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ELASTIC ELECTRON SCATTERING FROM He^4

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We discuss the influence of dynamical nucleon-nucleon correlations on elastic electron scattering from light nuclei. The numerical results for He^4 are presented.

The recent measurements[1,2] of the elastic electron scattering form factor for He^4 show definite deviations from the Gaussian model. Some attempts have been made to explain the data by introducing the short-range nucleon-nucleon correlations in the standard Gaussian wave function. In the work of Czyż[3] the correlations were treated by the Jastrow method,[4] and only the contributions from the one-correlated-pair part of the nuclear wave function were retained. The single-correlated-pair approximation has been questioned by Stovall and Vinciguerra[5] where it was shown that the calculations with six correlated pairs give quite a different result from that obtained in Ref. 3. In disagreement with this analysis Khanna[6] again suggests the validity of the approximation.

We are presenting a method (for more details see Małecki and Picchi[7] and Czyż, Leśniak, and Małecki[8]) which avoids the mentioned difficulties, as the summation of the troublesome series in the Jastrow method is now automatically performed. This allows us to make reliable calculations for nuclei heavier than He^4. In this paper the results for He^4 are presented; heavier nuclei will be discussed elsewhere.[7] The analysis[7] confirms the result of Ref. 5: Introducing similar correlations as in Refs. 3 and 5, one gets an analytical expression for the form factor which is consistent with Ref. 5 but not with Ref. 3.

The square of the elastic form factor is defined as the ratio of the experimental cross section to the Mott cross section. For spin-zero nuclei the elastically scattered electron interacts only with the charge of the nucleus. Using the nuclear charge operator as given by McVoy and Van Hove[9] we get the form factor

$$F_{\mathrm{ch}}(q_\mu^2)=(G_{Ep}+G_{En})\left(1+\frac{q_\mu^2}{8M^2}\right)e^{q^2/4A\alpha^2}\langle\psi_{SM}|\frac{1}{A}\sum_{j=1}^{A}\exp(i\vec{q}\cdot\vec{r}_j)|\psi_{SM}\rangle, \tag{1}$$

where G_E is the electric form factor[10] of the nucleon; M, q, and q_μ^2 are the nucleon mass, three-momentum transfer, and four-momentum transfer squared, respectively. The third term in (1) is the correction for the c.m. motion of the target,[11] evaluated in the shell model with oscillator potential for the nucleus. We shall use this model henceforth; α is the parameter of the oscillator well. The last term in (1) is called the shell-model elastic form factor of the nucleus F_{SM}, ψ_{SM} being the completely antisymmetrized shell-model ground-state wave function. In (1) the same number of protons and neutrons $Z=\frac{1}{2}A$ is assumed.

We can write F_{SM} as follows:

$$F_{SM}=\frac{1}{2A(A-1)}\langle\psi_{SM}|\sum_{j\neq k}^{A}[\exp(i\vec{q}\cdot\vec{r}_j)+\exp(i\vec{q}\cdot\vec{r}_k)]|\psi_{SM}\rangle$$

$$=\frac{1}{Z(2Z-1)}[4\sum_{ab}\langle a(1)b(2)|e^{i\vec{q}\cdot\vec{r}_1}+e^{i\vec{q}\cdot\vec{r}_2}|a(1)b(2)\rangle-\sum_a\langle a(1)a(2)|e^{i\vec{q}\cdot\vec{r}_1}+e^{i\vec{q}\cdot\vec{r}_2}|a(1)a(2)\rangle],\quad(2)$$

where in the second equation the summation over spin and isospin quantum numbers has been performed, a,b being the spatial single-particle quantum numbers.

In the case of harmonic-oscillator wave functions it is possible to define a transformation[12] from the motion of two particles about a common center to the relative and center-of-mass motion of the two particles. Following Moshinsky[12] we can write the two-particle state as follows:

$$|ab\rangle=\sum_{\lambda\mu mM}\sum_{nlNL}\langle l_am_al_bm_b|l_al_b\lambda\mu\rangle\langle lmLM|lL\lambda\mu\rangle\{nl,NL,\lambda|n_a,l_a,n_bl_b,\lambda\}|nlm\rangle|NLM\rangle,\quad(3)$$

where (n,l,m) are the quantum numbers of relative motion and (N,L,M) are the quantum numbers of the c.m. motion.

We introduce the nucleon-nucleon correlations in (3) by modifying the radial wave function of the relative motion:

$$|''nlm\rangle=\frac{g(r)}{(N_{nl})^{1/2}}R_{nl}''(r)Y_{lm}(\theta,\varphi),\quad N_{nl}=\int_0^\infty dr\,r^2R_{nl}{}^2g^2(r),\quad(4)$$

where $g(r)$ is a certain function.

Using Refs. 2 and 3 we obtain after some tedious, though straightforward algebra,[7] the following short-range correlations correction:

$$F_{SM}=\frac{e^{-t}}{Z(2Z-1)}\{[7Z-8-\frac{20}{3}(Z-2)t+(Z-2)t^2]\Delta\langle000|e^{i\vec{q}\cdot\vec{r}/\sqrt{2}}|000\rangle$$

$$+\tfrac{1}{2}(Z-2)\Delta\langle100|e^{i\vec{q}\cdot\vec{r}/\sqrt{2}}|100\rangle-(\tfrac{2}{3})^{\frac{1}{2}}(Z-2)t\Delta\langle100|e^{i\vec{q}\cdot\vec{r}/\sqrt{2}}|000\rangle\},\quad(5)$$

where $t=q^2/8\alpha^2$ and $\Delta(\cdots)$ denotes the difference between correlated and uncorrelated magnitudes. The formula (5) is valid for nuclei with two protons in the s shell and $Z-2$ protons in the p shell. It was assumed in (5) that the short-range correlations act on the relative s states only.[8]

For the He^4 nucleus we get from (2) and (3) a very simple result:

$$F_{SM}=\langle000_{\text{c.m.}}|e^{i\vec{q}\cdot\vec{R}/\sqrt{2}}|000_{\text{c.m.}}\rangle\langle000_{\text{rel}}|e^{i\vec{q}\cdot\vec{r}/\sqrt{2}}|000_{\text{rel}}\rangle.\quad(6)$$

Modifying the relative state $|000_{\text{rel}}\rangle$ according to (4) we have the correlated shell-model form factor

$$F_{SM}''=\frac{2}{q}e^{-q^2/8\alpha^2}\frac{\int_0^\infty ds\,se^{-\frac{1}{2}\alpha^2s^2}\sin(\frac{1}{2}qs)g^2(s)}{\int_0^\infty ds\,s^2e^{-\frac{1}{2}\alpha^2s^2}g^2(s)},\quad(7)$$

where s is the distance between nucleons.

The results of our analysis for He^4 are presented in Fig. 1. The curve 1 represents the uncorrelated charge form factor and has been calculated with $\alpha = 148.3$ MeV.[13] The curve 2 gives the form factor corrected for the short-range nucleon-nucleon correlations. We have modified the Gaussian wave function of the relative two-nucleon motion only at small distances, introducing a hard core, but we did not change it for large s —see the inset of Fig. 1 where we have presented the squared wave functions of the relative motion. The function $g(s)$ which "heals" the relative wave function at medium internucleon distances has been chosen as

$$g(s) = \alpha^2(s-r_c)^2 \exp[-\gamma\alpha^2(s-r_c)^2], \quad r_c \leq s \leq r_h, \tag{8}$$

where r_c is the radius of the hard core and r_h is the so-called healing distance. The curve 2 has been calculated with $\alpha = 148.3$ MeV, $r_c = 0.56$ F, and $\gamma = 1.0$; it gives $r_h = 2.37$ F. The curve fits the experimental data well down to the minimum at $q_\mu{}^2 = -10$ F^{-2}; also the position of this minimum is very well accounted for. Comparison of the curves 1 and 2 shows that the effect of the repulsive core in He^4 is very important.

For very large momentum transfers $q^2 > 10$ F^{-2}, curve 2 is no longer consistent with the experimental data. This suggests that one should modify the Gaussian relative wave function not only at small internucleon distances, but also at large s. In order to do this we have used the $g(s)$ function in the form (8) for $r_c \leq s < \infty$. Such a modification means that the internucleon forces at large distances between nucleons are more attractive than those implied by the oscillator model; at short distances the interaction contains a repulsive core. As a consequence the modification will make the surface of the He^4 nucleus less diffuse than indicated by the Gaussian model. This is in agreement with the result of an analysis done in Ref. 2. Using $\gamma = 0.74$ and the same values of α and r_c as before, we have calculated curve 3 in the figure. This curve is consistent with the experimental data up to $q^2 = 15$ F^{-2}, but falls still too rapidly with increasing q.

One could obtain a better fit than that of curve 3 by a simultaneous modification of both the relative and the center-of-mass motion of two nucleons. The latter modification could be performed, for instance, by changing the oscillator parameter in the c.m. wave function. Putting $\alpha_{c.m.}{}^2 = (1+\Gamma)\alpha^2$ with $\Gamma > 0$ one makes the surface of the nucleus still less diffuse than it was when only the relative motion was modified. Us-

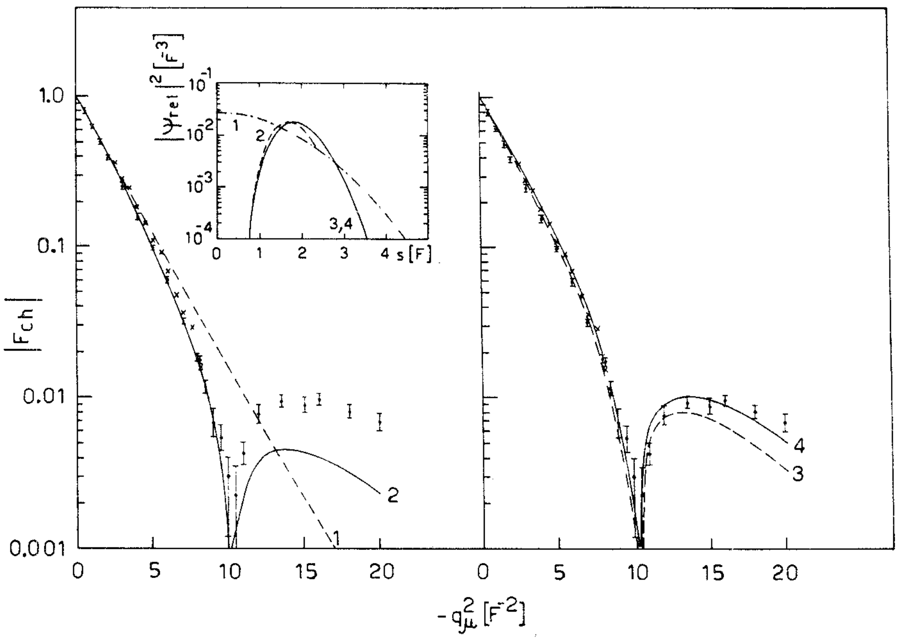

FIG. 1. Charge form factors of He^4. The experimental points marked by crosses are taken from Ref. 1; those marked by solid circles with error bars, from Ref. 2. Curves 1, 2, 3, and 4 have been calculated using the wave functions of the relative two-nucleon motion as shown by the corresponding curves in the inset. In addition, for curve 4 the c.m.-system motion of two nucleons has been modified.

ing $\Gamma = 0.1$ and the remaining parameters the same as for curve 3, we obtain curve 4 which reproduces the data very well. We do not want, however, to stress the importance of this fit, since the last modification introduces an additional parameter.

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