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Large-Angle p-p Scattering, Cerulus-Martin Bound and the Veneziano Model.

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A great deal of both experimental and theoretical efforts has been devoted in the past few years to study the p-p elastic scattering at high energies and large momentum transfers. By looking at the experimental differential cross-section $d\sigma/dt$ ([1-5]) one can distinguish three different regions.

In the first one, for values of $-t \lesssim (1 \div 2)$ (GeV/c)^2 the differential cross-section goes down as e^{At}. Theoretically this is completely understood in terms of diffractionlike scattering, and it is both qualitatively and quantitatively described by means of few Regge poles ([6]).

In the second region, for values of $-t$ lying between ~ 2 and ~ 6 (GeV/c)^2, it has been observed ([7]) that the fixed-s experimental data lie approximately on straight lines, when they are plotted as a function of $\sqrt{-t}$. This is shown in Fig. 1. Theoretical

(1) J. V. ALLABY, G. BELLETTINI, G. COCCONI, A. N. DIDDENS, M. L. GOOD, G. MATTHIAE, E. J. SACHARIDIS, A. SILVERMANN and A. M. WETHERELL: *Phys. Lett.*, **23**, 389 (1966).

(2) J. V. ALLABY, G. COCCONI, A. N. DIDDENS, A. KLOVNING, G. MATTHIAE, E. J. SACHARIDIS and A. M. WETHERELL: *Phys. Lett.*, **25** B, 156 (1967).

(3) C. W. AKERLOF, R. H. HIEBER, A. D. KRISH, K. W. EDWARDS, L. G. RATNER and K. RUDDICK: *Phys. Rev.*, **159**, 1138 (1967).

(4) J. V. ALLABY, A. N. DIDDENS, A. KLOVNING, E. LILLETHUN, E. J. SACHARIDIS, K. SCHLÜPMANN and A. M. WETHERELL: *Phys. Lett.*, **27** B, 49 (1968).

(5) J. V. ALLABY, F. BINON, A. N. DIDDENS, P. PUTEIL, A. KLOVNING, R. MOUNIER, J. P. PEIGNEUX, E. J. SACHARIDIS, K. SCHLÜPMANN, M. SPIGHEL, J. P. STROOT, A. M. THORNDIKE and A. M. WETHERELL: *Phys. Lett.*, **28** B, 67 (1968).

(6) See for instance V. BARGER: *Topical Conf. on High-Energy Collisions of Hadrons*, CERN, January 1968.

(7) M. GRECO: *Phys. Lett.*, **27** B, 578 (1968).

attempts [8-10] to explain these data in terms of Regge cuts have partially succeeded,

The transition from the second to the third region gives rise to the well-known break. The data still lie, at least roughly, on a straight line but the slope is different. To be more precise the following formula:

$$\frac{d\sigma}{dt} \sim \exp\left[-\frac{\sqrt{s}}{T}\right]\left\{\exp\left[-\frac{\sqrt{|t|}}{T}\right] + \exp\left[-\frac{\sqrt{|u|}}{T}\right]\right\}, \tag{1}$$

derived on the basis of general thermodynamical assumptions, has been shown [11] to be in very good agreement with the experiments.

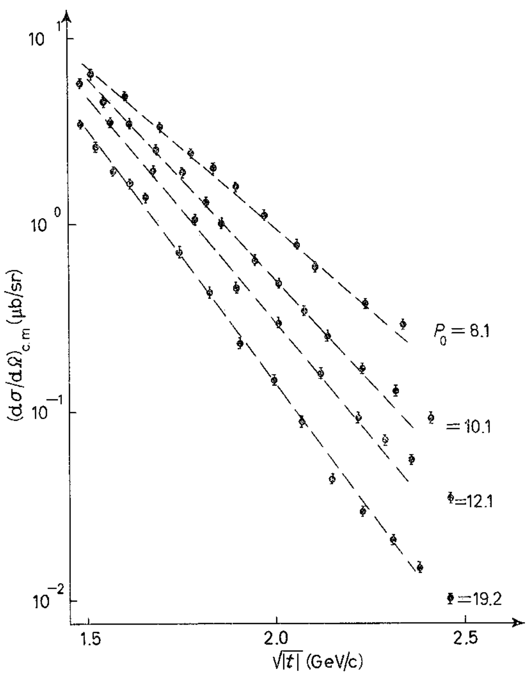

Fig. 1. – p-p differential cross-sections $(d\sigma/d\Omega)_{c.m.}$ in units of 10^{-30} cm²/sr, as functions of $\sqrt{|t|}$. The experimental points are taken from ref. [4,5]. Dashed lines connect the experimental points with the same value of P_0. • ALLABY *et al.*

The most interesting feature of eq. (1) is that, besides the t and u symmetry which takes care of the identical nature of the particles involved, the variables s and t play

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[8] K. HUANG and S. PINSKY: M.I.T. Center for Theoretical Physics Preprint n. 19; K. HUANG, C. E. JONES and V. L. TEPLITZ: *Phys. Rev. Lett.*, **18**, 146 (1967).

[9] S. PINSKY and J. S. TREFIL: *Phys. Lett.*, **27** B, 518 (1968).

[10] S. FRAUTSCHI and B. MARGOLIS: *Nuovo Cimento*, **56** A, 1155 (1968) and the references quoted there.

[11] M. GRECO: *Phys. Lett.*, **27** B, 234 (1968).

a completely symmetric role. This feature reflects the physical fact that at large angles the values of $|t|$ become comparable with s. Recalling now that the usual Regge theory is supposed to be good at small angles, where $|t| \ll s$, if we want to get a satisfactory description of elastic scattering over the full range of t, we have to be able to perform both limits, $s \to \infty$ for t fixed, and s, $|t| \to \infty$.

Recently a very interesting model for a crossing-symmetric relativistic scattering amplitude has been proposed by VENEZIANO [12]. Once linearly rising trajectories are assumed, the model exhibits Regge behaviour, saturation of finite-energy sum rules, daughters, etc. This model has been introduced to describe a very specific process (namely $\pi\pi \to \pi\omega$), but, due to the fact that all the properties listed above are supposed to be shared by any relativistic scattering amplitude, we are encouraged to guess that its structure has to have a very much wider validity. The great advantage of the Veneziano model compared with former Regge-pole models is that the kinematical variables s and t can be treated on the same footing.

In a pure Veneziano-like model the limits discussed above both go like e^{tk}, where k is a function of the scattering angle θ and, for the t-fixed limit, contain also a term like $\log s$. This fact is very unsatisfactory if we recall the phenomenological discussion at the beginning of this paper. From a theoretical point of view the reason of the failure can be related to the fact that the Cerulus-Martin [13] bound is not satisfied.

Recent works howewer [9,10] have shown how, still keeping linearly rising trajectories, these asymptotic limits can saturate the Cerulus-Martin bound by the introduction of Regge cuts.

Let us now consider the p-$\bar{\text{p}}$ elastic scattering, which is a process crossing-symmetric in the variables u and t, where u is the energy variable and t is the momentum transfer.

In the region of the high values of u and $|t|$ where the Pomeranchukon poles and its cuts can be supposed to be dominant, it is not unreasonable to approximate cuts far away to the left in the j-plane by poles at their branch points.

Assuming for the residues of these poles a parametrization consistent with the works on Regge-cut discontinuities [14,15] we suggest the following generalization of the Veneziano amplitude:

$$A(u, t) = \sum_{m,n=1}^{\infty} \frac{\Gamma[1-\alpha_m(u)]\,\Gamma[1-\alpha_n(t)]}{\Gamma[1-\alpha_m(u)-\alpha_n(t)]} (-\beta)^{m+n-2} a_{mn} , \tag{2}$$

where (*)

$$\alpha_n(x) = 1 + \frac{\alpha' x}{n} .$$

We note that being the s dependence missing in eq. (2), we are ignoring the contributions coming from the baryon number-two channel but this is considered to be a very reasonable approximation in this process.

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[12] G. VENEZIANO: *Nuovo Cimento*, **57** A, 190 (1968).
[13] F. CERULUS and A. MARTIN: *Phys. Lett.*, **8**, 80 (1964).
(*) The form of the a_{mn}'s has to be such that the signature factors come out properly, in case with the help of the baryon number-two channel contributions.
[14] V. N. GRIBOV, I. Y. POMERANCHUK and K. A. TER-MARTIROSYAN: *Phys. Rev.*, **139** B, 184 (1965).
[15] H. YABUKI: Kyoto University preprint, Sept. 1968.

The asymptotic limits are

$$\lim_{\substack{u\to+\infty \\ t\to-\infty}} A(u,t) = F_1(u,t)|t|^{\frac{1}{2}} \quad \exp\left[-2\sqrt{\alpha' \log(1/\beta)}\,\{\sqrt{|u|}f_1+\sqrt{|t|}g_1\}\right] \tag{3}$$

and

$$\lim_{\substack{u\to+\infty \\ t\ \text{fixed}}} A(u,t) = F_2(u,t)\,u|t|^{\frac{1}{2}} \exp\left[-2\sqrt{\alpha' \log(1/\beta)|t|\log[\alpha' u\log(1/\beta)]}\right], \tag{4}$$

where f_i and g_1 are two slowly varying, known functions of $\sqrt{|t|/u}$ only, while F_1 and F_2 contain also oscillating functions of $\sqrt{u}$ and $\sqrt{|t|}$. These results have been derived with a double use of the saddle point method where the $(-1)^{m+n}a_{mn}$ has been neglected. We shall discuss later this approximation.

Equation (4) is consistent with that obtained previously by many other authors discussing p-p scattering [9-10], while the result (3) is closely related to that given by GRECO [11] in the case of high-energy p-$\bar{\text{p}}$ elastic scattering.

Recalling from our previous discussion that formulae (3) and (4) have to describe two different regions of the $\text{d}\sigma/\text{d}t$ plot, a very simple explanation of the p-p « break » in terms of one parameter is suggested.

Our next step will be to consider the p-p scattering. This process is again symmetric in the variables t and u, where u now has the meaning of a momentum transfer, and we assume that it will still be described by the amplitude (2).

However we want to point out that to ignore the baryonic number-two channel contributions in both amplitudes, as we do, has different implications for p-p and p$\bar{\text{p}}$. Meanwhile eq. (2) should in principle describe p$\bar{\text{p}}$ scattering also in the diffraction region and its imaginary part in the forward direction should give the asymptotic total cross-section, on the contrary the amplitude we have guessed for the p-p case does not have any absorptive part in the energy and therefore neither can we expect to describe the diffraction peak nor can we give estimates for the total cross-sections.

The asymptotic behaviours of our amplitude are now:

$$\lim_{\substack{u\to-\infty \\ t\to-\infty}} A(u,t) = G_1(u,t)|t|^{\frac{1}{2}} \quad \exp\left[-2\sqrt{\alpha' \log(1/\beta)}\,\{\sqrt{|u|}f_2+\sqrt{|t|}g_2\}\right], \tag{5}$$

$$\lim_{\substack{u\to-\infty \\ t\ \text{fixed}}} A(u,t) = G_2(u,t)|u||t|^{\frac{1}{2}} \exp\left[-2\sqrt{\alpha' \log(1/\beta)|t|\log[\alpha'|u|\log(1/\beta)]}\right]. \tag{6}$$

Due to the different direction in which the u limit is taken, f_2, g_2, G_1 and G_2 are closely related to f_1, g_1, F_1 and F_2 introduced in (3) and (4), but not equal to them.

Just by inspecting these two formulae, one can see that all the previously discussed features of the large-t p-p experimental data are reproduced. More explicitly the different behaviour of our amplitude in the fixed t and in the large-angle regions give rise to the discontinuity of the differential cross-section observed experimentally [3,4]. But the agreement goes very much deeper than that. Equation (5), in fact, describes all the data in the 3rd region with an accuracy comparable with the experimental errors. This is shown in Fig. 2 where the experimental cross-sections are compared with eq. (5), evaluated for $\sqrt{\alpha' \log(1/\beta)} = 0.66\ (\text{GeV}/c)^{-1}$ (*).

(*) This value is obtained by fitting the 90° data.

Furthermore, with the value of $\sqrt{\alpha' \log(1/\beta)}$ just given, eq. (6) predicts the slopes of the differential cross-sections in the 2nd region. The agreement shown in Table I

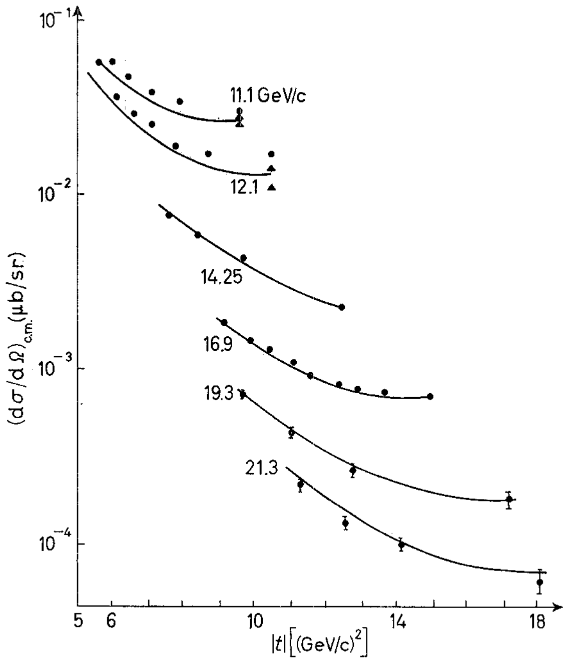

Fig. 2. – p-p differential cross-sections $(d\sigma/d\Omega)_{c.m.}$ in units of 10^{-30} cm²/sr, as functions of $|t|$. The full lines result from eq. (5). ● ALLABY *et al.*; ▲ AKERLOF *et al.*

is not any worse that other theoretical values available in the literature [10] but we want to stress that it has been obtained without free parameters and by considering G_1 and G_2 as constants.

TABLE I. – *Experimental and theoretical slopes of the curves of Fig.* 1 *vs. the incoming laboratory momentum* P_0. Theoretical values result from eq. (6).

P_0(GeV/c)	Experimental slope ((GeV/c)$^{-1}$)	Theoretical slope ((GeV/c)$^{-1}$)
8.1	3.9	3.2
10.1	5.0	3.5
12.1	5.6	3.9
19.2	6.3	4.5

We are aware that the oscillating terms that we have neglected play a very important role in modulating the amplitude ([16]). A more careful evaluation of these terms is now in progress by numerically evaluating the whole series. But we think that our results are promising enough to believe that the main features are independent from these details.

Recently ([10]) in the framework of a Glauber-type eikonal approximation to large-angle scattering the possibility has been reconsidered that the Pomeranchukon has a « normal » slope of about 1 $(\text{GeV})^{-2}$. We want to point out that, while our asymptotic formulae cannot give any direct information on α', the numerical evaluation of the whole series together with the above result for $\alpha' \log(1/\beta)$, once compared with the experiments, will give values for α' and $\log(1/\beta)$ to be checked against the asymptotic $p\bar{p}$ total cross-section.

In conclusion we want to stress that, our main ingredient being an « universal » object like the Pomeranchukon, any information about the specific structure of the proton seems to be missing, and this should strengthen the feeling that the high-momentum-transfer physics is sort of detached from the finer details of strong-interaction dynamics as the success of the thermodynamical approach would suggest.

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We should like to thank N. CABIBBO for his interest in our work and helpful criticism.

([16]) S. FRAUTSCHI, O. KOFOED-HANSEN and B. MARGOLIS: CERN preprint TH. 936 (1968).