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Dynamical Nucleon-Nucleon Correlations in the ^{6}Li, ^{12}C and ^{16}O Nuclei from Elastic Electron Scattering.

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Elastic electron scattering is usually analysed in terms of the elastic charge form factor F_{ch} defined as the Fourier transform of the ground-state charge density. Using the nuclear charge operator as given in ref. (1) we have the form factor

$$(1)\qquad F_{ch} = (G_{Ep} + G_{En})\left(1 + \frac{q_\mu^2}{8M^2}\right)\exp\left[\frac{q^2}{4A\alpha^2}\right]\langle\psi_{SM}|\frac{1}{A}\sum_{j=1}^{A}\exp[i\boldsymbol{q}\cdot\boldsymbol{r}j]|\psi_{SM}\rangle\,,$$

where G_E is the electric form factor (2) of the nucleon; $M, \boldsymbol{q}, q_\mu^2$ are the nucleon mass, three-momentum transfer and four-momentum squared, respectively. The third term in (1) is the correction for the center-of-mass motion of the target (3) evaluated in the shell model with oscillator potential for the nucleus. We shall use this model henceforth; α is the parameter of the oscillator well. The last term in (1) is called the shell-model elastic form factor of the nucleus F_{SM}; ψ_{SM} being the completely antisymmetrized shell-model ground-state wave function. In (1) the same number of protons and neutrons $Z = \frac{1}{2}A$ is assumed.

Recently the influence of the short-range nucleon-nucleon correlations on the elastic form factors has been discussed extensively (4-9). In ref. (5-8) the correlations were

(1) K. W. McVoy and L. Van Hove: *Phys. Rev.*, **125**, 1034 (1962).
(2) L. N. Hand, D. G. Miller and R. Wilson: *Rev. Mod. Phys.*, **35**, 335 (1963); T. Janssens, R. Hofstadter, E. B. Hughes and M. R. Yearian: *Phys. Rev.*, **142**, 922 (1966).
(3) L. J. Tassie and F. C. Barker: *Phys. Rev.*, **111**, 940 (1958). If one assumes for the s-p shell nuclei the oscillator model in which the s- and p-nucleons move in different wells ($\alpha_s \neq \alpha_p$) the parameter in the c.m. correction is given by $A\alpha^2 \to 4\alpha_s^2 + 2(Z-2)\alpha_p^2$.
(4) Y. C. Tang and R. C. Herndon: *Phys. Lett.*, **25** B, 307 (1967).
(5) W. Czyż and L. Lesniak: *Phys. Lett.*, **25** B, 319 (1967).
(6) C. Ciofi degli Atti: *Nuovo Cimento*, **55** B, 570 (1968); Sanità preprint ISS 68/10 (1968).
(7) F. C. Khanna: *Phys. Rev. Lett.*, **20**, 871 (1968).
(8) T. Stovall and D. Vinciguerra: Orsay preprint LAL 1191.
(9) A. Małecki and P. Picchi: Frascati report LNF-68/27 (1968); *Phys. Rev. Lett.*, to be published.

introduced by using a Jastrow-type nuclear density of the form [10]

$$(2)\qquad |\tilde{\psi}(\boldsymbol{r}_1 \dots \boldsymbol{r}_A)|^2 = |\psi_{\text{SM}}(\boldsymbol{r}_1 \dots \boldsymbol{r}_A)|^2 \prod_{j<k}^{A} [1 - h(s_{jk})] = |\psi_{\text{SM}}|^2 \left[1 - \sum_{j<k} h(s_{jk}) + \sum_{j<k,\, l<m} h(s_{jk}) h(s_{lm}) - \dots \right].$$

The function $h(s_{jk})$ introduces correlations of the (j, k) pair; in order to simulate at small relative distances $s_{jk} = |\boldsymbol{r}_j - \boldsymbol{r}_k|$ the hard-core repulsion between nucleons the Gaussian form of $h(s_{jk}) = \exp[-\frac{1}{2} \Delta^2 s_{jk}^2]$ was assumed.

Because of the complexity of (2) the expression for the correlated form factor (with $\tilde{\psi}$ instead of ψ_{SM}) can be evaluated exactly only for simple systems like ^{4}He. In fact, in ref. [5-7] only the contributions from the one correlated pair part of $\tilde{\psi}$, *i.e.* terms in the series (2) with powers of $h(s_{jk})$ smaller than two were retained. The single correlated pair approximation (s.c.p.a.) has, however, been questioned in ref. [8] where it was shown that the exact calculations for ^{4}He give quite a different result from that obtained in ref. [5] with the help of this approximation.

Does the s.c.p.a. fail also for heavier nuclei? In order to answer this question we perform the correlation calculations for ^{6}Li, ^{12}C and ^{16}O. As it would be almost impossible to perform the exact Jastrow-type calculations for these nuclei we use a method which has been described in our previous paper [9].

In ref. [9] the shell-model form factor F_{SM} was expressed in terms of the matrix elements between the two-particle states. In the case of harmonic-oscillator wave functions one can go [11] over from the motion of two particles about a common center to a description of the relative and c.m. motion of the two particles. The nucleon-nucleon correlations were introduced in ref. [9] by modifying the radial wave function $R_{nl}(r)$ of the relative two-nucleon motion:

$$(3)\qquad |\widetilde{nlm}\rangle = \frac{g(r)}{\sqrt{N_{nl}}} R_{nl} Y_{lm}, \qquad N_{nl} = \int_0^\infty \mathrm{d}r\, r^2 R_{nl}^2 g^2(r),$$

where $g(r)$ is a certain function.

Emplying the Moshinsky technique [11] we obtain after some algebra the following correlation correction:

$$(4)\qquad \Delta F_{\text{SM}} = \frac{1}{Z}\Bigg\{6 \exp[-t_s]\, \Delta(000, 000; s) + (Z-2)\exp[-t_p] \cdot$$

$$\cdot \Bigg[(3 - 4t_p + t_p^2)\, \Delta(000, 000; p) + \frac{1}{2} \Delta(100, 100; p) - \sqrt{\frac{2}{3}}\, t_p \Delta(100, 000; p) +$$

$$+ \Delta \sum_m (01m, 01m; p) - 2t_p \Delta(011, 011; p) + \frac{1}{2} \Delta \sum_m (02m, 02m; p) + \frac{2}{\sqrt{3}} t_p \Delta(020, 000; p)\Bigg] +$$

$$+ 4(Z-2)\exp[-t_{sp}] \Bigg[\left(1 - \frac{2}{3} t_{sp}\right) \Delta(000, 000; sp) + \frac{1}{3} \Delta \sum_m (01m, 01m; sp)\Bigg]\Bigg\},$$

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[10] R. J. JASTROW: *Phys. Rev.*, **98**, 1479 (1955).
[11] M. MOSHINSKY: *Nucl. Phys.*, **13**, 104 (1959).

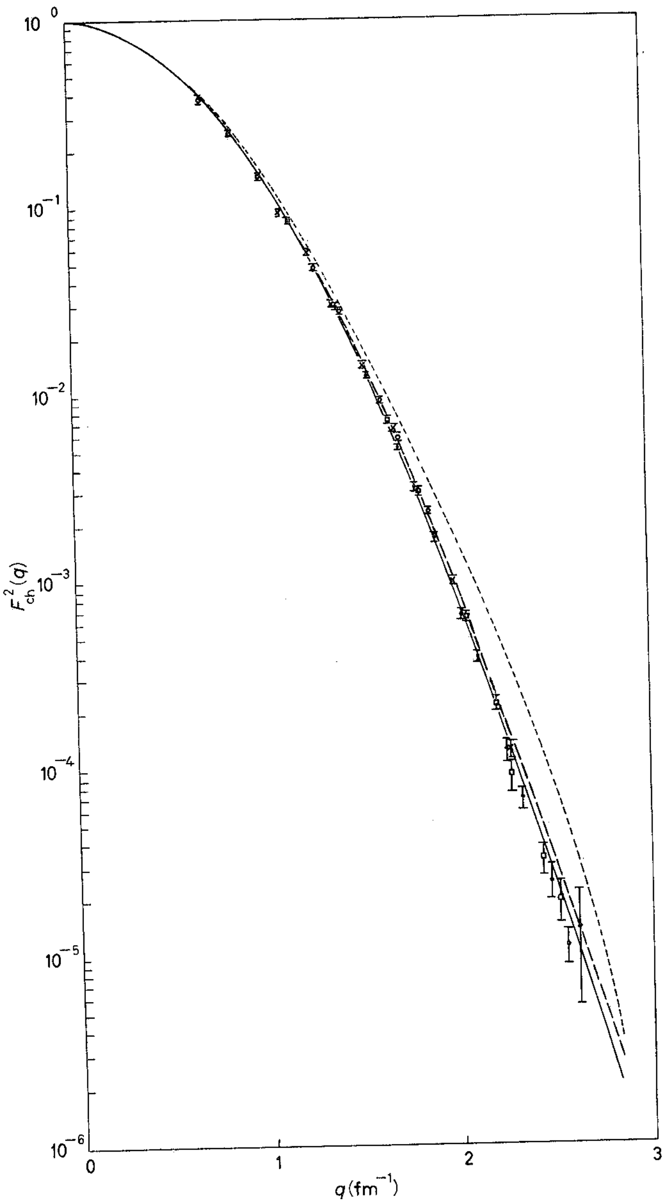

Fig. 1. – Elastic form factor of ^{6}Li. The experimental points are from ref. ([15]). The broken line represents the uncorrelated form factor with $\alpha_s = 118$ MeV, $\alpha_p = 106$ MeV. The full line was obtained with the same α's and with the correlation parameter $\Lambda = 4\ \text{fm}^{-1}$. The dashed line, calculated with $\alpha_s = 127.5$ MeV, $\alpha_p = 100.6$ MeV and $\Lambda = 1.878\ \text{fm}^{-1}$, is given for a comparison with ref. ([6]).

where

$$t_c = \frac{q^2}{8\alpha_c^2}\,, \qquad c = s, p, sp, \qquad (nlm, n'l'm'; c) = \langle (nlm)_c|\exp\left[\frac{1}{\sqrt{2}}\,qz\right]|(n'l'm')_c\rangle$$

and $\Delta(...)$ denotes the difference between correlated and uncorrelated magnitudes [12]. Formula (4) is valid for nuclei with two protons in the s-shell and $Z-2$ protons in

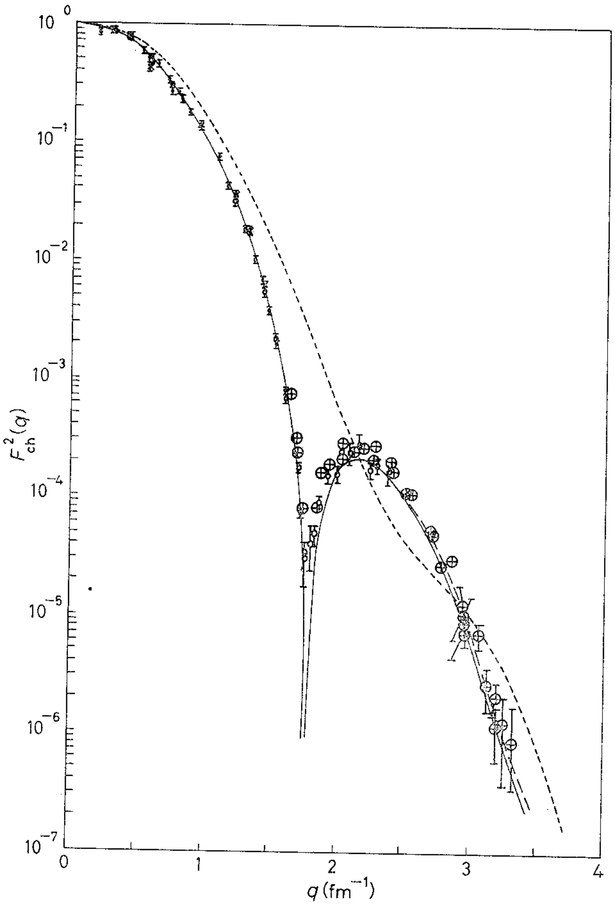

Fig. 2. – Elastic form factor of ^{12}C. The experimental data are those reported in ref. [6]. The broken line represents the uncorrelated form factor with $\alpha_s = \alpha_p = \alpha = 119.5$ MeV. The full line was obtained with the same α and with the correlation parameter $\Lambda = 5\ \text{fm}^{-1}$. The dashed line, calculated with $\alpha_s = 168.2$ MeV, $\alpha_p = 126.2$ MeV and $\Lambda = 2.156\ \text{fm}^{-1}$, is given for the comparison with the result of ref. [6].

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(12) We use Moshinsky's notation for the radial quantum number n. The more usual n' can be obtain by adding unity to Moshinsky's values. In order to obtain orthonormality of all the correlated states involved in calculations for the $|100\rangle$ state a radial dependence slightly different from (3) should be introduced [13]:

$$\tilde{R}_{10} = 6^{\frac12}\pi^{-\frac14}\alpha^{\frac32}\,\frac{g(r)}{\sqrt{N_{10}}}\left(1+\delta-\frac{2}{3}\,\alpha^2r^2\right)\exp\left[-\frac{1}{2}\,\alpha^2r^2\right],$$

where $N_{10} = -\frac{3}{2}(N_{01}^2/N_{00}) + \frac{5}{2}N_{02}$ and $\delta = N_{01}/N_{00} - 1$.

the p-shell; the oscillator parameters for the two shells are assumed to be different. The index c denotes the source of the correlation correction: two correlated nucleons from the s (p)-shell yield the terms with $c = s$ (p) while the correlation between two nucleons from the different shells introduces the terms with $c = sp$ (14).

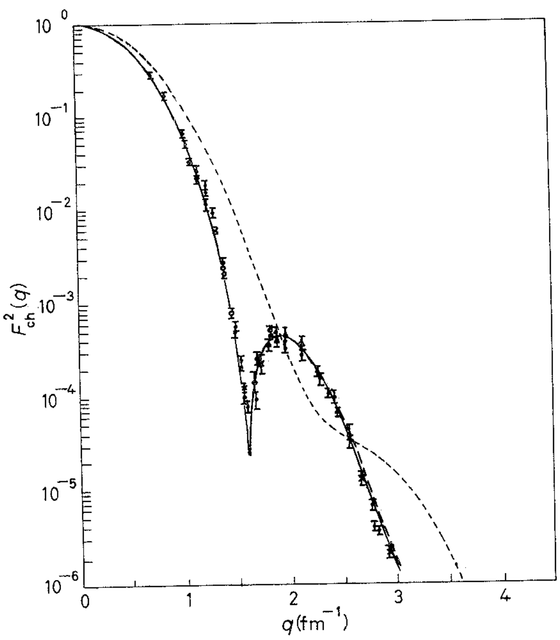

Fig. 3. – Elastic form factor of ^{16}O. The experimental data are those reported in ref. (6). The broken line represents the uncorrelated form factor with $\alpha_s = \alpha_p = \alpha = 109$ MeV. The full line was obtained with the same α and with the correlation parameter $\Lambda = 5\ \mathrm{fm}^{-1}$. The dashed line, calculated with $\alpha_s = 175.1$ MeV, $\alpha_p = 122.3$ MeV and $\Lambda = 2.426\ \mathrm{fm}^{-1}$, is given for the comparison with the result of ref. (6).

We use the following form of $g(s)$ function (s being the distance between two nucleons):

$$g(s) = \sqrt{1 - \exp\left[-\tfrac{1}{2}\Lambda^2 s^2\right]}\,. \tag{5}$$

Thus we are able to compare our results with those (6) obtained with help of the Jastrow method and s.c.p.a. The results of our analysis for the ^6Li, ^{12}C and ^{16}O nuclei are presented in Fig. 1, 2 and 3.

Our conclusions and comments are the following:

a) In the case of ^6Li the elastic electron scattering turns out to be sensitive to the nucleon-nucleon correlations only at large momentum transfers. The low-momentum-transfer data (up to $q = 2\ \mathrm{fm}^{-1}$) for this nucleus are well explained in the oscillator

(13) W. Czyż, L. Lesniak and A. Małecki: *Ann. of Phys.*, **42**, 119 (1967).

(14) The average oscillator parameter α_{sp} was assumed to be

$$\alpha_{sp} = \sqrt{5Z - 4}\,[6\alpha_s^{-2} + 5(Z-2)\alpha_p^{-2}]^{-\frac{1}{2}}\,.$$

(15) L. R. Suelzle, M. R. Yearian and H. Crannel: *Phys. Rev.*, **162**, 992 (1967).

shell model provided one assumes that the s- and p-shell protons move in different wells. At large momentum transfers there is, however, a deviation from the uncorrelated shell model. Introducing the correlations one improves the situation. We have obtained a good fit to the experimental data over the wide range of momentum transfer (see Fig. 1). Our calculations predict a diffraction minimum for ^{6}Li at $q = 3.7\ \text{fm}^{-1}$.

b) In the elastic electron scattering from ^{12}C and ^{16}O the short-range correlations are less important. The experimental results for these nuclei are fairly well explained in the oscillator shell model without the correlations. The introduction of the correlations introduces only small changes of the elastic form factor at large momentum transfers (see Fig. 2 and 3).

c) Our calculations are in a drastic disagreement with the results ([6]) which were based on the Jastrow method with the s.c.p.a. Using the oscillator and correlation parameters as given in ref. ([6]) we have evaluated the dashed lines in Fig. 1, 2 and 3. These curves are inconsistent with the experimental results while in ref. ([6]) applying the same parameters, a good accord with experiment was obtained. This comparison allows us to state that the single correlated pair approximation is wrong. In our opinion, this approximation considerably over-estimates the effect of the short-range correlations.

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