Contribution to the
"IXth Intern. Conf. on
High Energy Accelerators"
Stanford - May 2-7, 1974

COMITATO NAZIONALE PER L'ENERGIA NUCLEARE
Laboratori Nazionali di Frascati

LNF-74/19(P)
23 Aprile 1974

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RESONANT METHODS FOR BEAM SIZE CONTROL IN STORAGE RINGS

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Abstract.

Three methods for varying the magnetic structure of a storage ring at constant betatron number Q_x, in order to vary the natural beam dimensions, are discussed. It is also shown that beam-beam interaction in the thin lens approximation, and dipole or gradient errors can be interpreted as magnetic structure modifications of the kind discussed.

Introduction.

In colliding beams storage rings physics, the function[1]

$$H(s) = \frac{1}{\beta(s)}\left[\psi^2 + (\beta\psi' - \frac{1}{2}\beta'\psi)^2\right] \tag{1}$$

ψ being the periodic solution (continuous with its derivative) of equation

$$x'' + K(s)\,x = \frac{1}{\varrho(s)} \quad , \tag{2}$$

is particularly important. The local value of H(s) at the crossing point is simply given, due to the symmetry of the optical functions, by

$$H_o = \frac{\psi^2}{\beta} \tag{3}$$

and its average value in the bending magnets is given by

$$\bar{H} = \frac{1}{2\pi}\int \frac{ds}{\varrho(s)}\,H(s) \tag{4}$$

The importance of $\bar{H}$ and H_o stems from the dependence of beam dimensions at the crossing point on these parameters (we assume $J_s = 2$, $J_x = J_z = 1$), namely

$$\sigma_x = \sigma_p\sqrt{\beta_x\left[\frac{2\bar{H}}{1+\varepsilon^2} + H_o\right]} \tag{5}$$

$$\sigma_z = \sigma_p\sqrt{\beta_z\,\bar{H}\,\frac{2\varepsilon^2}{1+\varepsilon^2}} \tag{6}$$

ε being the coupling parameter between radial and vertical betatron oscillations ranging from 0 (no coupling) to 1 (maximum coupling).

In order to obtain a satisfactory behaviour of luminosity in the low-energy range, it is necessary to increase the beam cross-section at the interaction point. It is apparent from 5) and 6) that this can be obtained by the increase of $\bar{H}$ or of H_o (high ψ). In this paper we propose a resonant method to control parameter $\bar{H}$.[2]

It is convenient to transform formulas (2) and (4) by means of the Floquet transformation

$$\begin{cases} \eta = \dfrac{\psi}{\sqrt{\beta}} \\ \phi = \dfrac{1}{Q_x}\displaystyle\int \frac{ds}{\beta} \end{cases} \tag{7}$$

Formulas (2) and (4) become:

$$\frac{d^2\eta}{dQ^2} + Q_x^2\,\eta = Q_x^2\,\beta^{3/2}\left(\frac{1}{\varrho}\right) \tag{8}$$

$$\bar{H} = \frac{1}{2\pi}\int \frac{ds}{\varrho}\left[\eta^2 + \frac{1}{Q_x^2}\left(\frac{d\eta}{d\phi}\right)^2\right] \tag{9}$$

Starting from a symmetry point, the right hand side of (8) can be written as

$$Q_x^2\,\beta^{3/2}\,\frac{1}{\varrho} = f(\phi) = \sum_{k=0}^{\infty} A_k \cos k\phi \tag{10}$$

and the solution of equation (8) is:

$$\eta = \sum_{k=0}^{\infty} \frac{A_k}{Q_x^2 - k^2}\cos\ \phi \tag{11}$$

It should be kept in mind that the independent variable in formula (11) is ϕ and not s.

Resonant methods.

It can be seen from (9) and (10) that, keeping Q_x constant, $\bar{H}$ is a function of coefficients A_k, in particular of those corresponding to k values near to Q_x: this observation suggests that, to control $\bar{H}$, it is necessary to have a degree of freedom in the ring magnetic structure allowing to modify the value of coefficients A_k near to Q_x.

The machine magnetic structure can be varied in three different ways, which will be illustrated referring as an example to the structure of the preliminary project of Super Adone [3], and namely:

A) Variation of the standard cells and the low-β insertion relative contributions to Q_x.

B) Modification of the standard cell dipole term $1/\varrho(s)$[4].

C) Modulation of the standard cell gradient function k(s).

In our example we have 32 standard cells, each beginning from the center of a straight section in between two defocusing quadrupoles Q_D and two inser tions defined by the conditions to which they must satisfy.

A) Starting from a configuration with $Q_x = 10.2$ (8.2 from the standard cell and 2.0 from the two low-β insertions), the total standard cell contribution to Q_x is raised to 9.2 and that of the two insertions decreased from 2.0 to 1.0, thus keeping the total value of 10.2 constant. We also assume that the insertions do not contain bending magnets.

In addition to producing the low-β at the interaction points, the insertions must allow to obtain:

1) Variable Q_x values
2) Fixed Q_z values.

In this case we do not want any matching between the ψ values at the end of the low-β insertion and at the beginning of the standard cells: the periodic solution of equation (2) must be calculated on the whole machine.

Let us consider the quarter of the ring, extending from symmetry point A to low-β point B at the center of the experimental straight section. Function ψ can be considered as being the sum of two terms: the first (ψ_1) is a solution of eq. (2), as applied to the standard cell only: if ψ_1 were propagated inside the low-β insertion, it would have a non-zero derivative at the crossing point; the second term (ψ_2) is a free betatron oscillation, whose amplitude at point A, is such that at points B its derivative has the opposite sign with respect to the derivative of ψ_1. The sum of the two terms is still a solution of eq. (2) and represents the total ψ. The increase of $\bar{H}$ is due to ψ_2.

The behaviour of $\bar{H}$ and of the momentum compaction α_c as a function of Q_{xp} (contribution to Q_x of standard cells) is shown in Fig. 1. In this case the degree of freedom is Q_{xp}.

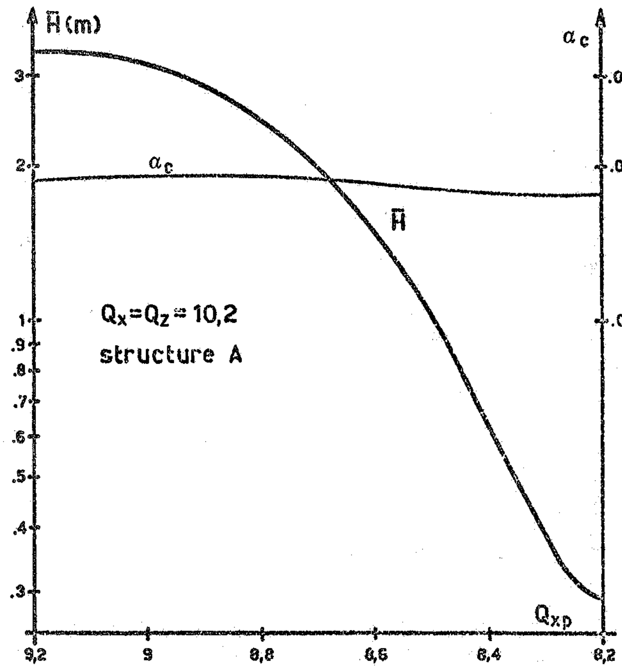

FIG. 1

B) The variation of the standard cell dipole term $1/\varrho$ can be obtained by replacing parts of the bending magnets with straight sections and viceversa. This can be achieved by introducing either a high superperiodicity (e.g. a superperiod of 4 standard cells in which the length of each magnet is slightly changed) or a low superperiodicity, equal to the number of insertions in which some whole standard cell has the bending magnets replaced by straight sections.

As an example we refer to the periodic structure described above and consider a 4-cell superperiod. We assume that the magnets of the first and of the fourth cell are .2 m longer, and those of the second and third cell .2 m shorter than those of the normal structure. The natural degree of freedom of this second kind of magnetic structure would be the lengthening of the magnets, and is obviously impossible to adopt in practice: the lengthening (and shortening) of the magnets will therefore be kept fixed and equal to .2 m; as one degree of freedom we choose the standard cells contribution to Q_x, the total Q_x being kept constant, by properly varying the low-β insertion contribution to Q_x. The same 2 conditions of case A) and a third condition, namely (3) matching of the standard cells ψ value to that of the low-β insertion have to be satisfied.

This condition requires that some bending magnets exist in the low-β insertion: the bending magnet radius must therefore be changed, to compensate for the additional bending angle of the insertion magnets.

The behaviour of $\bar{H}$ and of the momentum compaction α_c as a function of Q_x are shown in Fig. 2.

C) The variation of the standard cell gradient function K(s) is suggested by the observation that in formula (10) function $f(\phi)$ can be modified either by changing the dipole term $1/\varrho(s)$ or function $\beta(s)$ (which is influenced by the gradient function K(s)). It should be pointed out that according to (7) the change in $\beta(s)$ entails a change of function $\phi(s)$ so that functions $1/\varrho(\phi)$ and $\beta(\phi)$ are both modified.

We introduce (as in case b)) a superperiod of 4 cells. In this case a degree of freedom is obtained in the following way: if we call K_F and K_D the values of the gradient function in the focusing and defocusing quadrupoles of the first and the fourth cell, the corresponding values in the second and third cell are replaced by:

$$K_F(1-p)$$

$$K_D(1+p)$$

p can be easily varied by properly varying the quadrupole currents, and the low-β insertions are very simple, because is sufficient for the insertions to correspond to an identity matrix to obtain the proper matching of functions $\beta(s)$ and $\psi(s)$.

Among the three possible kinds of structure modification, this seems to be the most promising, and will therefore be analyzed in some detail.

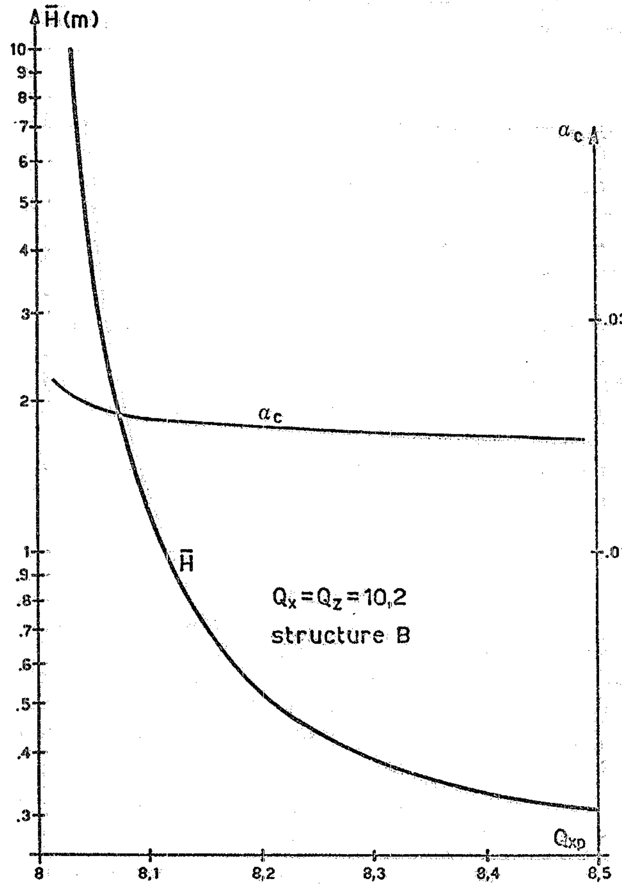

FIG. 2

We assume $Q_x = Q_z = 10.2$. The behaviour of $\bar{H}$ and α_c as a function of p are shown in Fig.3, while σ_x, the maximum r.m.s. radial dimension divided by the energy (in GeV) along the periodic structure, and σ_z , the maximum r.m.s. vertical dimension on the coupling resonance are shown in Fig. 4. In order to get an idea of the requirements on the low-β insertions functions β_x, β_z and the off-energy function ψ, evaluated at the beginning of the low-β insertion are shown, as a function of p, in Fig. 5. The same quantities for $Q_x = Q_z = 11.2$ are shown in Figs. 6, 7, 8. It can be seen from Fig. 5 and Fig. 8 that beam dimensions can be easily increased by factors of the order of 4.

Effect of beam-beam interaction on beam dimensions.

Let A and B be the same points defined above. Following the optical model, we assume that beam-beam interaction can be described by means of a thin lens defined by the approximate tune shifts ξ_x and ξ_z in the radial and vertical planes.

Labelling with an asterisc the functions modified by beam-beam interaction, it is easy to obtain the following formulas[5].

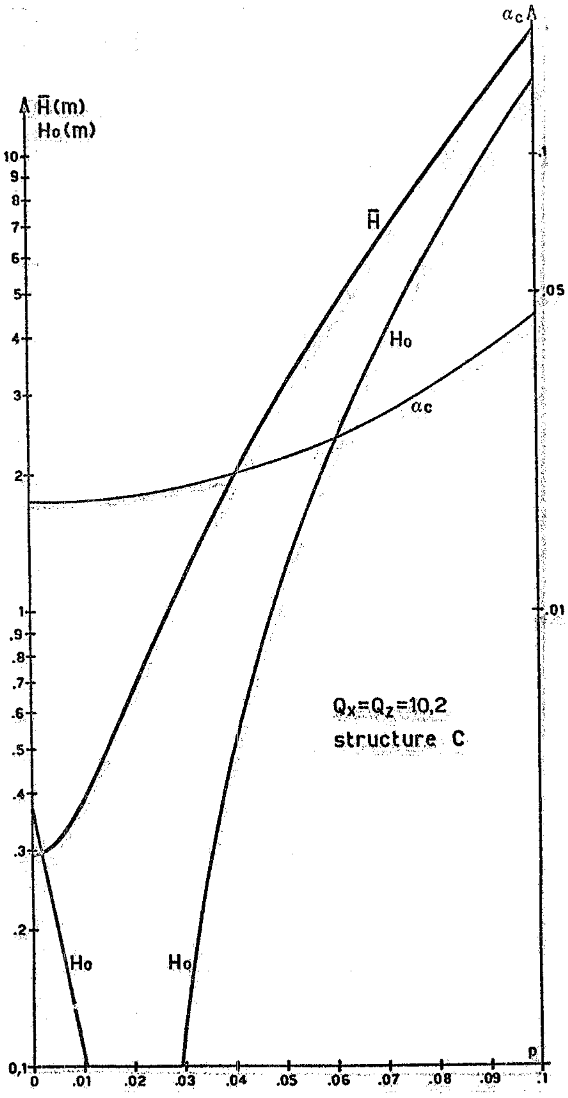

FIG. 3

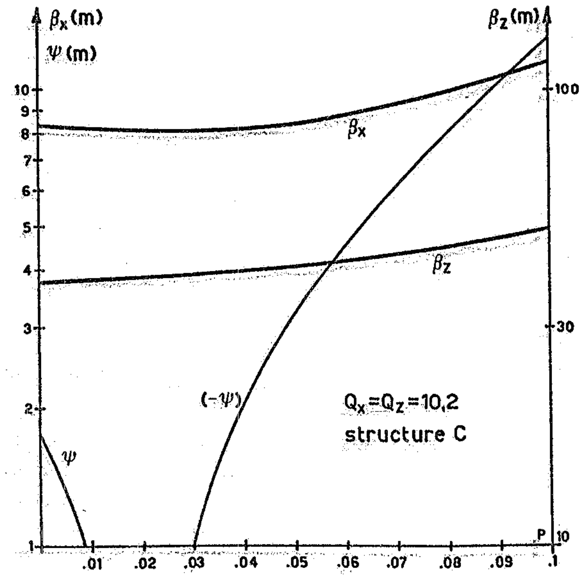

FIG. 4

$$\begin{cases} \psi^{*}_{A} = \psi_A - \psi_B \left(\frac{\beta_A}{\beta_B}\right)^{1/2} \dfrac{2\pi\xi_x}{\operatorname{sen}\frac{\pi}{2}Q_x + 2\pi\xi_x \cos\frac{\pi}{2}Q_x^{*}} & (13) \\ \psi^{*}_{B} = \dfrac{\psi_B}{1+2\pi\xi_x \operatorname{cotg}\frac{\pi}{2}Q_x} & (14) \\ \dfrac{d\psi^{*}_{B}}{ds} = \dfrac{2\pi\xi_x}{\beta_x}\psi^{*}_{B} & (15) \end{cases}$$

$$\begin{cases} \cos\pi Q^{*}_{x,z} = \cos\pi Q_{x,z} - 2\pi\xi_{x,z} \operatorname{sen}\pi Q_{x,z} & (16) \\ \beta^{*}_{A_{x,z}} = \beta_{A_{x,z}} \left(\dfrac{\operatorname{tg}\frac{\pi}{2}Q_{x,z}}{\operatorname{tg}\frac{\pi}{2}Q^{*}_{x,z}}\right) & (17) \\ \beta^{*}_{B_{x,z}} = \beta_{B_{x,z}} \left(\dfrac{\operatorname{sen}\pi Q_{x,z}}{\operatorname{sen}\pi Q^{*}_{x,z}}\right) & (18) \end{cases}$$

We want to show how the interaction entails a variation of $\bar{H}$ with a mechanism similar to A). At point B there is a thin lens with an intensity given by $4\pi\xi_x/\beta$: the derivative of function $\psi^{*}(s)$ at point B must therefore be given by eq. (15) in order to mantain the required symmetry. On the other hand, nothing has changed in the ring, magnetic structure except for point B, so that the difference between functions ψ and ψ^{*} can only be a free betatron oscillation, symmetric with respect to A and with an amplitude such as to have the right derivative in B. The difference between ψ_A and ψ^{*}_{A} and the variation of function $\beta(s)$ modify the value of $\bar{H}$. The variation of $\bar{H}$, ψ_B and β_B shows that beam-beam interaction (in the thin lens approximations) strongly changes the beam dimensions. It should be pointed out that for $\xi_x = 8\times10^{-2}$ the quantity $2\pi\xi_x$ has a value of 0.5. All these effects strongly change if Q_x is near an even rather than to an odd integer; the choice between an even and an odd integer is therefore difficult: as an example with Q_x even, the longitudinal beam-beam limit is less severe but the value of ψ at the crossing point is strongly decreased.

Effect of dipole or gradient errors on beam dimensions.

For completeness, we want to point out that errors in the dipole function $1/\varrho(s)$ and in K(s) can be interpreted as applications of methods B) and C): it would seem that the methods we have described do not apply to the case of a single dipole perturbation, which gives an error orbit in the form of a betatron oscillation along the ring, but it is sufficient to make a transformation into the reference system of the error orbit to get a term $1/\varrho(s)$ on the right hand side of eq. (2), which represents the bending effect of the off-axis quadrupole modulated by the betatron oscillation: the situation is therefore similar to that described in B).

In Adone a vertical error orbit[6], due to a single dipole perturbation with a maximum amplitude of 2 cm, gives, for $Q_z = 3.05$ a value of $\bar{H}_z$ leading to the same vertical dimensions as full coupling. For the case of gradient errors, it can be shown that the maximum tolerated gradient error in the experimental magnetic detector (MEA), which is equivalent to a ξ_x value of 3.10^{-3}, entails a 12% variation in the function ψ for $Q_x = 3.05$: this can be seen from (14), taking into account that there is only one perturbation. This perturbation therefore excites a betatron oscillation in ψ, with an amplitude equal to 12% of the unperturbed ψ. The ψ value at two successive crossing regions, about half a betatron wavelength apart, differ therefore by 25%.

Conclusions.

In this paper some resonant methods to control beam dimensions through the variation of the parameter $\bar{H}$ defined by (1) and (4) have been illustrated. Among the 3 proposed methods, the third, namely a modulation of the gradient function K(s), seems, to be the most promising.

Studies on this subject have been also carried out at SLAC, by D. Helm[7].

Obviously, this solution for beam dimension control must be compared or integrated with other proposed methods, like the high ψ value at the crossing point and the variation of Q_x with energy, as proposed initially by F. Amman[8] for the Super Adone project, and adopted for the SLAC 15 GeV electron-positron storage ring project[9] and for EPIC[10], for which the variation of J_x and J_z, (partition numbers of betatron damping constants) or a vertical $\bar{H}_z$ are also proposed.

The first and the third methods, in particular the third, can also be applied to existing machines. In ref. 11 experiments on Adone with a configuration of 6 shunted quadrupoles are described. It would be desirable to make similar experiments at other machines in order to look for possible difficulties not at present foreseen.

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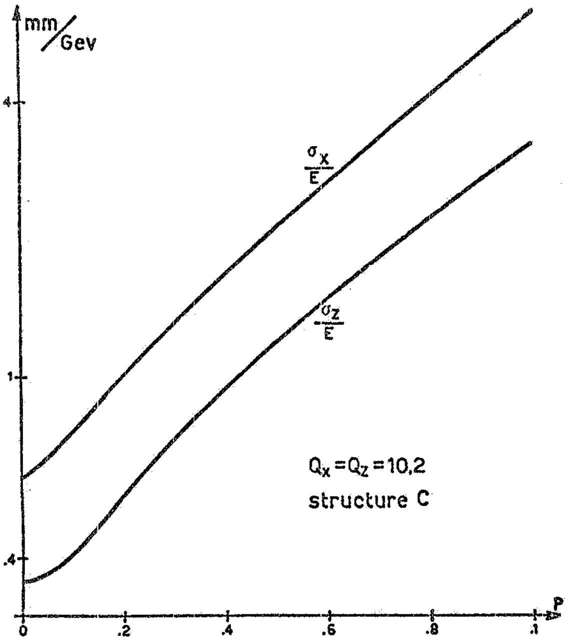

FIG. 5

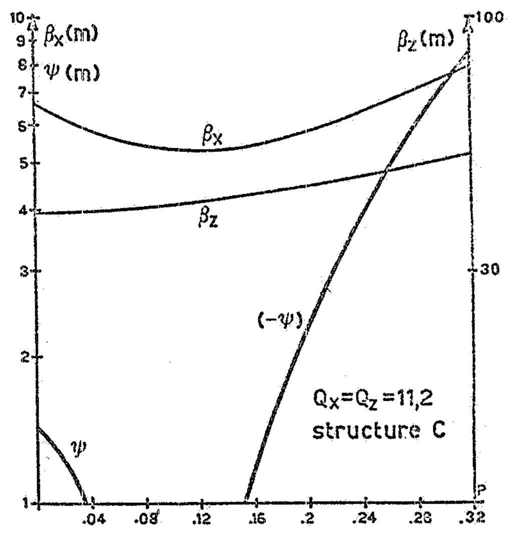

FIG. 7

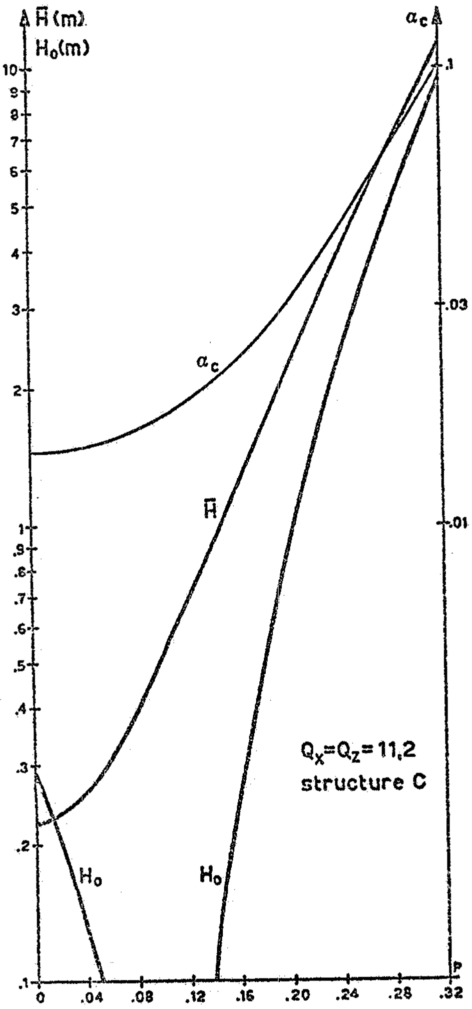

FIG. 6

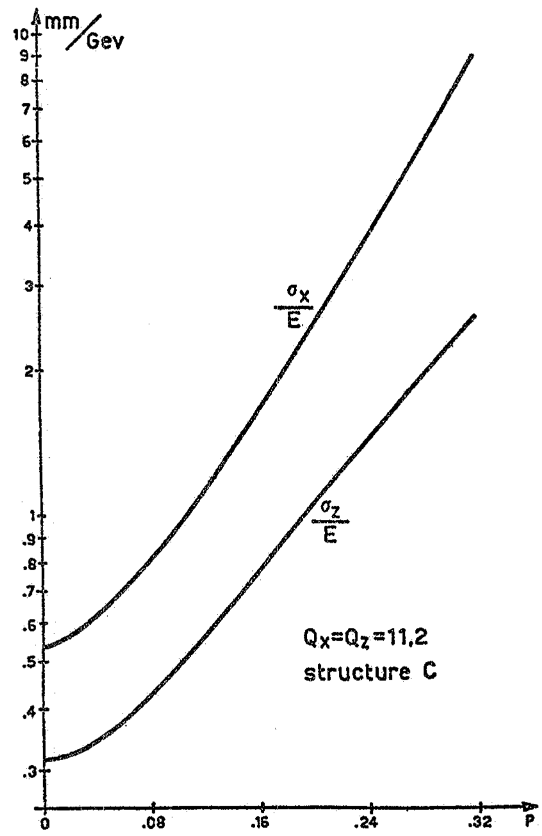

FIG. 8