/E$), the soft correction would be a complicated function of E, M, $\Delta\omega$ and Γ raised to the power $\frac{4\alpha}{\pi}\ln\frac{2E}{m}$, of the order of 50%.

Another important electromagnetic effect is the presence of a substantial radiative tail. Due to the emission of a hard photon from the initial state, the radiative tail is expected to radically modify the angular asymmetries in $e^+e^- \rightarrow \mu^- \mu^+$. It follows therefore that a detailed calculation of the e.m. radiative corrections is of primary importance for the forthcoming experiments around the Z_0-mass.

In this paper we present a detailed study of the above electromagnetic effects in the reaction $e^+e^- \rightarrow Z_0 \rightarrow \mu^+\mu^-$ and give simple analytical prescriptions of immediate experimental application. A brief account of our results has already been given[8].

We consider the infrared factors to all orders in perturbation theory. This is achieved by using the coherent state formalism developed previously in ref. (7) for a purely resonant cross-section as well as for the interference with a QED background. Simple expressions for the infrared factors are thus obtained, which depend on E, M, Γ and $\Delta\omega$. Finite corrections of order α have also been computed, for the case of unpolarized as well as transversely polarized $e^{\pm}$ beams. The weak effects are considered only to lowest order, the weak boson Z_0 being taken into effect as a resonance of mass M and width Γ in the s-channel. Hard photon effects, other than the tail effect, are not included.

In sect. 2 we define our notation and collect the relevant formulae. The infrared corrections are discussed in detail in sect. 3, while a careful analysis of diagrams contributing to first order in α is presented in sect. 4. A discussion of our results as well as some numerical applications are finally presented in sect. 5.

2. NOTATION AND FORMULAE.

For the reaction:

$$e^-(p_1) + e^+(p_2) \longrightarrow \mu^-(p_3) + \mu^+(p_4)$$

we have

(2.1a) $$s = W^2 = 4E^2 = (p_1+p_2)^2,$$

(2.1b) $$z = \cos\theta = \hat{p}_1\cdot\hat{p}_3 = \hat{p}_2\cdot\hat{p}_4,$$

(2.1.c) $$a = \sin\theta/2\ , \qquad b = \cos\theta/2\ ,$$

(2.2a) $$\beta_e = \frac{4\alpha}{\pi}\left[\ln\frac{W}{m_e} - \frac{1}{2}\right],$$

(2.2b) $$\beta_\mu = \frac{4\alpha}{\pi}\left[\ln\frac{W}{m_\mu} - \frac{1}{2}\right],$$

(2.2c) $$\beta_{int} = \frac{4\alpha}{\pi}\ln\left(\tan\frac{\theta}{2}\right),$$

and $$\Delta \equiv \frac{\Delta\omega}{E} = \text{fractional energy resolution.}$$

The weak boson Z_0 is taken to be a resonance of mass M and total width Γ, such that the phase shift δ_R is

(2.3) $$\tan\delta_R(s) = \frac{M\Gamma}{M^2-s}\ .$$

In the standard Weinberg-Salam model(2), with only one doublet of Higgs fields one has

(2.4) $$M = \left(\frac{\pi\alpha}{\sqrt{2}\,G_F}\right)^{\frac{1}{2}} \frac{1}{\sin\theta_W \cos\theta_W},$$

and

(2.5) $$\Gamma \equiv \Gamma(Z_0 \to \text{all}) \simeq \frac{G_F M^3}{24\sqrt{2}\,\pi} \left\{ \left[1+(1-4\sin^2\theta_W)^2\right] N_l + 2N_\nu + 3\left[1+(1-\tfrac{8}{3}\sin^2\theta_W)^2\right] N_u + 3\left[1+(1-\tfrac{4}{3}\sin^2\theta_W)^2\right] N_d \right\},$$

where N_l, N_ν, N_u and N_d are the numbers of charged leptons, neutrinos, charge (2/3) and (-1/3) quarks respectively, with masses appreciably smaller than $M/2$. Using $\sin^2\theta_W \simeq 1/4$ and $N_l=N_\nu=N_u=N_d=3$ eqs. (2.4-5) give $M \simeq 86.4$ GeV, $\Gamma \simeq 2.2$ GeV and $\Gamma_e \equiv \Gamma(Z_0 \to e\bar{e}) = \Gamma/32$.

We also define

(2.6a) $$r_V = \frac{g_V^2}{g_V^2+g_A^2} = \frac{(1-4\sin^2\theta_W)^2}{1+(1-\sin^2\theta_W)^2},$$

(2.6b) $$r_A = \frac{g_A^2}{g_V^2+g_A^2} = \frac{1}{1+(1-4\sin^2\theta_W)^2},$$

(2.6c) $$r_{AV} = \frac{g_A\, g_V}{g_V^2+g_A^2} = \frac{4\sin^2\theta_W - 1}{1+(1-4\sin^2\theta_W)^2},$$

with $g_V = (4\sin^2\theta_W - 1)/2\sin(2\theta_W)$, $g_A = 1/2\sin(2\theta_W)$ and

(2.7a) $$\Gamma_e = \frac{\alpha M}{3}(g_V^2 + g_A^2),$$

(2.7b) $$\chi(s) = \left(\frac{3\Gamma_e}{\alpha M}\right)\left(\frac{s}{s-M^2+iM\Gamma}\right).$$

The differential cross section can be written as follows:

(2.8) $$\left(\frac{d\sigma}{d\Omega}\right)^{corr} = C_{infra}^{res}\left(\frac{d\sigma_{res}}{d\Omega}\right)(1+C_F^{res}) + C_{infra}^{int,V}\left(\frac{d\sigma_{int,V}}{d\Omega}\right) \cdot (1+C_F^{int,V}) + C_{infra}^{int,A}\left(\frac{d\sigma_{int,A}}{d\Omega}\right)(1+C_F^{int,A}) + C_{infra}^{int,VA}\left(\frac{d\sigma_{int,VA}}{d\Omega}\right)(1+C_F^{int,VA}) + C_{infra}^{QED}\left(\frac{d\sigma_{QED}}{d\Omega}\right)(1+C_F^{QED}),$$

where the Born cross sections are given by[9]

(2.9a) $$\left(\frac{d\sigma_{QED}}{d\Omega}\right) = \frac{\alpha^2}{4s}\,(1+z^2-(P(\Theta,\emptyset)),$$

(2.9b) $$\left(\frac{d\sigma_{int,V}}{d\Omega}\right) = \frac{\alpha^2}{4s}\,(1+z^2-P(\Theta,\emptyset))\,.(2\mathrm{Re}\,\chi)r_V\,,$$

(2.9c) $$\left(\frac{d\sigma_{int,A}}{d\Omega}\right) = \frac{\alpha^2}{4s}\,(2\,z)\,(2\,\mathrm{Re}\,\chi)\,r_A,$$

(2.9d) $$\left(\frac{d\sigma_{int,VA}}{d\Omega}\right) = \frac{\alpha^2}{4s}\,Q(\Theta,\emptyset)(2\,\mathrm{Im}\chi)\,r_{VA}\,,$$

(2.9e) $$\left(\frac{d\sigma_{res}}{d\Omega}\right) = \frac{\alpha^2}{4s}\left\{1+z^2+8z\,r_A r_V - P(\Theta,\emptyset)\,.\,(r_V^2-r_A^2)\right\}|\chi|^2\,,$$

with $P(\Theta,\emptyset) \equiv /\,\xi_+\,\xi_-/\,\sin^2\Theta\cos\emptyset$ and $Q(\Theta,\emptyset) \equiv /\,\xi_+\,\xi_-/\,\sin^2\Theta\sin 2\emptyset$.

In the above $\xi_{(\pm)}$ is the polarization of the $e^{(\pm)}$ beam along the direction of the magnetic field and $(\Theta,\emptyset)$ refer to the μ^- with respect to the e^-.

The $C_{infra}^{res,\ int,QED}$ are the infrared factors associated with each of the respective cross sections, while $C_F^{(i)}$ (i = res, int(V, A, VA), QED) incorporate the rest of the finite corrections. The C_{infra} factors are obtained using the techniques developed in ref. (7) for narrow resonances, and are discussed in detail in section 3. They take the form

(2.10a) $$C_{infra}^{QED} = \left(\frac{\Delta\omega}{E}\right)^{\beta_e+\beta_\mu+2\beta_{int}},$$

(2.10b) $$C_{infra}^{int,(i)} = \left(\frac{\Delta\omega}{E}\right)^{\beta_\mu+\beta_{int}}\cdot\frac{1}{\cos\delta_R}\,.$$
$$.\,\mathrm{Re}\left\{e^{i\delta_R}\left(\frac{\Delta}{1+\Delta\left(\frac{s}{M\Gamma}\right)e^{i\delta_R}\sin\delta_R}\right)^{\beta_e}\left(\frac{\Delta}{\Delta+\left(\frac{M\Gamma}{s}\right)\frac{e^{-i\delta_R}}{\sin\delta_R}}\right)^{\beta_{int}}\right\},$$

(i=V,A,VA) ,

(2.10c) $$C_{infra}^{res} = \left(\frac{\Delta\omega}{E}\right)^{\beta_\mu}\left|\frac{\Delta}{1+\Delta\left(\frac{s}{M\Gamma}\right)e^{i\delta_R}\sin\delta_R}\right|^{\beta_e}\,.$$
$$\left|\frac{\Delta}{\Delta+\left(\frac{M\Gamma}{s}\right)\frac{e^{-i\delta_R}}{\sin\delta_R}}\right|^{2\beta_{int}}.\left[1-\beta_e\delta(s,\Delta\omega)\cot\delta_R\right].$$

with $\delta\,(s,\Delta\omega)$ given by eq (3.8).

Finally the $C_F^{(i)}$ factors are:

(2.11a) $$C_F^{QED} = \frac{13}{12}\,(\beta_e+\beta_\mu) + \frac{2\alpha}{\pi}\left(\frac{\pi^2}{3}-\frac{17}{18}\right) + \delta_{VP} + X_V\,(\Theta,\emptyset)\,,$$

(2.11b) $$C_F^{int,V} = \frac{11}{12}(\beta_e+\beta_\mu) + \frac{2\alpha}{\pi}\left(\frac{\pi^2}{3} - \frac{13}{18}\right) + \frac{1}{2}\delta_{VP} + \frac{1}{2}X_V(\theta,\phi),$$

(2.11c) $$C_F^{int,A} = \frac{11}{12}(\beta_e+\beta_\mu) + \frac{2\alpha}{\pi}\left(\frac{\pi^2}{3} - \frac{13}{18}\right) + \frac{1}{2}\delta_{VP} + \frac{1}{2}X_A(\theta,\phi),$$

(2.11d) $$C_F^{int,VA} = \frac{11}{12}(\beta_e+\beta_\mu) + \frac{2\alpha}{\pi}\left(\frac{\pi^2}{3} - \frac{13}{18}\right) + \frac{1}{2}\delta_{VP} + \frac{1}{2}X_{AV}(\theta,\phi),$$

(2.11e) $$C_F^{res} = \frac{3}{4}(\beta_e+\beta_\mu) + \frac{2\alpha}{\pi}\left(\frac{\pi^2}{3} - \frac{1}{2}\right) - \frac{2\alpha}{3}\left(\frac{\alpha M^2\Gamma}{\Gamma_e s}\right) \cdot \frac{r_V Y_V(\theta,\phi) + r_A Y_A(\theta,\phi)}{1+z^2 + 8z\, r_V r_A - (r_V^2 - r_A^2)\cdot P(\theta,\phi)},$$

where

(2.12a) $$X_V(\theta,\phi) = -\frac{\alpha}{\pi}\left\{ z\left(\frac{\ln^2 a}{b^4} + \frac{\ln^2 b}{a^4}\right) - \left(\frac{\ln a}{b^2} - \frac{\ln b}{a^2}\right) + 4\left[\ln^2 b - \ln^2 a + \frac{1}{2}Li_2(a^2) - \frac{1}{2}Li_2(b^2)\right] + \frac{2z}{1+z^2-P(\theta,\phi)} \cdot \left[z\left(\frac{\ln^2 a}{b^4} - \frac{\ln^2 b}{a^4}\right) - \left(\frac{\ln a}{b^2} + \frac{\ln b}{a^2}\right)\right]\right\},$$

(2.12b) $$X_A(\theta,\phi) = -\frac{2\alpha}{\pi}\left\{ Li_2(a^2) - Li_2(b^2) - (\ln a)^2 + (\ln b)^2 - \frac{b^2}{z}\ln a - \frac{a^2}{z}\ln b - \frac{P(\theta,\phi)}{4z} \cdot \left[z\left(\frac{\ln^2 a}{b^4} - \frac{\ln^2 b}{a^4}\right) - \left(\frac{\ln a}{b^2} + \frac{\ln b}{a^2}\right)\right]\right\},$$

(2.12c) $$X_{AV}(\theta,\phi) = -\frac{2\alpha}{\pi}\left\{ z\frac{\ln^2 b}{a^4} + \frac{\ln b}{a^2} + 2(\ln^2 b - \ln^2 a) + Li_2(a^2) - Li_2(b^2) - \pi\frac{Re\chi}{Im\chi}\left[z\frac{\ln b}{a^4} - 2\ln\left(\frac{a}{b}\right) + \frac{1}{1-z} + \frac{1}{3}\bar{R}\right]\right\},$$

(2.12d) $$Y_V(\theta,\phi) = \left\{\left[z - 2z(\ln a + \ln b) + 2(1+z^2)\left(\ln\frac{a}{b} - \frac{\bar{R}}{6}\right)\right] - P(\theta,\phi) \cdot \left[\left(2\ln\frac{a}{b} - \frac{\bar{R}}{3}\right) - \frac{1}{2}z\left(\frac{\ln a}{b^4} + \frac{\ln b}{a^4}\right) - \frac{z}{1-z^2}\right]\right\},$$

(2.12e) $$Y_A(\theta,\phi) = \left\{\left[2z\left(\ln\frac{a}{b} - \frac{\bar{R}}{3}\right) + 1\right] + P(\theta,\phi) \cdot \left[\frac{1}{1-z^2} - \frac{1}{2}z\left(\frac{\ln a}{b^4} - \frac{\ln b}{a^4}\right)\right]\right\},$$

with $\bar{R} = \sum_{\substack{i=\text{leptons}\\+\text{quarks}}} Q_i^2$ and δ_{VP} is given in sect. 4 (eq. 4.33).

This completes the list of our formulae. Their derivation and discussion follow in the coming sections.

3. MULTIPLE SOFT PHOTONS EFFECTS.

In this section we will discuss the collective effect of multiple soft photons emission in the framework of the coherent state formalism, leading to the expressions of the infrared factors given above.

The use of coherent states in the infrared problem in QED is rather well known[10]. This technique was generalized in ref. (7) to discuss the radiative effects generated in presence of a narrow resonance like the J/ψ , whose decay width is much smaller than the energy resolution of the experiment. To our purposes, as well as for the reader's convenience, we will briefly review those results, without any restriction on the relative size of Γ and $\Delta\omega$.

When the reaction

$$a + b \longrightarrow c + d + \dots \tag{3.1}$$

proceeds through the production of a resonance in the s channel, the classic current associated to the external charged particles is modified as follows:

$$j_\mu^R(k) = \frac{(W-M)+\frac{1}{2}\,i\Gamma}{(W-M-k)+\frac{1}{2}\,i\Gamma}\, j_\mu^{(i)}(k) + j_\mu^{(f)}(k) \tag{3.2}$$

where $j_\mu^{(i)}(k)$ and $j_\mu^{(f)}(k)$ are the usual classical currents associated to the initial and final particles respectively, namely

$$j_\mu^{(i,f)}(k) = \frac{ie}{(2\pi)^{3/2}} \sum_{1}^{(i,f)} \varepsilon_1 \frac{p_\mu^{(1)}}{(p_1\cdot k)} . \tag{3.3}$$

This modification takes into account the finiteness of the time interval between the formation of the resonance and the creation of the final state, as can be seen from the Fourier transform of (3.3). In pertubation theory this is equivalent to write the matrix element for the emission of one soft photon in the reaction (3.1) as

$$M_\mu^{(1\gamma)}(k) = j_\mu^{(i)}\, M^\circ(W-k) + j_\mu^{(f)} M^\circ(W), \tag{3.4}$$

and accounts for the shift in the c.m. energy when the radiation is emitted from the initial state.

Let us introduce the action Λ_R relative to the distribution of classical current $j^R(k)$ of eq. (3.2)

$$(3.5) \qquad \Lambda_R = \frac{1}{(2\pi)^4} \int d^4k \; j^R_\mu(k) \; A^\mu(-k) ,$$

where A_μ (k) is the quantized electromagnetic field. Then, for a pure resonant process, the matrix element

$$(3.6) \qquad \bar{M}_R = \frac{1}{\sqrt{N}} \langle f/e^{-i\Lambda_R^+} S/ i \rangle,$$

with $N = \langle e^{-i\Lambda_R^+} e^{i\Lambda_R} \rangle$, can be shown(7) to be: (i) finite and does not therefore possess an infrared divergence, (ii), separable in the infrared factors and (iii) directly comparable with the observable cross section which results proportional to $|\bar{M}_R|^2$.

Without entering into the details of the derivation, one obtains for $e\bar{e} \to R \to \mu^+\mu^-$

$$(3.7) \qquad d\sigma^{res}_{corr} = d\sigma^{res}_o \left(\frac{\Delta\omega}{E}\right)^{\beta_\mu} \left| \frac{2\Delta\omega}{\sqrt{s}} \; \frac{M_R^2-s}{M_R^2-s+2\sqrt{s}\Delta\omega} \right|^{\beta_e} \cdot \left| \frac{2\sqrt{s}\,\Delta\omega}{M_R^2-s+2\sqrt{s}\Delta\omega} \right|^{2\beta_{int}} \left\{ 1+ \frac{s-M^2}{M\Gamma} \beta_e \delta(s,\Delta\omega)(1+C_F^{res}) \right\},$$

where $M_R^2 = M^2 - iM\Gamma$,

$$(3.8) \qquad \delta(s,\Delta\omega) = \mathrm{arctg}\, \frac{2\sqrt{s}\Delta\omega - (s-M^2)}{M\Gamma} + \mathrm{arctg}\, \frac{s-M^2}{M\Gamma} ,$$

and C_F^{res} accounts for the rest of the finite corrections and has to be calculated perturbatevely.

Let us briefly discuss our result. First, it is easily seen than the infrared factors appearing in eq. (3.7) reduce to the standard one $(\Delta\omega/E)^{\beta_e+\beta_\mu+2\beta_{int}}$ in the limit $\Delta\omega \ll |(M_R^2-s)/2\sqrt{s}|$, as they should.

On the other hand, in the case of a narrow resonance like the J/Ψ, for which in a typical experiment $\Delta\omega \gg |(M_R^2-s)/2\sqrt{s}|$, the $\Delta\omega$ dependence drops completely out, namely the width of the resonance provides a natural cut-off in damping the energy loss in the initial state. Furthermore, the β_{int} dependence in eq. (3.8) also cancels out, giving no interference between the soft emission from the initial and final states.

Finally the term proportional to δ (s, $\Delta\omega$), with the arctangent defined to have values in the interval $(-\pi/2, \pi/2)$, gives the radiative tail of the resonance. In fact for narrow resonances $\delta\,(s, \Delta\omega)$ reduces to $\delta_R(s)$, the usual phase shift of the Breit-Wigner resonance.

With the standard definition (see eq. 2.3)

$$\frac{s}{M_R^2 - s} \equiv \frac{s}{M\Gamma} \sin\delta_R \, e^{i\delta_R} \,, \tag{3.9}$$

eq. (3.7) easily leads to the infrared resonant factor given in section 2 (eq. 2.10c).

We would like to discuss now the radiative effect arising from the interference of a resonant term with a pure QED term. As shown in detail in ref. (7), the procedure to follow is quite clear. One starts with the sum of two infrared finite matrix elements, $\bar{M}_{QED}$ and $\bar{M}_R$, as

$$\bar{M} = \bar{M}_{QED} + \bar{M}_R = \langle f/e^{-i\Lambda_c} S_{QED}/i\rangle + \frac{1}{\sqrt{N}} \langle f/e^{-i\Lambda_R^+} S_R/i\rangle, \tag{3.10}$$

where Λ_c is the action relative to the distribution of pure classical currents

$$j_\mu^c(k) = j_\mu^{(i)}(k) + j_\mu^f(k) \,, \tag{3.11}$$

which describes the infrared properties of a pure QED process [10]. More explicity Λ_c is given by eq. (3.5) when $j_\mu^R(k) \longrightarrow j_\mu^c(k)$. The full observed cross section is then proportional to $|\bar{M}|^2$. While $|\bar{M}_{QED}|^2 \propto d\sigma_{corr}^{QED}$ and $|\bar{M}_R|^2$ $d\sigma_{corr}^{res}$, the interference term comes only from that part of $\langle f/\exp(-i\Lambda_c)$ which overlaps with $\langle f/\exp(-i\Lambda_R^+)/\sqrt{N}$. Introducing therefore a matrix element $\bar{M}_{QED}^{int}$ as

$$\bar{M}_{QED}^{int} = \frac{1}{\sqrt{N}} \langle f/e^{-i\Lambda_R^+} S_{QED}/i \rangle \,, \tag{3.12}$$

all the interference effects will then come from $\mathrm{Re}\,(\bar{M}_{QED}^{int} \; \bar{M}_R^*)$, as one can see by comparison with perturbation theory.

With that in mind, one finds

$$d\sigma_{corr}^{int} = d\sigma_o^{int} \left(\frac{\Delta\omega}{E}\right)^{\beta_\mu + \beta_{int}} \frac{1}{\cos\delta_R} \cdot$$
$$\cdot \mathrm{Re}\left\{ e^{i\delta_R} \left(\frac{\Delta}{1 + \Delta \frac{s}{M_R^2 - s}}\right)^{\beta_e} \left(\frac{\Delta}{\Delta + \frac{M_R^2 - s}{s}}\right)^{\beta_{int}} \right\} \,, \tag{3.13}$$

which, using eq. (3.9), leads finally to eq. (2.10b).

Again, taking the limit $\Delta \ll (\Delta \gg) \left|(M_R^2 - s)/s\right|$ one gets the usual QED, or narrow resonance-like behaviour respectively.

This concludes our discussion of the multiple soft photons effect. From our formulae it is clear that, in experiments studying forward-backward asymmetries, dips and similar subtle effects, the Γ and $\Delta\omega$ dependence in $C_{infra}^{(i)}$ factors has to be taken into account.

We now proceed to present the first order corrections which, together with the results of this section, give to the formulae of sect. 2.

4. BORN TERMS AND FIRST ORDER CORRECTIONS.

The Born terms are given by the two diagrams of Fig. 1, for which the scattering amplitude reads

$$T = M_A(W) + M_B(W) =$$

(4.1)
$$= \frac{e^2}{s}(J'_\mu J^\mu) - \frac{e^2}{M_R^2 - s}(g_V J'_\mu + g_A A'_\mu)(g_V J^\mu + g_A A^\mu)),$$

where

(4.2)
$$J_\mu = \bar{v}(q)\gamma_\mu u(p)\ ; \qquad A_\mu = \bar{v}(q)\gamma_\mu \gamma_5 u(p)$$
$$J'_\mu = \bar{u}(p')\gamma_\mu v(q')\ ; \qquad A'_\mu = \bar{u}(p')\gamma_\mu \gamma_5 v(q')$$

and $M_R^2 = M^2 - i M\Gamma$. In the standard model(2) one has (see sect.2) $g_V^e = g_V^\mu = (4\sin^2\theta_W - 1)/2\sin(2\theta_W)$ and $g_A^e = g_A^\mu = 1/2\sin(2\theta_W)$.

The differential cross ssection is given by

(4.3)
$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}\left(\frac{1}{4}\right)\sum_{spin}(TT^+) .$$

Since we consider only transverse polarization for the $e^\pm$ and we neglect all lepton masses, in doing the trace it suffices to insert

$$\sum_{spin} u(p)\bar{u}(p) = \not{p}(1 + \not{s}_1\gamma_5)$$
$$\sum_{spin} v(q)\bar{v}(q) = \not{q}(1 + \not{s}_2\gamma_5)$$

where

(4.4)
$$s_1^\mu = (0, \xi_-, 0, 0) \text{ and } s_2^\mu = (0, \xi_+, 0, 0) ,$$

with $\xi_+\xi_- < 0$.

In the Appendix, we list all the traces and two useful identities for the calculations to follow. We have written these down explicitely to facilitate a check of our resulting formulae by the reader.

Using eqs. (4.1) and (4.3) and upon performing the relevant traces we find that the Born cross-section may be written in the following way:

(4.5)
$$\left(\frac{d\sigma}{d\Omega}\right)_{Born} = \left(\frac{\alpha^2}{4s}\right)\left\{\left[(1+\cos^2\theta)A(s) + 2B(s)\cos\theta\right] - \right.$$
$$\left. - \left|\xi_-\xi_+\right|\sin^2\theta\left[C(s)\cos 2\phi - D(s)\sin 2\phi\right]\right\}$$

where

(4.6a) $$A(s) = 1+2s\, g_V^2\, \mathrm{Re}\left(\frac{1}{s-M_R^2}\right) + \frac{s^2}{|s-M_R^2|^2}\,(g_V^2+g_A^2)^2,$$

(4.6b) $$B(s) = 2s\, g_A^2\, \mathrm{Re}\left(\frac{1}{s-M_R^2}\right) + \frac{4s^2}{|s-M_R^2|^2}\, g_V^2\, g_A^2,$$

(4.6c) $$C(s) = 1+2s\, g_V^2\, \mathrm{Re}\left(\frac{1}{s-M_R^2}\right) + \frac{s^2}{|s-M_R^2|^2}\,(g_V^4 - g_A^4),$$

(4.6d) $$D(s) = 2s\, g_V g_A\, \mathrm{Im}\left(\frac{1}{s-M_R^2}\right),$$

which coincides with eqs. (2.9), once eqs. (2.7) are taken into account.

We now consider the first order virtual corrections given by the interference of diagrams in Fig. 1 with the box diagrams of Figs. 2 and 3.

The 2-photon exchange diagrams have been calculated earlier many times(11,12,13). We can thus omit the details and simply record the result:

(4.7) $$M_C = \left(\frac{2\alpha^2}{s}\right)\left\{ (J'_\mu J^\mu)(V_1+2\pi i\, V_2)+(A'_\mu A^\mu)(A_1+2\pi i\, A_2)\right\},$$

where

(4.8a) $$V_1 = -8 \ln\frac{a}{b} \ln\frac{2E}{\lambda} - z\left(\frac{\ln^2 a}{b^4} + \frac{\ln^2 b}{a^4}\right) + \left(\frac{\ln a}{b^2} - \frac{\ln b}{a^2}\right),$$

(4.8b) $$V_2 = 2 \ln\left(\frac{a}{b}\right) - \frac{z}{2}\left(\frac{\ln a}{b^4} + \frac{\ln b}{a^4}\right) - \frac{z}{(1-z^2)},$$

(4.8c) $$A_1 = -z\left(\frac{\ln^2 a}{b^4} - \frac{\ln^2 b}{a^4}\right) + \left(\frac{\ln a}{b^2} + \frac{\ln b}{a^2}\right),$$

(4.8d) $$A_2 = \frac{z}{2}\left(\frac{\ln a}{b^4} - \frac{\ln b}{a^4}\right) + \frac{1}{(1-z^2)},$$

and λ denotes the mass of the photon.

The diagrams in Fig. 3 have been previously computed for a resonance with vector coupling only(12,13), e.g. J/Ψ. We need to include axial-vector coupling as well. For example, the first diagram gives

$$M_{D_1} = (-ie^4)\int \frac{d^4k}{(2\pi)^4}\ \cdot$$

(4.10)
$$\cdot\ \frac{\bar{u}(p')\gamma_\mu(\not p'+\not k+m_\mu)\,\gamma_\mu(g_V+g_A\gamma_5)v(q')\bar{v}(q)\gamma^\nu(g_V+g_A\gamma_5)(\not p+\not k+m_e)\gamma^\mu u(p)}{(k^2-\lambda^2)\left[(k+p+q)^2-M_R^2\right]\left[(p+k)^2-m_e^2\right]\left[(p'+k)^2-m_\mu^2\right]}$$

As before, we neglect all masses in the numerator. Also, we make the standard approximation of setting $k \to 0$ in the numerator. The terms

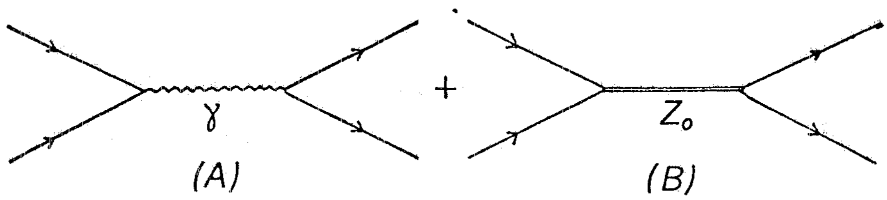

FIG. - 1 Born diagrams with γ and Z_o in the s-channel.

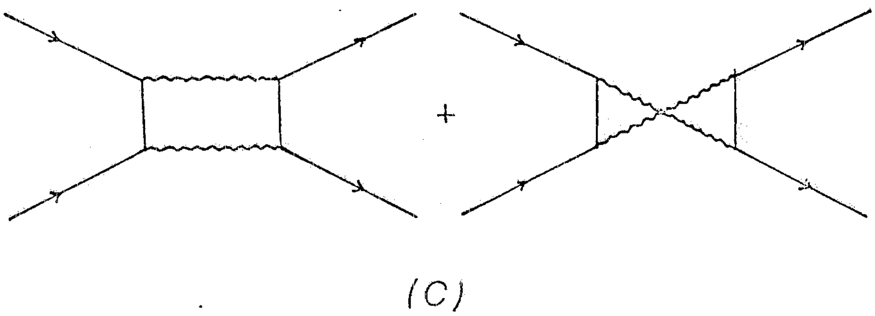

FIG. - 2 2 photon diagrams.

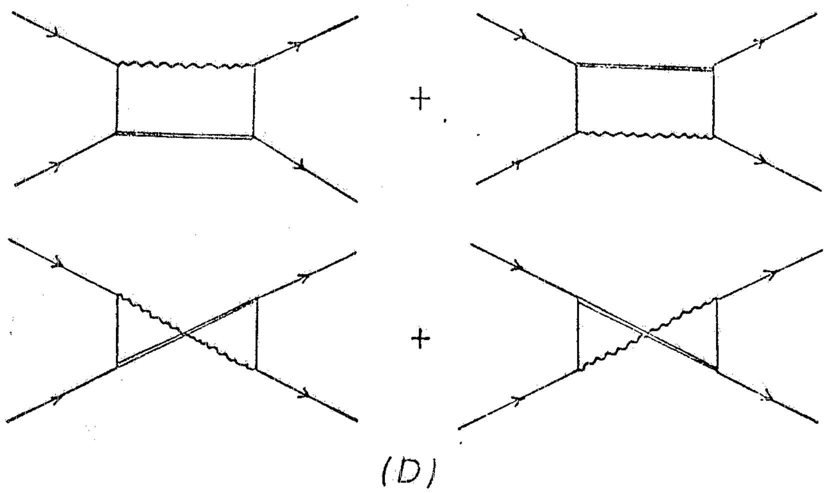

FIG. - 3 $(\gamma+Z_o)$ intermediate states.

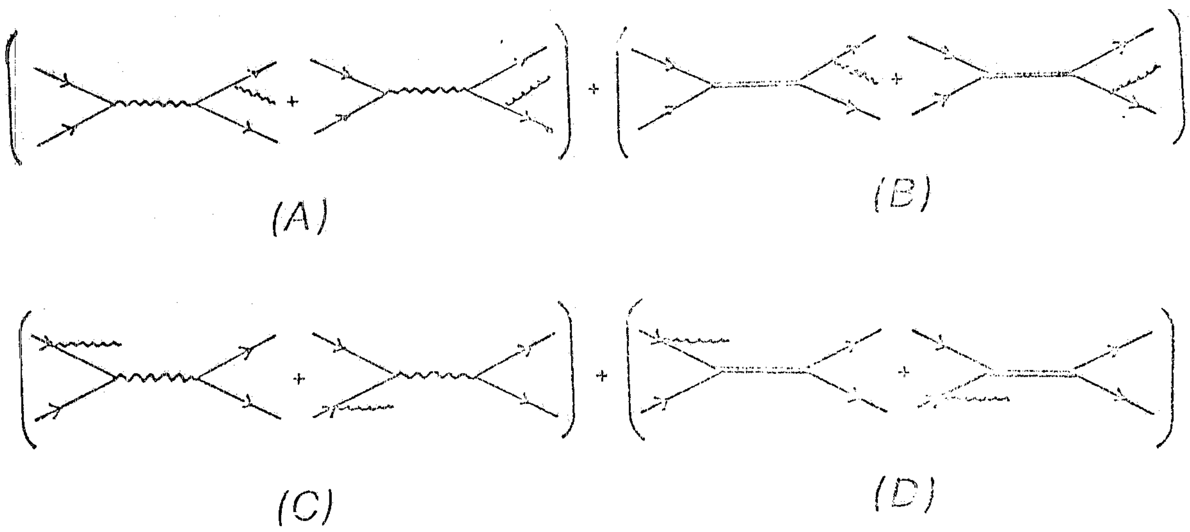

FIG. - 4 Bremsstrahlung diagrams.

proportional to k_μ , which will be of order $(M_R^2/s)\ln(M_R^2/s)$, can in fact be neglected safely in the vicinity of the resonance. This simplifies the calculation tremendously. Using the identities (A-1) and (A-2) of the Appendix, we find in this limit:

(4.11) $$M_{D_1} = (-ie^4)(4p.p')(g_V J'_\mu + g_A A'_\mu)(g_V J^\mu + g_A A^\mu)\, I_1$$

where

(4.12) $$I_1 = \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2-\lambda^2)\left[(k+p+q)^2-M_R^2\right]\left[(p+k)^2-m_e^2\right]\left[(p'+k)^2-m_\mu^2\right]}$$

Adding up all 4 diagrams of Fig. 3 we finally obtain

(4.13) $$M_D = M_B \cdot \frac{2\alpha}{\pi} \cdot \left\{ (\ln^2 a - \ln^2 b) + \frac{1}{2}\left[Li_2(b^2) - Li_2(a^2)\right] + \ln\left(\frac{a}{b}\right)\ln\left[\frac{(M_R^2-s)^2}{\lambda^2 s}\right] \right\},$$

where the dilogaritm is defined as $Li_2(z) = -\int_0^z \frac{dx\,\ln(1-x)}{x}$.

Now we turn to the calculation of the real emission (bremsstrahlung) diagrams. These may be separated (see Fig. 4) into 2 classes, according as to whether the emission is from the initial legs, type B_i or from the final legs, type B_f. In the following, we shall indicate the product of two sets of diagrams with the symbol $B_i \otimes B_i$ or $B_i \otimes B_f$, etc. When evaluating the cross-section, the products $B_i \otimes B_i$ and $B_f \otimes B_f$ in the soft photon approximation, do not depend upon the details of the interactions (i.e. whether γ or Z_o is exchanged) and thus we may simply take over the earlier results for these[14,15]. These terms generate corrections factors which are independent of the scattering angles (θ, ϕ) and are included in the first four terms of $C_F^{(i)}$ (eqs. 2.11), discussed at the end of this section. On the other hand the angle dependent parts are generated by the interference terms $B_i \otimes B_f$, a discussion of which follows.

Following ref. (13), we shall give separately the results for the sets $A \otimes C$, $B \otimes C$ and $(A+B) \otimes D$, where A, B, C and D all refer to the diagrams of Fig. 4.

(i) <u>$A \otimes C$:</u> This term contributes to the purely QED cross-section and one obtains, in the soft photon approximation,

$$\left(\frac{d\sigma}{d\Omega}\right)^{real}_{A\otimes C} = \frac{1}{64\pi^2 s} \cdot 2e^2 \int_{\omega \leq \Delta\omega} \frac{d^3k}{2\omega(2\pi)^3}\left(\frac{p_\mu}{p.k} - \frac{q_\mu}{q.k}\right) \cdot \left(\frac{p'^\mu}{p'.k} - \frac{q'^\mu}{q'.k}\right) \cdot \frac{1}{4}\, Tr\left[M_A^+(W)\, M_A(W)\right],$$

where $M_A(W)$ refers to Fig. 1 (eq.4.1). Then one gets where $M_A(W)$ refers to Fig. 1 (eq.4.1). Then one gets

$$(4.14)\qquad \left(\frac{d\sigma}{d\Omega}\right)^{real}_{A\otimes C} = \left(\frac{d\sigma_{QED}}{d\Omega}\right) . \frac{4\alpha}{\pi}\left[F(z) - F(-z)\right] ,$$

where

$$(4.15)\qquad F(z) = 4\pi^2 \int_{\omega\lesssim\Delta\omega} \frac{d^3k}{2\omega(2\pi)^3} . \frac{(p.p')}{(p.k)(p'.k)} \simeq$$

$$\simeq 2\ln\left(\frac{2\Delta\omega}{\lambda}\right)\ln a + \ln^2 a + \frac{1}{2}Li_2(b^2) .$$

Clearly ($z \longleftrightarrow -z$) implies ($a \longleftrightarrow b$).

To this we must add the elastic part of $A\otimes C$ (Figs. 1-2) which, through eqs. (4.1), (4.7) and (4.8) is obtained to be

$$(4.16)\qquad \left(\frac{d\sigma}{d\Omega}\right)^{elastic}_{A\otimes C} = \frac{1}{64\pi^2 s}2\left(\frac{e^2}{s}\right)\left(\frac{2\alpha^2}{s}\right) .$$

$$. \frac{1}{4}Re\left\{Tr\ \ (\overline{J'.J})\left[(V_1+2\pi i V_2)(J'.J)+(A'.A)(A_1+2\pi i A_2)\right]\right\} .$$

Summing eqs. (4.14) and (4.16) and performing the traces, we find

$$(4.17)\qquad \left(\frac{d\sigma}{d\Omega}\right)_{A\otimes C} = \frac{\alpha^2}{4s}\left[1+z^2-P(\theta,\phi)\right]\left[1-\left(\frac{8\alpha}{\pi}\right)\ln\left(\frac{a}{b}\right)\ln\left(\frac{E}{\Delta\omega}\right)+X_V(\theta,\phi)\right] .$$

where[16]

$$(4.18)\qquad X_V(\theta,\phi) = -\frac{\alpha}{\pi}\left\{4(\ln^2 b-\ln^2 a)+2\left[Li_2(a^2)-Li(b^2)\right]+\right.$$

$$+ z\left(\frac{\ln^2 a}{b^4}+\frac{\ln^2 b}{a^2}\right) - \left(\frac{\ln a}{b^2}-\frac{\ln b}{a^2}\right) +$$

$$\left. + \frac{2z}{1+z^2-P(\theta,\phi)}\left[z\left(\frac{\ln^2 a}{b^4}-\frac{\ln^2 b}{a^4}\right)-\left(\frac{\ln a}{b^2}+\frac{\ln b}{a^2}\right)\right]\right\}$$

In eq. (4.17) the infrared factor dependent on $\ln\left(\frac{E}{\Delta\omega}\right)$, is the first order expansion of $\left(\frac{\Delta\omega}{E}\right)^{2\beta int}$ (see eq.(2.10a). Eq. (4.18) gives the finite factor of eq. (2.12a).

(ii) <u>B ⊗ C</u>: For the real part, we obtain (Fig. 4)

$$(4.19)\qquad \left(\frac{d\sigma}{d\Omega}\right)^{real}_{B\otimes C} = \frac{1}{64\pi^2 s} Re\left\{\frac{1}{4}Tr(M_A^+(W)M_B(W)) .\right.$$

$$\left. . \frac{2e^2}{(2\pi)^3}\int_{\omega\lesssim\Delta\omega}\frac{d^3k}{2\omega}\left(\frac{p_\mu}{pk}-\frac{q_\mu}{qk}\right)\left(\frac{p'_\mu}{p'k}-\frac{q'_\mu}{q'k}\right)\right\} ,$$

which, in analogy to eq. (4.14), gives

(4.20)
$$\left(\frac{d\sigma}{d\Omega}\right)^{real}_{B\otimes C} = \left\{\frac{d\sigma_{int,V}}{d\Omega} + \frac{d\sigma_{int,A}}{d\Omega} + \frac{d\sigma_{int,VA}}{d\Omega}\right\} \cdot \frac{2\alpha}{\pi}\left[F(z) - F(-z)\right]$$

The corresponding elastic part is obtained through eqs. (4.1), (4.7) and (4.8). We find

(4.21)
$$\left(\frac{d\sigma}{d\Omega}\right)^{elastic}_{B\otimes C} = \frac{1}{64\pi^2 s}\, 2\,\mathrm{Re}\left\{\frac{1}{4}\,\mathrm{Tr}\left[(M_B^+(W)M_C(W)\right]\right\} =$$
$$= \frac{\alpha^3}{4\pi s}\left[V_1 \mathrm{Re}\chi + 2\pi V_2 \mathrm{Im}\chi\right]\left[r_V(1+z^2-P(\theta,\phi))+2r_A z\right] +$$
$$+\left[A_1\mathrm{Re}\chi + 2\pi A_2 \mathrm{Im}\chi\right]\left[r_A(1+z^2+P(\theta,\phi)+2r_V z\right] +$$
$$-\left[2\pi(V_2-A_2)\mathrm{Re}\chi-(V_1-A_1)\mathrm{Im}\chi\right]\left[r_{AV}Q(\theta,\phi)\right] ,$$

where $\chi(s)$ is defined in eq. (2.7b).

(iii) $(A+B)\otimes D$: For this case there is an almost complete cancellation between the real and virtual diagrams, as in the pure vector case[7,12,13]. For the real ones, we have

(4.22)
$$\left(\frac{d\sigma}{d\Omega}\right)^{real}_{(A+B)\otimes D} = \frac{1}{64\pi^2 s} \cdot 2e.^2 \mathrm{Re}\left\{\int \frac{d^3k}{2\omega(2\pi)^3}\left(\frac{p_\mu}{p.k}-\frac{q_\mu}{q.k}\right)\right.$$
$$\left.\left(\frac{p'_\mu}{p'.k} - \frac{q'_\mu}{q'.k}\right).\frac{1}{4}\left[(M_A(W)+M_B(W)\right]^+ M_B(W-k)\right\}.$$

Using eqs. (4.1) and (4.22) we find

(4.23)
$$\left(\frac{d\sigma}{d\Omega}\right)^{real}_{(A+B)\otimes D} = \frac{1}{64\pi^2 s}\left(\frac{\alpha}{\pi}\right)\mathrm{Re}\left\{\mathrm{Tr}\left[(M_A(W)+M_B(W))^+ M_B(W)\right].\right.$$
$$\left.\cdot\left[F'(z) - F'(-z)\right]\right\},$$

where

(4.24)
$$F'(z) = 4\pi^2 \int_{\omega\leq\Delta\omega} \frac{d^3k}{2\omega(2\pi)^3}\, \frac{(pp')}{(p\,k)\,(p'k)}\left\{\frac{1}{1+4E\omega/(M_R^2-s)}\right\}$$
$$= 2\ln a \ln\frac{2\Delta\omega}{\lambda\left[1+4E\Delta\omega/(M_R^2-s)\right]} + \ln^2 a + \frac{1}{2}\,\mathrm{Li}_2(b^2) .$$

For the corresponding elastic part (Figs. 1-3)

$$\left(\frac{d\sigma}{d\Omega}\right)^{elastic}_{(A+B)\otimes D} = \frac{2}{64\pi^2 s}\cdot\frac{1}{4}\,\mathrm{Re}\left\{\mathrm{Tr}\left[(M_A(W)+M_B(W))^+ M_D(W)\right]\right\},$$

we obtain, through eqs. (4.1) and (4.13),

$$(4.25)\quad \left(\frac{d\sigma}{d\Omega}\right)^{elastic}_{(A+B)\otimes D} = -\left(\frac{1}{64\pi^2 s}\right)\left(\frac{\alpha}{\pi}\right) \mathrm{Re}\left\{ \mathrm{Tr}\left[(M_A(W)+M_B(W))^+ M_B(W)\right] \cdot \left[2\ln\left(\frac{a}{b}\right)\ln\left(\frac{M_R^2-s}{2\lambda E}\right) + \frac{1}{2}\left(\mathrm{Li}_2(b^2)-\mathrm{Li}_2(a^2)\right) + \ln^2 a - \ln^2 b\right]\right\}$$

Summing the real and virtual contributions given by eqs. (4.23) and (4.25) we get the final result

$$(4.26)\quad \left(\frac{d\sigma}{d\Omega}\right)_{(A+B)\otimes D} = \left(\frac{1}{64\pi^2 s}\right)\left(\frac{\alpha}{\pi}\right) \mathrm{Re}\left\{ \mathrm{Tr}\left[(M_A(W)+M_B(W))^+ M_B(W)\right] \cdot \left[2\ln\left(\frac{a}{b}\right)\ln\left(1+\frac{M_R^2-s}{4E\Delta\omega}\right)\right]\right\} .$$

which is negligible when $\Delta\omega \gg \left| (M_R^2-s) \right| /4E$.

The total angle-dependent first order correction factor is given by the sum of eqs. (4.17), (4.20), (4.21) and (4.26). It is easily verified that it checks with eqs. (2.10a,b and c), when the infrared factors are expanded to first order in β_{int}, and with the expressions for X_i(i=A,VA) and Y_j(j=V,A) given in eqs. (2.12 a+e) upon factoring out the corresponing Born terms. The only exception is given by the $\bar{R}$-terms, which arise from the imaginary part of the propagator correction and are discussed below in connection with the vacuum polarization factor.

The first order correction $\Pi(s,m_i^2)$ to the photon propagator $\Pi(s)$, due to a fermion loop of charge eQ_i and mass m_i, is given in the relativistic limit ($s \gg m_i^2$) by

$$(4.27)\qquad \Pi(s) = 1 + \Pi(s, m_i^2)$$

with

$$(4.28)\qquad \mathrm{Re}\ \Pi(s, m_i^2) \simeq \frac{\alpha}{3\pi}\left[\ln\left(\frac{s}{m_i^2}\right) - \frac{5}{3}\right]$$

and

$$(4.29)\qquad \mathrm{Im}\ \Pi(s, m_i^2) \simeq -\frac{\alpha}{3} Q_i^2 .$$

Thus, the vacuum polarization correction due to the i-th fermion loop is

$$(4.30)\quad \left(\frac{d\sigma}{d\Omega}\right)^{(i)}_{VP} = \left(\frac{1}{64\pi^2 s}\right) 2\,\mathrm{Re}\left\{\left[\mathrm{Re}\ \Pi(s,m_i^2) - i\ \mathrm{Im}\ \Pi(s,m_i^2)\right] \cdot \frac{1}{4}\mathrm{Tr}\left[M_A^+(W)(M_A(W)+M_B(W))\right]\right\} ,$$

which leads to

$$(4.31)\quad \left(\frac{d\sigma}{d\Omega}\right)^{(i)}_{VP} = \left(\frac{\alpha^2}{4s}\right)\Big\{ 2\,\mathrm{Re}\;\Pi(s,m_i^2)\Big\{(1+z^2-P(\theta,\phi)) + \mathrm{Re}\chi\,. \left[r_V(1+z^2-P(\theta,\phi))+2z\,r_A\right] + \mathrm{Im}\chi\,.\,r_{AV}\,Q(\theta,\phi)\Big\} + +2\,\mathrm{Im}\;\Pi(s,m_i^2)\Big\{\mathrm{Im}\chi\left[r_V(1+z^2-P(\theta,\phi))+2z\,r_A\right] - -\mathrm{Re}\chi\,.\,r_{AV}\,Q(\theta,\phi)\Big\}\Big\}\,.$$

In the total Re $\Pi(s)$ we shall include the contributions of the leptons e, μ, τ as well as the hadronic part, which has been calculated in refs. (17,18). In particular explicit use of the experimental data has been made in the dispersive integral for $\Pi_{had}(s)$ at low energies ($\sqrt{s'} \lesssim 9$ GeV), while the high energy part has been estimated through the six quarks u, d, s, c, b and t. This leads to a total correction factor

$$(4.32)\quad \delta^{tot}_{VP} = 2\,\mathrm{Re}\left[\Pi(s)-1\right] = 2\sum_i \mathrm{Re}\,\Pi(s,m_i^2) = =\delta_{VP}(m_e^2) + \delta_{VP}(m_\mu^2) + \delta_{VP}$$

where

$$(4.33)\quad \delta_{VP} = (1.86\ln W - 1).10^{-2} - \frac{2\alpha}{3\pi}\left[\frac{1}{3}I_b(W) + \frac{4}{3}I_t(W)\right],$$

$$I_i(W) = \frac{5}{3} + \frac{4m_i^2}{W^2} - \sqrt{1-\frac{4m_i^2}{W^2}}\left(1+\frac{2m_i^2}{W^2}\right)\ln\left\{\frac{W}{2m_i}\left[1+\sqrt{1-\frac{4m_i^2}{W^2}}\right]\right\},$$

and we have separated out the electron and muon terms.

On the other hand the total contribution to Im $\Pi(s)$ is simply obtained as $(-\alpha\bar{R}/3)$ where $\bar{R} = \sum_{\substack{i=\text{leptons}\\+\text{quarks}}} Q_i^2$. With all that in mind eqs. (4.31) lead to the vacuum polarization factors included in eqs. (2.11) and (2.12).

Finally, finite vertex corrections and bremsstrahlung terms independent of the scattering angle are contained in the first four terms of $C_F^{(i)}$ (see eqs. 2.11), which also include, as discussed above, the electron and muon vacuum polarization factors $\delta_{VP}(m_e^2)$ and $\delta_{VP}(m_\mu^2)$. The former terms are well known and can be found, for instance, in refs. 7 and 15.

This completes our discussion of finite first order corrections.

5. DISCUSSION AND NUMERICAL RESULTS.

We have presented a detailed study of higher order electromagnetic effects in the reaction $e^+e^- \rightarrow Z_o \rightarrow \mu^+\mu^-$, in the framework of the Weinberg-Salam model, for the case of unpolarized as well as transversely polarized electron positron beams. Infrared factors have been considered to all orders in α, together with full finite first order corrections. Only hard bremstrahlung effects, other than the tail effect, have not been taken into account. The weak effects are considered to lowest order, the weak boson Z_o being taken as a resonance of mass M and width Γ. Our results are then given by simple analytical expressions in terms of s, M, Γ and $\Delta\omega$, of immediate experimental application. This formalism, which certainly describes with very good accuracy the regions below, around and not too far above the Z_o, can be extended at larger energies by adding first order pure weak corrections(4).

In order to explicitly show the importance of the radiative effects we have calculated, we present in Figs. 5-6-7 some numerical results for $(d\sigma/d\Omega)$, σ and the integrated forward-backward asymmetry, in case of unpolarized beams and for the following choice of the parameters.

We have considered 6 quarks and 6 leptons and assumed for simplicity $\sin^2\theta_W = 1/4$. This gives $M \simeq 86.4$ GeV, $\Gamma \simeq 2.2$ GeV and $\Gamma(Z_o \rightarrow e\bar{e}) \simeq 70$ MeV. Furthermore we have used for $\Delta = \Delta\omega/E$ the values 5×10^{-3}, 10^{-2}, 5×10^{-2} and 10^{-1}. The comparison is always made with the Born terms called "naive" in the figures.

From inspection of Figs. 5-6-7 it is clear that e.m. radiative corrections change substantially the naive results. The effect on the asymmetry, in particular, is rather impressive for energies above the Z_o mass. It follows therefore that one has to taken full account of radiative effects even for planning experiments at LEP energies.

Similar consideration of course apply to all reactions proceeding from e^+e^- annihilation in the vicinity of the Z_o, including purely hadronic final states.

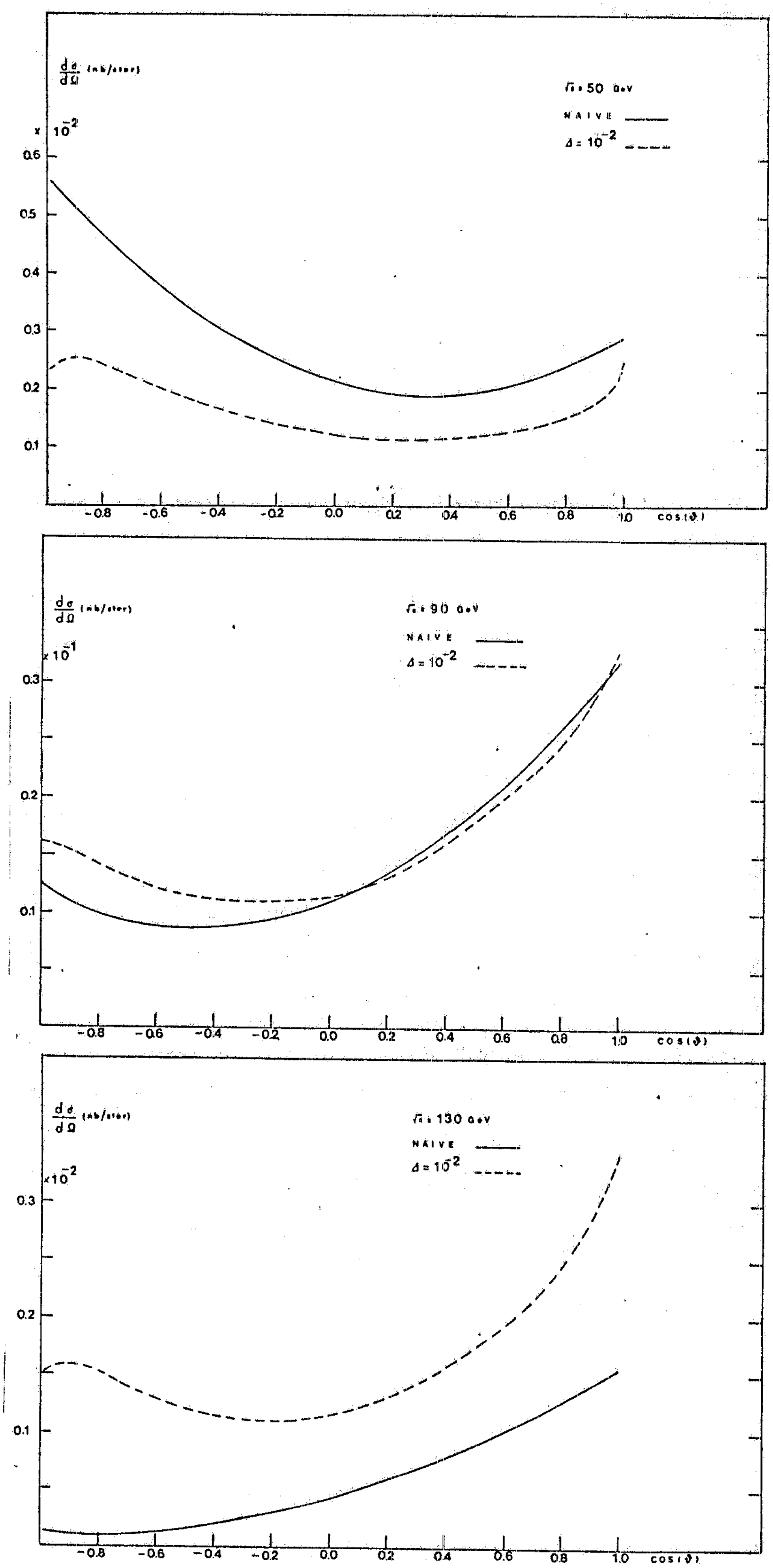

FIG. - 5 Differential cross-section $d\sigma/d\Omega$ vs. cos θ, for various energies, with and without radiative corrections (naive).

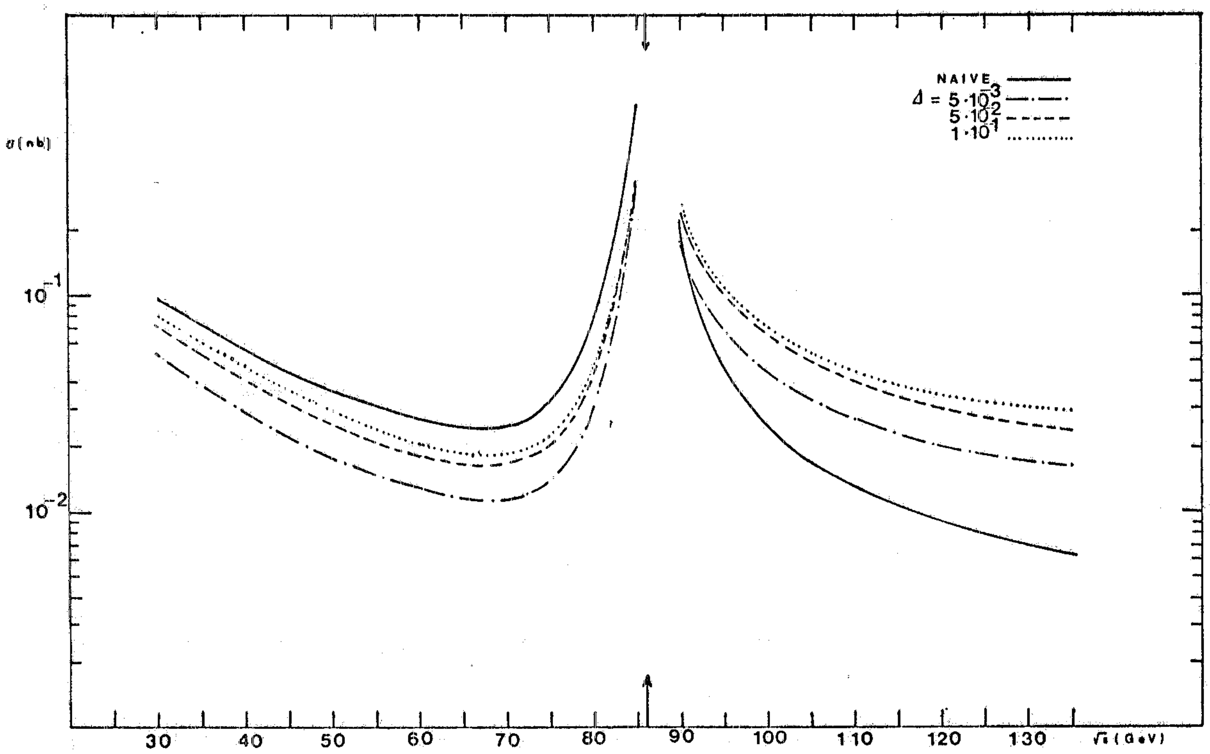

FIG. - 6 Total cross-section σ vs. $\sqrt{s}$ with and without radiative corrections (naive). Various values of the experimental resolution $\Delta\omega/E = \Delta$ are considered.

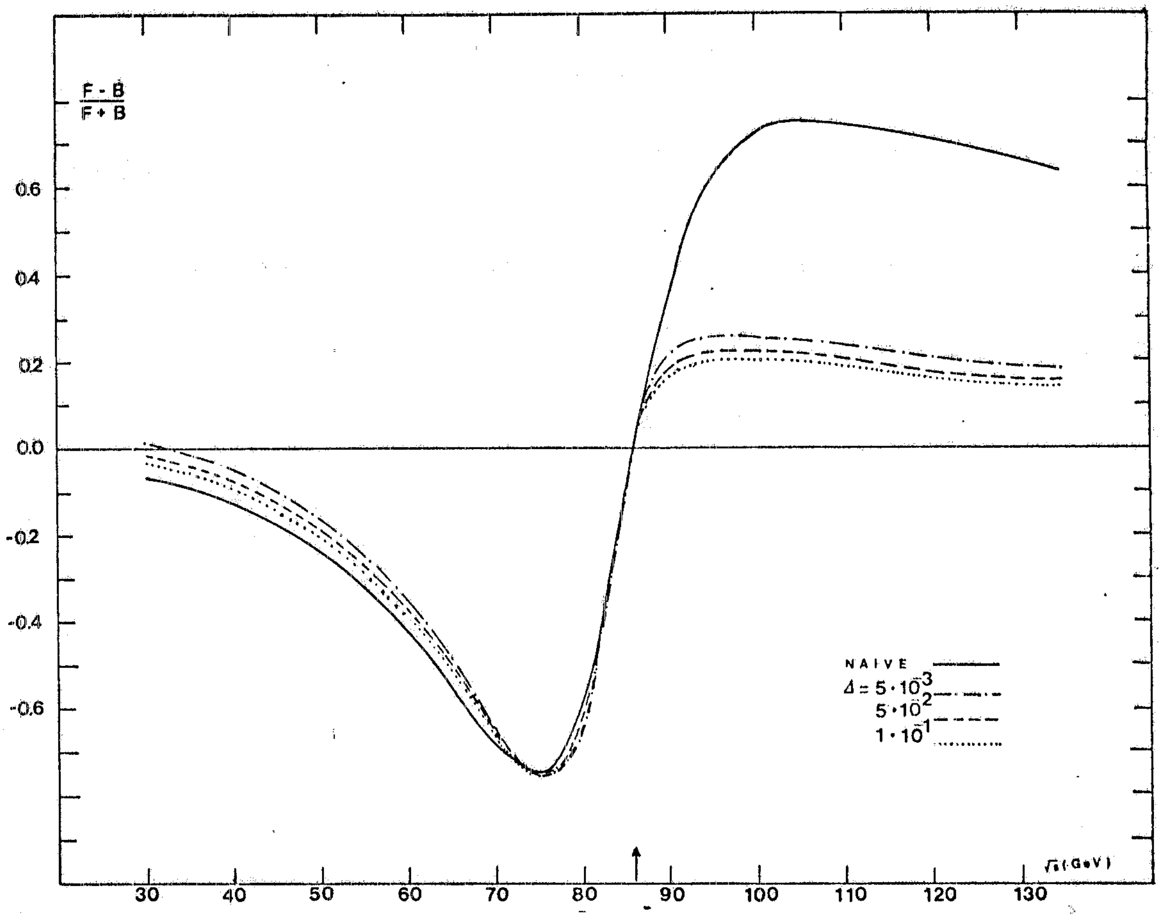

FIG. - 7 Integrated asymmetry vs. $\sqrt{s}$, for various values of the experimental resolution (dotted lines) and without radiative corrections (full line).

APPENDIX

Here we record two identities and the trace formulae which are needed to obtain our final expressions.

For massless spinors the following identity

$$(A.1) \qquad (p'.J)(p.J')+(p.A')(p'.A)=(p.p')\left[(J'.J)+(A'.A)\right]$$

can be found in refs. (12) and (13). For our purposes, we need another identity involving (J'.A) type terms. We have obtained the following, which we quote here, without proof:

$$(A.2) \qquad (p.J')(p'.A)+(p.A')(p'.J)=(p.p')\left[(J'.A)+(A'.J)\right]$$

Eqs. (A.1) and (A.2) are useful in reducing the bow diagram amplitues of eq. (4.10) to the form given in eq. (4.11).

In the following we write down all the expressions we used for computing the traces:

$$(A.3) \qquad \mathrm{Tr}\left\{(\overline{J'.J})(J'.J)\right\} = \mathrm{Tr}\left\{(\overline{A'.J})(A'.J)\right\} = 4s^2(1+z^2-\left|\xi_+\xi_-\right|\sin^2\theta\cos 2\phi) ;$$

$$(A.4) \qquad \mathrm{Tr}\left\{(\overline{A'.A})(A'.A)\right\} = \mathrm{Tr}\left\{(\overline{J'.A})(J'.A)\right\} = 4s^2(1+z^2+\left|\xi_+\xi_-\right|\sin^2\theta\cos 2\phi) ;$$

$$(A.5) \qquad \mathrm{Tr}\left\{(\overline{J'.A})(J'.J)\right\} = \mathrm{Tr}\left\{(\overline{A'.A})(A'.J)\right\} = +(4is^2)\left|\xi_+\xi_-\right|\sin^2\theta\sin 2\phi ;$$

$$(A.6) \qquad \mathrm{Tr}\left\{(\overline{J'.J})(A'.A)\right\} = \mathrm{Tr}\left\{(\overline{J'.A})(A'.J)\right\} = 8s^2z ;$$

$$(A.7) \qquad \mathrm{Tr}\left\{(\overline{A'.J})(J'.J)\right\} = \mathrm{Tr}\left\{(\overline{J'.A})(A'.A)\right\} = 0 .$$

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