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M. V. Ricci: INTRODUCTION NOTES ON THE STABILIZATION OF SUPERCONDUCTING MATERIALS. -

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M. V. Ricci: INTRODUCTORY NOTES ON THE STABILIZATION OF SUPERCONDUCTING MATERIALS. -

(Lectures given at the Intern. School of Applied Physics "Developments of Applied Superconductivity", Erice, April 1969; to be published on the Proceedings of the School).

I. - INTRODUCTION. -

The construction of superconducting magnets and other electrical machines with zero steady-state energy dissipation, was made possible, at the beginning of the Sixties, by the discovery of hard superconductors.

The importance of the attainment of this goal is illustrated by the following example: to generate a magnetic field of the order of 10^5 G in a volume of few cubic centimeters, in the case of a superconductive magnet one needs some tens of liters of liquid helium and a current of the order of 100 A, in the case of a room-temperature magnet, a power in the range of megawatts is required, as well as a cooling water flux of some thousands of liters per minute.

However the use of superconductors for winding magnets has been strongly retarded by the fact that, until recently, designing a superconductive magnet had nothing to do with the application of physics and engineering principles.

The only way to be sure that a newly planned magnet had the necessary characteristics (namely ability to carry the current required to produce the wanted maximum field) consisted in making it identical, or at least very similar to one already tested. Small changes in dimensions or shape resulted in the impossibility to predict the performance sometimes within 50%. This may be accepted when building small coils, but is intolerable in larger systems, where investiments in materials and labor are considerable.

Let us now consider which are the difficulties that one encounters.

In a superconductor, the current flows without dissipation only if it is smaller than the critical current I_c. I_c depends on many parameters some of which are intrinsic to the material (type of superconductor, thermal and metallurgical treatment ...) and some are connected to its environment (temperature, magnetic field). For a given material, I_c is usually measured as a function of the field H at a fixed temperature (in most cases at boiling liquid helium temperature: $4.2^{o}K$). To do this, a short piece of wire or ribbon of the superconductor is put in a transversal magnetic field, and the voltage at its ends is monitored while an increasing current is driven through it. The short-sample-critical-current is defined as the one above which a measurable voltage develops across the sample. Repeating the measure for different values of the field, a curve $I_c(H)$ is obtained. (e. g. Fig. 1)[1].

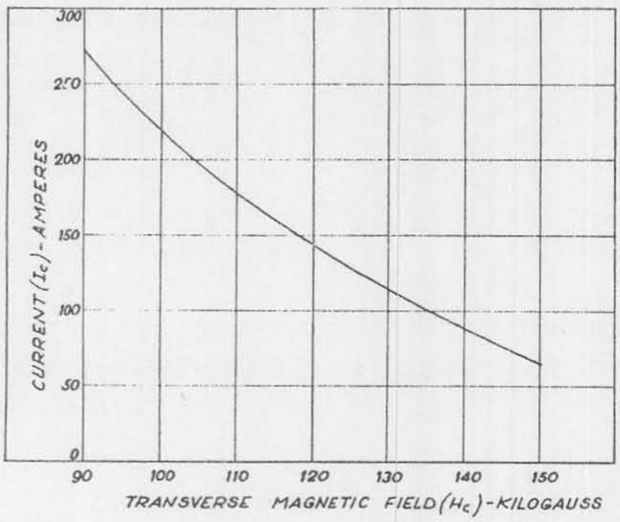

FIG. 1 - Critical current versus transverse magnetic field of a Nb_3Sn ribbon[1].

Having this plot, it would seem possible to design a superconducting magnet in a very straightforward way: after reading from the plot the maximum superconducting current $I_c(H_o)$ for that wire in the required magnetic field H_o, the number of turns necessary to obtain H_o is readily calculated. But if a magnet is built in this way, one finds that the transition to the normal (dissipative) state, occurs at a current much lower than $I_c(H_o)$ so that also the maximum field obtainable is much smaller than H_o. For example, a heat treated Nb-Zr wire that in short sample carried 220 A at 20 KG, when wound in a coil carried no more than 2 A[2]. This effect, called degradation, increases with the dimensions of the magnet, as well as with the complexity of its structure (split-coil, quadrupole ...). These notes intend to illustrate how far our understanding of this phenomenon is today and to show how the various sources of degradation of coil-wound superconductors can be eliminated or at least their effects reduced to a level which allows the achievement of two results:

1) that the coils, independently of size or shape, can work close to the short sample critical current;
2) elimination of uncontrolled transitions to the normal state which mean the destruction of the magnetic field and the vaporization of liquid helium.

The importance of point 2) is connected with the fact that the electrical conductivity of a superconductor in the normal state is very low, about one thousandth of that of pure copper (it is precisely the decrease in the free path of the electrons which leads in the final analysis to the increase of the critical fields of these superconductors).

Let us consider what happens when a portion of the winding of a magnet goes over into the normal state. The Joule-heat generated in the normal zone raises the temperature of the adjoining turns above the critical temperature; the normal zone grows until the whole energy of the magnetic field is expended. The maximum temperature at the center of this zone depends on the decay time of the current in the solenoid, the heat conduction and the heat capacity of the windings and can amount, even in the case of small solenoids, to hundreds of degrees(3). This makes possible irreversible changes (annealing), overvoltages, destruction of the insulation as well as of part of the winding. A large quantity of liquid helium is vaporized and this may create problems of overpressure. (One liter of liquid helium corresponds to about 700 liters of gas).

Superconductors which satisfy the requirements 1) and 2) are called stabilized. The performance of magnets wound with them is predictable within the usual engineerging tolerance.

II. - DEGRADATION. -

What can be the reasons for degradation? How do the conditions of the wire in a winding differ from those under which measurements of the critical currents of a short length of the same wire are made?

The first difference lies in the fact that when $I_c(H)$ is measured for a short sample, one of the parameters (in most cases H) is kept constant, whereas in a solenoid I and H change simultaneously. Indeed measurements on short samples with proportionally increasing current and field have been made(4), and showed a considerable amount of degradation. A second difference is due to the electromagnetic influence of neighbouring turns (proximity effect). Finally one must take into account that the wire in a coil is not as effectively cooled as a sample freely immersed in liquid helium. This last difference indicates rather clearly the possibility of dissipative processes in the superconductor. What happens inside a superconducting coil that is able to produce heat?

II. 1. - Mechanical effects. -

In a coil, the conductors are strongly pressed together by the electromagnetic forces. If, owing to a change of the magnetic field or a thermal contraction, two conductors slip one over the other, the heat li berated in friction may be enough to rise the temperature above the critical temperature. If a load of 100 Kg/cm^2 corresponding to a magnetic field of 50 KG, slips by 0.1 mm, the liberation of heat is 0.1 Joule per cm^2 of slipping surface. This is able to raise the temperature of 1 cm^3 of copper by some 10^oK at liquid helium temperature(5). Care must then be taken, during the winding of a solenoid, to fasten the turns as well as possible, compatibly with cooling requirements. This is usually achieved impregnating the solenoid with grease during the construction, layer by layer, to fill partly the gaps between the wires. The grease, which of course solidifies at liquid helium temperature, has a good thermal conductivity compared to other materials. It seems possible to improve this technique(6) by suspending silica grains in the grease, thus improving the heat transfer as well as the mechanical strength.

II. 2. - Flux Jumps. -

It is well known that the capability of hard superconductors to carry high current densities is connected to the fact that they do not expell completely the magnetic field(7). Above the first critical field H_{c1}, which is relatively small, the field penetrates into the bulk of the material in the form of quantized vortex lines. At the center of these vortices the magnetic field is close to the external field and the material is in the normal state. Inhomogeneities, like boundaries, inclusions or dislocations provide the so-called pinning centers, that is potential wells that trap the vortices preventing them from moving under the action of the Lorentz force due to the current. As long as the vortices do not move, the current flows in the superconducting regions around the vortices and no heat is generated. If the Lorentz force overcomes the pinning forces and the vortices begin to move, the current penetrates partly into the normal cores and the usual release of Joule heat occurs(8). The power dissipated per unit volume is(9):

$$\vec{J} \times \vec{E} = \frac{\phi_o}{c} \vec{N} \times (\vec{V} \wedge \vec{J})$$

where $\phi_o = hc/2e = 2.067 \; 10^{-7}$ G cm^2 is the fluxoid quantum, $\vec{V}$ is the average velocity of the vortex lines, $\vec{N} = \vec{B}/\phi_o$ and $\vec{J}$ is the current density.

The probability that a vortex is detached from a pinning point because of thermal excitation is finite and this results in the phenomenon called "flux creep". However sporadically another phenomenon occurs, called "flux jump": a large number of vortex lines move with high velocities ($10^2 \div 10^4$ cm/sec). This process is quasi-adiabatic and causes a large local rise of temperature and sometimes loss of superconductivity. The cause of a flux jump is connected to the fact that the pinning force and the Lorentz force are both decreasing functions of temperature, but the former decreases more rapidly then the latter. Then a local rise of temperature can be able to trigger an avalanche-like process. This initial rise may be due to several cases: flux creep, a sudden variation of field or current, annihilation of two fluxoids with oppositely oriented moments.. ..

III. - STABILIZATION. -

A quantitative and complete theory of stabilization has not yet been made; at the present time there are only some partial solutions which however made already possible the construction of magnets of large dimensions that work reliably.

The stabilization can be complete or partial. Let us consider a superconducting solenoid, with a current I flowing, and H be the field generated. Suppose that an instability or an external disturbance drives the whole winding (or a portion of it) normal. The magnet will be defined completely stabilized if, during this disturbance, it is still able to carry the same current I (so that the field H will remain constant) and if, once the disturbance is over, it reverts to the superconducting state. The stabilization is partial if, by means of suitable artifices, the temperature spikes due to flux jumps are kept below the level which drives the superconductor normal, or if the occurence of flux jumps is completely avoided.

In steady conditions, a partially stabilized magnet will not quench, exactly like a completely stabilized one, but while the latter can withstand any perturbation, the former will not recover to the superconducting state once driven normal by an external disturbance.

We will see that a very important role, on the stability problem, is played by the heat transfer characteristics of boiling liquid helium. Depending on the temperature difference between the surface of a heated solid and the liquid helium, two regimes are possible: nucleate - boiling and film-boiling.

Fig. 2 shows heat transfer data obtained for a cable in liquid helium[10]. The temperature difference ΔT remains small (nucleate - - boiling) as long as the power dissipated is below a critical value q_n at which a sudden transition occurs to the film-boiling regime, characterized by a much higher ΔT. This happens because the cable gets covered by a continuous film of gas which makes the heat transfer coefficient drop to a lower value. Reducing the power again, nucleate boiling is resumed at a power $q_f < q_n$, called minimum film boiling heat flux.

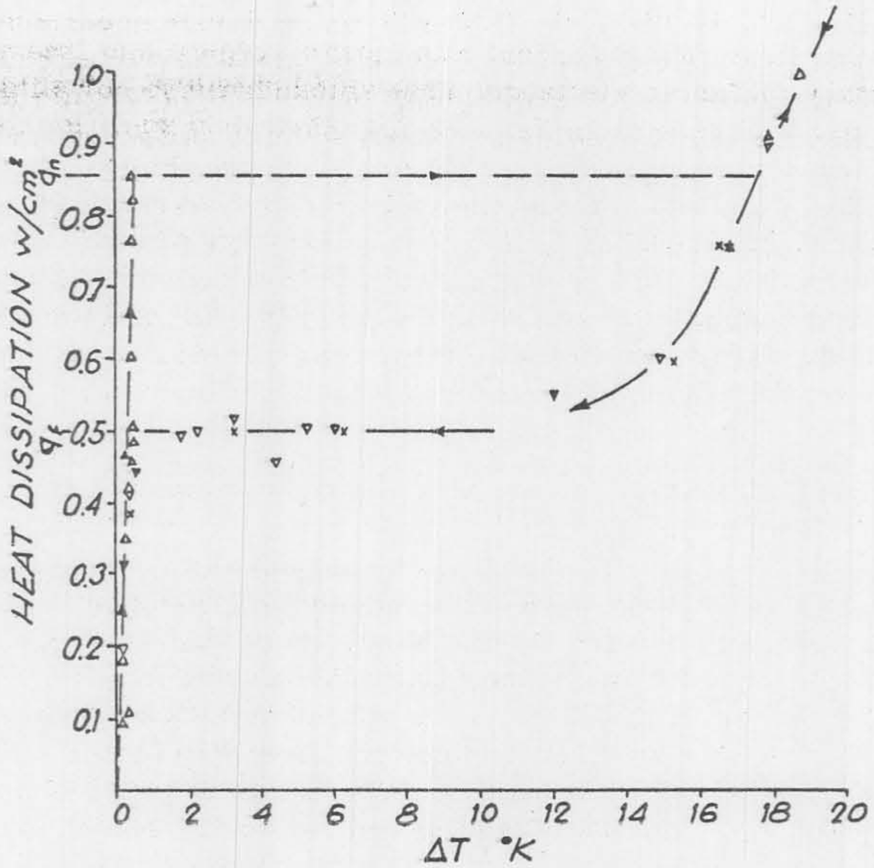

FIG. 2 - Heat transfer from a cable to liquid helium

IV. - COMPLETE STABILIZATION. -

In the technique of complete stabilization, the superconductor is heavily coated with a normal material of high electrical and thermal conductivity (like copper or aluminum), in order to reduce the average resistance of the conductor and to increase the heat conduction. When a flux jump raises the temperature (which the high thermal conductivity contributes to keep in limits) and reduces the current-carrying capacity of the superconductor, the current (or part of it) will be carried by the normal material. If in these conditions the joule heating is low and the cooling is efficient, as soon as the disturbance is over, the temperature will drop again and the current will switch back to the superconductor.

Let us see what is the effect of the normal metal (for brevity we will always say copper, although, as we have already said other metals are also suitable) on the current-voltage characteristic of the superconductor.

Let us put a sample of bare superconductor in a magnetic field H. When we increase the current I that flows through the sample no vol tage across its ends appears until a value of the current is reached at which a sudden transition to the normal state occurs (Fig. 3a). This is due to the fact that the resistivity of the wire, in the normal state, is very high (see Introduction); as soon as a small normal region is created by the current, the Joule heating is sufficient to quench the whole sample. Heat is dissipated by film-boiling. Increasing still the current, we find an I-V characteristic which is linear with a slope determined by the normal resistance R_n.

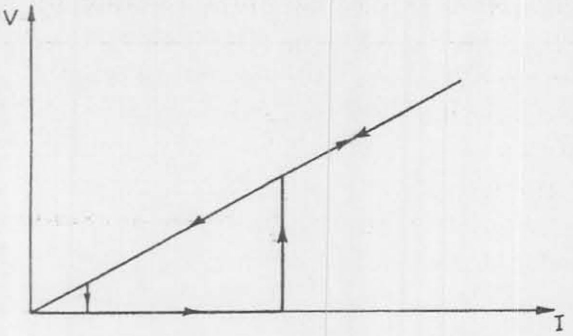

FIG. 3a - Current-voltage characteristics of a bare superconductor.

On decreasing the current, the wire remains normal as long as $I^2R_n > q_f$. As we said R_n is large, and therefore the passage to the nucleate-boiling regime ($I^2R_n < q_f$) occurs for a very low value of I. Once in the nucleate-boiling regime, the temperature is low (see Fig. 2) and allows the wire to go back into the superconducting state (V=0). It is thus clear that this wire is unstable for practically any value of the current: if for any reason it should quench, it is not able to recover.

Let us now coat it with a thin layer of copper. The I-V characteristic becomes that of Fig. 3b. Up to I_c all the current is carried by the superconductor. At this stage the superconductor begins to develop resistance. As we said, this is due to the fact that the Lorentz force, at I_c, equals the pinning force. Vortices begin to move, jumping from a pinning center to another, with a rather low mean velocity ($\sim 10^{-2}$ cm/sec). The heat hereby generated produced a small rise in temperature which reduces the current-carrying capacity of the superconductor; the excess current flows in the copper giving rise to a small voltage. As the total current is further increased the proportion of current in the copper increases. When $I = I_q$ the heat generated is so high that the conductor becomes thermally unstable: transition to the film boiling regime occurs, and the current switches completely over to the copper. Now however the heat produced is only I^2R_{cu}, which can be made much

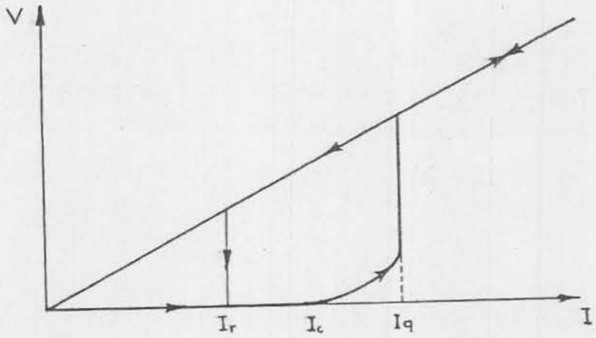

FIG. 3b - Current-voltage characteristics of a stabilized superconductor.

lower than I^2R_n; so that the recovery to the superconducting state happens at a current I_r higher than in the case of Fig. 3a. It should be clear that for values of the current lower than I_r the conductor is completely stabilized: if for any reason it should quench the heat transfer to the helium is so low that the wire will recover its superconducting properties as soon as the disturbance is over.

Increasing the quantity of copper, I_r increases. Clearly the optimum conditions are achieved when $I_r \simeq I_c$.

Let us see now, how the calculation of the quantity of copper necessary to have complete stabilization of a superconductor, up to its critical current, can be made[11].

Let us consider a composite conductor and be A and ρ the cross sectional area and resistivity of the normal metal (substrate). If we define f the fraction of total current I that flows in the substrate, then the voltage per unit length of the conductor is:

(1) $$V = \rho\, If/A$$

In the nucleate boiling regime (see Fig. 2 at low ΔT's) the power diffused to the helium can be written as

(2) $$\frac{dQ}{dt} = h\, P\, (T - T_B) = VI$$

where T is the conductor temperature, T_B is the cooling bath temperature, P is the cooled perimeter of the conductor, and h is a constant heat transfer coefficient. Combining 1) and 2):

(3) $$T - T_B = \rho\, I^2 f/hPA$$

The current I_s that flows in the superconductor is determined by the field H and temperature T. For a given H, a good approximation is:

(4) $$I_s/I_{cH} = 1 - (T - T_B)/(T_{cH} - T_B)$$

where I_{cH} is the short sample critical current in the field H and at the temperature of the bath T_B and T_{cH} is the critical temperature in the same field. Moreover:

$$I = I_s + f\, I$$

Combining these equations we get:

(5) $$f = \frac{I/I_{cH} - 1}{I/I_{cH}\,(1 - \alpha\, I/I_{cH})}$$

(6)
$$\frac{VA}{\rho I_{cH}} = \frac{I/I_{cH} - 1}{1 - \alpha (I/I_{cH})}$$

where α is the stability parameter, defined as,

(7)
$$\alpha = \frac{\rho I_{cH}^2}{hPA\,(T_{cH} - T_B)}$$

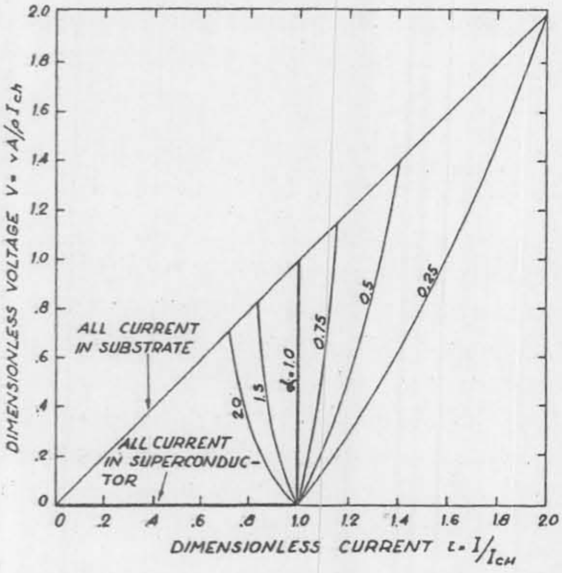

FIG. 4 - Current-voltage characteristic of a composite conductor for several values of α .

Eq. 6) is plotted in Fig. 4 for several values of α .

For $\alpha < 1$, no voltage appears as long as $I \leq I_{cH}$, and for $I > I_{cH}$ the voltage increases gradually with current. The characteristic is everywhere single valued and the wire is completely stable.

For $\alpha > 1$ two regions exist:

$$0 < I < I_{cH}/\sqrt{\alpha}$$

single valued characteristic with all the current in the superconductor ($V = 0$);

$$I_{cH}/\sqrt{\alpha} \leq I \leq I_{cH}$$

double valued operation; all the current in the superconductor or all the current in the substrate (the negative resistance part of the curve is unstable. For $I > I_{cH}$ all the current flows again in the substrate.

It is clear that the conductor is completely stable for currents lower than $I_{cH}/\sqrt{\alpha}$. Above $I_{cH}/\sqrt{\alpha}$, it is still possible for all the current to flow in the superconductor, but should the conductor suffer a disturbance, all the current will immediately switch to the substrate. We have thus found that a conductor is completely stable up to the critical current only if $\alpha < 1$. From the definition of α , we see the convenience of using a low resistivity metal and a cooled perimeter as large as possible. The requirement $\alpha < 1$ is equivalent to the condition:

$$\rho I_{cH}^2 < hPA\,(T_{cH} - T_B)$$

which reads that the heat produced when a current equal to the critical one flows in the substrate, must be lower than the power that can be transfered to the liquid helium when the conductor temperature is just the critical temperature.

We have now examined the operation of a conductor in an external field. To solve the problem of stabilizing completely a coil[11], we must take into account the fact that the local magnetic field varies from point to point so that I_{cH} and T_{cH} vary, and that the substrate may exhibit magnetoresistive effects. Moreover it has been shown that eq. (2) is a crude approximation[12] and this must be taken into account as well.

To test if a coil is completely stable, a common technique is to put a heater inside the winding. While the magnet is working, the heater is switched on, so that part of the winding is quenched. If the magnet is stable, it will carry the same current (and H will remain stable), and will return superconducting as soon as the heater is switched off.

The complete stabilization has been applied to the construction of magnets of large dimensions, among which the Argonne National Laboratory bubble chamber magnet: 18 KG with a working space of 4 m of diameter and 20. 000 liters of volume.

Since a completely stabilized magnet is quite reliable, why isn't this technique always applied? The reason is that, to reach the complete stabilization a large copper to superconductor ratio (e. g. 100 : 1) is required. This lowers considerably the overall current density in the winding. For example in the magnet just mentioned, the current flows in the superconductor (Nb_3Sn) with a current density of 2×10^5 A/cm^2, but the overall current density is only 900 A/cm^2.

There are many applications that require higher current densities, like bending or focusing magnets, lenses. Besides the overall current density plays an important role in determining the cost of the magnet and of cooling. Defining:

l = length of the solenoid
D = diameter of its bore
K = filling factor
λ = current density
N = total number of turns
L = total length of wire in the magnet
V = volume of the coil

we have:

$$H = C\,\frac{NI}{l} \tag{8}$$

$$L = \frac{Hl}{CI}\,\pi\left(D + \frac{H}{K\lambda}\right) \tag{9}$$

$$V = \frac{Hl}{K\lambda}\,\pi\left(D + \frac{H}{K\lambda}\right) \tag{10}$$

When the coil diameter is large ($D \gg H/K\lambda$), the required length L of conductor is independent of the current density λ. This is one more reason to use a completely stabilized superconductor in large magnets. However the volume V of the magnet, which determines the cryogenic expenses, is always dependent on λ. If D is small, both types of costs increase with decreasing λ.

V. - PARTIAL STABILIZATION. -

In a completely stabilized magnet, recovery of its superconducting properties will occur even if the whole winding is driven normal. In practice however, all the types of instability we have seen, are localized, that is the normal regions generated are small and are a transient effect. As we have said, with the partial stabilization, either flux jumps are prevented from occuring or normal zones are prevented from propagating.

We emphasize once more that with this technique an external disturbance or a sudden variation of the current may still have a destructive effect on the field.

VI. - MINIMUM PROPAGATING CURRENT. -

Let's see first how we can affect the propagation velocity of a normal region[13].

To do this, consider, a simplified system: an infinitely long current-carrying film of unit width, deposited on an insulating substrate. If a normal region is formed and starts to propagate it can be assumed that the phase boundary velocity will reach a constant value, that is the temperature profile, shown schematically in Fig. 5, will drift rigidly with a velocity v.

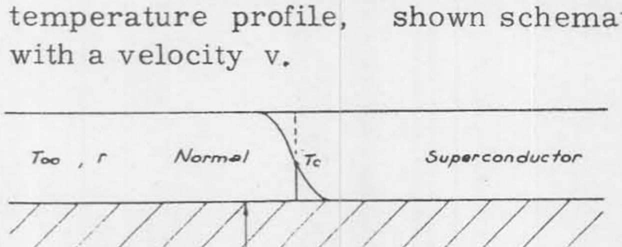

FIG. 5 - Schematic of the temperature profile moving from left to right during thermal propagation.

Nomenclature:

T_B Cooling bath temperature;
T_∞ Maximum film temperature;
T_c Film critical temperature;
D, ρ, S,K Thickness, density, specific heat and thermal conductivity of the substrate;
r film resistence per square.

The thermal conductivity and specific heat of the film we assume to be negligible with respect to those of the substrate.

Let us consider a normal region of length Δx and at temperature T. The power generated is $i^2 r \Delta x$, the heat loss to the helium is $2 \Delta x h (T - T_B)$. The heat flow across any surface at temperature T and position x in the substrate is $KD(\partial T/\partial x)$. Hence the heat increase in the volume of unit width and length Δx is: $KD(\partial^2 T/\partial x^2)\Delta x$. The heat inflow that produces a temperature rise $\partial T/\partial t$ is $D \Delta x S \rho (\partial T/\partial t)$.

Combining all these terms and simplifying we find that the thermal equilibrium is given by:

(11) $$\frac{\partial^2\theta}{\partial x^2} - A\frac{\partial\theta}{\partial t} - B\theta + c = 0$$ in the normal region

(12) $$\frac{\partial^2\theta}{\partial x^2} - A\frac{\partial\theta}{\partial t} - B\theta = 0$$ in the superconducting region

where:

$$\theta = T - T_B \qquad B = 2h/KD$$
$$A = \rho S/K \qquad c = i^2 r/KD$$

With our assumption that the interphase boundary moves with a constant velocity v, the solution must be of the form $\theta = \theta(x - vt)$. Using a reference frame moving at the boundary velocity, and introducing the variable $\xi = (x - vt)$ so that $\theta = \theta(\xi)$,

$$\frac{\partial\theta}{\partial t} = -v\frac{\partial\theta}{\partial\xi}$$

the two equations become:

(13) $$\frac{\partial^2\theta}{\partial\xi^2} + Av\frac{\partial\theta}{\partial\xi} - B\theta + c = 0 \qquad \xi < 0$$

(14) $$\frac{\partial^2\theta}{\partial\xi^2} + Av\frac{\partial\theta}{\partial\xi} - B\theta = 0 \qquad \xi > 0$$

These two equations must satisfy the requirements:

$$\theta \rightarrow \theta_\infty = T_\infty - T_B \quad \text{as} \quad \xi \rightarrow -\infty$$
$$\theta \rightarrow 0 \quad \text{as} \quad \xi \rightarrow +\infty$$

The solutions are:

(15) $$\theta = F\exp\left\{\sigma_1\xi\right\} + \theta_\infty \qquad \xi < 0$$

(16) $$\theta = E \exp\left\{ \sigma_2 \xi \right\} \qquad \xi > 0$$

where

(17) $$\sigma_1 = -\frac{1}{2}\left\{ Av - (A^2 v^2 + 4B)^{1/2} \right\}$$

(18) $$\sigma_2 = -\frac{1}{2}\left\{ Av + (A^2 v^2 + 4B)^{1/2} \right\}$$

and F and E are constants which are fixed by the condition that at the boundary between the two phases θ equals $\theta_c = T_c - T_B$ and $\partial\theta/\partial\xi$ be continuous, i. e.

(19)
$$\theta_c = F + \theta_\infty = E$$

$$\sigma_1 (\theta_c - \theta_\infty) = \sigma_2 \theta_c$$

Combining (17), (18) and (19), we get:

(20) $$v = \frac{1}{A\theta_\infty} (\theta_\infty - 2\theta_c)(A^2 v^2 + 4B)^{1/2}$$

This shows that v can be made positive, zero or negative (that is a normal region will propagate, remain in equilibrium or collapse) according to whether θ_∞ is made greater, equal or smaller than $2\theta_c$.

The problem we have just solved, was a simplified one; in a composite conductor, the substrate is conductive, so joule heat is generated in it as well; the specific heat and thermal conductivity are temperature dependent; only a perimeter P is in contact with the helium.

Furthermore not only the heat transfer equation $\dot{q} = hP(T-T_B)$ is approximated, but it has been shown (14) that h is time-dependent: it takes some time, of the order of few milliseconds, to pass from nucleate to film-boiling. Therefore during short transients heat can still diffuse in the nucleate-boiling regime (that is with a high heat transfer coefficient) even if the temperature difference is much higher than those usually associated with this regime.

Taking into account some of these factors we get(15) for the current below which a normal zone collepses:

$$I_m = \left[\frac{2hP\,(T_c - T_B)}{\rho n/S} \right]^{1/2}$$

ρ_n is the normal resistance of the conductor and S its cross-sectional area. ρ_n/S is the linear resistivity of the conductor and can be substantially reduced by increasing the quantity of normal material around the superconductor. The higher is the conductivity, the lower will be the amount of metal required to get a given stabilized current.

The most suitable metals are Copper and Aluminium. Aluminium has the advantage of a lower magneto-resistive coefficient, but it has worse mechanical properties.

I_m is called "minimum propagating current" and is experimentally determined by two main methods[16]. The first consists in putting a heater in intimate contact with a sample of the conductor (Fig. 6a). For a fixed value of H and I a current pulse is sent through the heater which generates a normal region. Only if $I > I_m$ the normal region propagates and a voltage is measured between points A and B.

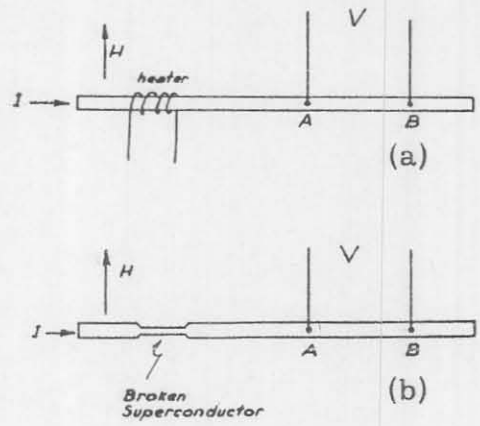

FIG. 6 - The heat-pulse method (a) and the broken-superconductor method (b) to obtain the minimum-propagating-current.

In the broken-superconductor method one measures the V-I characteristic of a short sample, with a length of the superconductor removed from the middle of the sample (Fig. 6b). The break in the superconductor is an artificially induced steady-state normal region which, for its length, forces complete transfer of current into the stabilizing copper. As the current is increased above I_m, a voltage will appear across the taps A and B. This determines I_m. As the current is increased further, current sharing between the superconductor and the copper occur more and more, until the joule heat generated in the copper is so large as to induce the nucleate to film boiling transition. The heat flux at which this runaway occurs depends on the shape and surface conditions of the copper. So the broken-superconductor method, in addition to I_m, gives information about the stabilizing copper.

If the copper is not enough, the film-boiling threshold is reached before propagation and the I-V characteristic is that of Fig. 2a. In this case it can be of help to reduce the bath temperature. In fact it has been shown that below the λ-point I_m is higher[17].

An alternative method of partial stabilization, called Enthalpy stabilization, consists in avoiding the generation of flux jumps that can create normal regions.

There are many solutions of the stability problem, more or less approximated, but they all fall within a range of 20% of each other[18].

For these notes we have chosen Lange's[19] criterion because of its analogy with the classical stability analysis. Following Williams[20], we consider a long cylinder of hard superconductor with an indefinitely small axial bore.

If, starting from a large value, the external field is reduced to zero, a current equal to the critical current flows in the cylinder and the field trapped in the bore obeys the formula:

$$H_i^2/8\pi = \alpha_c\, w/10 \tag{21}$$

where w is the wall thickness and α_c is the Kim, Hempstead and Strnad parameter[21] that depends on temperature like

$$\alpha_c = \alpha_{co}\,(1 - T/T_c)^2 \tag{22}$$

At distance x from the inner wall (here virtually from the axis) the field H obeys the formula:

$$H^2/8\pi = \alpha_c\,(w - x)/10 \tag{23}$$

By integration we can find the stored magnetic energy per unit volume $\mathcal{E}_m$:

$$\mathcal{E}_m = \frac{1}{3}\,(H_i^2/8\pi) = \alpha_c\, w/30 \tag{24}$$

Lange's method consists in considering the change, during a small displacement of the system, in the sum of magnetic (that is potential) and dissipative energies; if the sum falls as a result of the displacement, the system is unstable. The second term is the energy dissipated when the flux slips out against the "friction" of the pinning points, and it is, per unit volume:

$$e_h = \int_0^T \rho\, S(T)\, dT \tag{25}$$

where ρ is the density and S(T) the specific heat. Putting $e = e_m + e_h$, the stability condition is: $de/dT > 0$ whence

$$\rho\, S(T) + de_m/dT > 0 \tag{26}$$

From (24)

$$de_m/dT = \frac{w}{30}\,\frac{d\alpha_c}{dT} = -\,\frac{w}{15}\,\alpha_c/(T_c - T)$$

And (26) becomes

(27) $$\frac{w\alpha_c}{10} < \frac{3}{2}\varsigma S(T)(T_c - T)$$

Since the left hand side is $H_i^2/8\pi$,

(28) $$H_i^2 < 12\pi\rho S(T)(T_c - T)$$

which can be read: the cylinder (coil) is stable up to its critical current (which corresponds to H_i) only if ς , S(T) and $T_c - T$ are such to satisfy that inequality.

Taking for S(T) the expression $S(T) = aT^3$, it is found that the right hand side of (28) has a maximum for $T_M = 3/4\ T_c$, at which temperature:

$$H_i^2 = 3\pi\varsigma T_c\ S(3/4\ T_c).$$

Experiments[22] have actually shown that the current-carrying ability of coils rises as temperature falls, only to fall sharply below a critical temperature.

Based on one-dimensional models, Hancox gives the following result: a coil is stable up to its critical current if

(29) $$JD < (3\ 10^9 \varsigma\ S\ T_o/4\pi)^{1/2}$$

where J is the current density, D the diameter of the wire and T_o is a temperature lower than T_c:

$$T_o = -J_c \frac{dJ_c}{dT}$$

Since I_c is a decreasing function of the magnetic field, materials with high current densities can be stabilized by an externally applied field. This effect was demonstrated by Schrader et al. [23] who showed that a small coil of Nb_3Sn was severely degraded when energized on its own, but carried an increased current when situated in a background magnetic field. This effect can be utilized by constructing a magnet in two or more concentric sections; the inner sections are stabilized by the field of the outer ones and can thus be made with less stabilizing copper. The main disadvantage consists in the increased number of power-supplies and current leads.

From (29) we can see that it is also possible to stabilize a superconductor by increasing the specific heat of the conductor. This can be achieved in several ways. One is the use of a sintered superconductor, in order to take advantage of the specific heat of the liquid helium which impregnates such a conductor, since it is 10^2 times larger than that of metals at that temperature.

A second alternative is to cover the superconductor with lead or indium that have comparatively high specific heats. Since now the stability does not depend directly on the transfer of heat to the cooling bath, operation is possible in gas or even in vacuum, and one can fully impregnate the magnet with grease or epoxy to improve the rigidity of the winding.

Another parameter in (29) that can be varied is the diameter D. Reducing D, the energy dissipated by the flux motion is reduced and the stability improved. If D is small enough, the superconductor is stable even if no normal metal is added to it (Intrinsic Stabilization). It is found however that, for the most common materials, this method requires diameters smaller than 0.1 mm, so that the construction of both the conductor and the magnet is difficult[24].

VII. - OTHER METHODS OF STABILIZATION. -

Experimentally it has been found that partial stabilization can be achieved by interposing a Mylar - copper - Mylar foil-sandwich, or an anodized aluminium foil, between layers of the winding. The higher the electrical conductivity and the thinner the insulator, the better is the stabilization. At first this result was attributed to the damping of the flux motion. In fact, owing to the good conducting wall in close proximity, the magnetic diffusion time can become greater than the thermal diffusion time so that any local heating will be reduced by thermal conduction. It has been recently proved however[25], that the stabilizing effect is primarily thermal in nature rather than electrical. The heat locally generated in the superconductor is transfered to the copper through the Mylar. This heat diffusion mechanism from a solid to a solid, works without the limitation of the nucleate-boiling film-boiling transition which occurs when using liquid helium.

Work has been directed toward finding a thin inorganic insulator (like silver sulphide) in substitution of Mylar to improve the thermal conductivity, and a metal with both higher specific heat and higher thermal conductivity than copper (like cadmium).

Finally we will describe briefly another method of stabilization which has been proposed[26] but not yet applied to magnets. We have seen that jumps occur because the pinning force decreases faster than the Lorentz force with increasing temperature. This situation could be reversed if the pinning centers themselves were weakly superconducting and were driven normal with increasing temperature more rapidly than the matrix. The increasing dissimilarity between the phases would then increase the pinning force, possibly more rapidly than the increased thermal activation of flux out of the pinning centers.

In conclusion we have seen that there are several alternative approaches to the problem of stabilizing a superconducting magnet. So far only the complete stabilization has been used for the construction of large magnets. Conductors whose minimum propagating current is equal to their critical current have been used in several intermediate sized magnets, but some of the finer details of this technique need further study. Enthalpy stabilization has been used in small coils. It must be pointed out that it is difficult to relate theories and experiments since a particular conductor can be stabilized from the point of view of more than one of the theories exposed. This can be of advantage in coil design, but complicates the interpretation of results. Only a better understanding of the complex effects occurring in superconducting coils will help us to push further the optimization of their performance[18].

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