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QUARKS STATISCS AND ASYMPTOTIC BEHAVIOUR
OF E. M. FORM FACTORS OF NUCLEI. -

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ABSTRACT. -

The asymptotic behaviour of the e.m. form factor of nuclei is related to the quark statistics. This allows a check of the color hypothesis in the physics of nuclei.

In a recent experiment[1] the elastic e.m. form factor of the deuteron has been measured for values of t^2 up to 160 fm^{-2}, and the data agree well with the asymptotic behaviour predicted by Brodsky and Farrar[2] in a quark model of the deuteron, $F(t) \sim t^{-5}$. Brodsky and Farrar obtain in general for a system with minimum number of constituents N

$$F(t) \sim t^{-(N-1)}, \tag{1}$$

without taking into account the restrictions due to statistics. Such restrictions are irrelevant to the quark model of hadrons because of the small number of quarks involved, but come into play in the quark model of nuclei, as we are going to show.

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Let us first consider the non relativistic case, where the spin-averaged elastic e. m. form factor is given by

$$F^2(q) \sum_{ss'} \sum_{\alpha\alpha'\beta\beta'} \int d\underline{k} \int d\underline{k}' \int d\underline{p} \int d\underline{p}' \, \varphi^*_{\underline{Q},s}(\underline{k}\,\alpha)\, \varphi_{\underline{Q}+\underline{q},s'}(\underline{k}',\alpha') \cdot$$

$$\cdot\, \varphi^*_{\underline{Q}+\underline{q},s'}(\underline{p}\,\beta)\, \varphi_{\underline{Q},s}(\underline{p}'\,\beta')\, G(\underline{k},\alpha,\underline{k}'\,\alpha',\underline{p},\beta,\underline{p}',\beta',\underline{q}), \tag{2}$$

where $(\underline{k}\,\alpha)$, $(\underline{k}'\,\alpha')$, $(\underline{p}\,\beta)$ and $(\underline{p}'\,\beta')$ are shorthand for the intrinsic mo.· menta and intrinsic quantum numbers ·of the constituents" s, s' are the z-projections of the spin of the system and $\underline{Q}$ is the total mOlnentum of the system. The intrinsic non relativistic wave function $\varphi_{\underline{Q},s}$ is of course independent of $\underline{Q}$, which has been written having in mind the relativistic case to be discussed later.

If the potentials describing the interaction among the constituent:s have a Fourier transform with a power fall off, the function G .has such a structure[3] that the momenta $\underline{k}$, $\underline{k}'$, $\underline{p}$, $\underline{p}'$ only' appear in linear combination with the momentum transfer $\underline{q}$. This allows to replace in the Taylor expan,sion of G with respect to $\underline{k}$, $\underline{k}'$, $\underline{p}$, $\underline{p}'$, derivatives with respect to the components of these vectors by derivatives with respect to the components of $\underline{q}$ so that one has for G the asymptotic expansion[3]

$$G \sim \sum_{\substack{m,m',n,n' \\ \mu,\mu',\nu,\nu'}} q^{-4\gamma-m-m'-n-n'} P^{(m)}_{\mu}(\underline{k},\alpha)\, P^{(m')}_{\mu'}(\underline{k}',\alpha') \cdot$$

$$\cdot\, P^{(n)}_{\nu}(\underline{p},\beta)\, P^{(n')}_{\nu'}(\underline{p}',\beta'), \tag{3}$$

where $P^{(n)}_{\nu}(\underline{k}\,\alpha)$ is a homogeneous polynomial in the components of $\underline{k}$ of degree n and γ is a number depending on the number of the constituents and the form of the potentials. A similar expansion *in* homogeneous polynomials can be done for the $\varphi_{\underline{Q},s}$[4]. The leading term in eq. (1) is determined by the minimum possible value of the exponent n (denoted by K_{min}) appearing in the expansion of the Ψ_Q's

$$F(q) \sim q^{-2\gamma - 2K_{min}} . \tag{4}$$

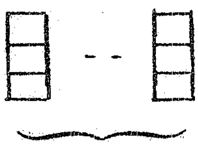
For boson constituents $K_{min} = 0$, while for fermion constituents K_{min} is determined by the requirement that $P^{(K_{min})}$ should be completely anti-symmetric in the variables of the constituents and should have the Same total quantum numbers of Ψ_Q's. It is precisely throughout K_{min} that the statistics of the constituents affects the asymptotics of the form factors.

In the relativistic case eq. (2) still holds, with the replacement of momenta by four momenta, and also the asymptotic expansion (3) can be written down provided the relativistic function G has the same properties of the non relativistic G. What follows refers to relativistic models for which this is true, so that

$$F(t) \sim t^{-(\gamma + K_{min})} . \tag{5}$$

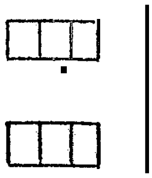
The value of K_{min} depends on the number of degrees of freedom of the constituents. This fact can be used in order to obtain a check in the physics of nuclei of the color hypothesis introduced in the quark model of hadrons[5]. According to this hypothesis each nucleon is composed of three fermion quarks whose color wave function belongs to the fundamental re-presentation of SU3. A nucleus of Z protons and A-Z neutrons is then composed by A + Z u quarks and 2A - Z d quarks. The total nuclear wave function must be completely antisymmetric and its color part must be an SU3 singlet. The same requirement must be of course imposed on the structure of the homogeneous polynomials $P_\nu^{(n)}$.

The only way to have a colour singlet out of the 3A coloured single quark wave functions is to combine them according to the Yang pattern



A times

It then follows that fully antisymmetric polynomials can be obtained if their space, spin, isospin parts have the permutational symmetry of the conjugate pattern



A times.

Kmin is the minimum degree in the components of the quarks momenta of a polynomial satisfying the above requirement and also having the same total spin and isospin as the nuclear wave function.

In the following we evaluate Kmin imposing only the constraints due to statistics. Enforcement of the conditions on the total spin and isospin can only increase the value of K_{min} making the effect of statistics more pronounced.

Let us denote by N_n ($n = 0, 1, \ldots$) the number of independent (including spin) homogeneous polynomials of degree n in the components of the momentum of a single quark. The maximum values of n necessary for u and d quarks respectively are the minimum values ν_u and ν_d respectively satisfying the unequalities

$$\sum_{n=0}^{\nu_u} N_n \geq \left[\frac{A+Z+2J}{3}\right], \qquad \sum_{n=0}^{\nu_d} N_n \geq \left[\frac{2A-Z}{3}\right], \tag{6}$$

where $[x]$ = integral part of x.

K_{min} is then given by

$$K_{min} = 3\nu_u\left[\frac{A+Z+2}{3}\right] + 3\nu_d\left[\frac{2A-Z}{3}\right] - 3\sum_{n=0}^{\nu_u-1}(\nu_u - n)N_n - 3\sum_{n=0}^{\nu_d-1}(\nu_d - n)N_n, \qquad Z < \frac{A}{2}. \tag{7}$$

The value of N_n depends on the relativistic model to be used. If the relativistic wave function depends on three-vectors[6], $N_n = (n+1)(n+2)$. If on the contrary the relativistic wave function depends on four vectors" $N_n =$ $= \frac{1}{3}(n+1)(n+2)(n+3)$. In both cases $K_{min} = 0$ for $A \lesssim 4$. As an example we quote that for the ligthest stable nucleus with $A > 4$, i. e" ^6Li, $K_{min} = 6$ in both cases. Applying for instance eq.(1) to ^6Li, one would obtain $F(t) \sim t^{-17}$, compared to $F(t) \sim t^{-23}$ including the effect of the stan sties.

REFERENCES. -

1 - R. G. Arnold, B. T. Chertok" E. B. Dally" A. Grigorian" C. L. Jordan" W. p. Schültz, R. Zdarko, F. Martin and B. A. Mecking" Phys. Rev. Letters 35, 776 (1975).

2 - S. J. Brodsky and G. R. Farrar, Phys. Rev. Letters 31, 1153 (1973).

3 - I. M. Narodetsky" F. Palum.bo and Yu. A. Simonov" Phys. Letters 58B" 125 (1975).

4 .. Yu. A. Simonov, in !The Nuclear Many Body Problem!1 (F. Calogero and Ciofi degli Atti, Editors), Roma (1972).

5 - O"W. Greenberg, Phys. Rev. Letters 13" 598 (1964).

6 - R"Blankenbecler and R.Sugar" Phys. Rev. 142, 1051 (1966); C. Itzykson, V. G" Kadyshevsky and I. T. Todorov, Phys. Rev. D1, 2623 (1970).