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Simple but New Calculations on Single and Double Charge-Exchange of Pions by Light Nuclei.

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Summary. — The cross-section for single and double charge-exchange nuclear reactions with pions for $1p$-shell nuclei is calculated using the Glauber method. The results are obtained for the reactions of pions of $(80\div 200)$ MeV by boron $(\pi^+ + {}^{11}B \to \pi^0 + {}^{11}C)$ and lithium $(\pi^+ + {}^{7}Li \to \pi^- + {}^{7}B)$.

1. – Introduction.

Nuclear reactions by pions can give valuable information on the many-body structure of a nucleus.

The single charge exchange

$$\pi^{\pm} + \begin{pmatrix} A \\ Z \end{pmatrix} \to \pi^0 + \begin{pmatrix} A \\ Z \pm 1 \end{pmatrix}, \tag{1.1}$$

and the double charge exchange

$$\pi^{\pm} + \begin{pmatrix} A \\ Z \end{pmatrix} \to \pi^{\pm} + \begin{pmatrix} A \\ Z \pm 2 \end{pmatrix}, \tag{1.2}$$

are two examples; (1.1) can be used to check nuclear models, (1.2) for nuclear correlation study (two nucleons, at least, take part in the reaction). Very little is known about the single-charge exchange. The only theoretical work on (1.1) dealt primarily with the Fermi-gas model in the impulse approximation (1). A great deal of theoretical and experimental work on the other hand has been devoted to the double charge exchange (2).

The purpose of the present article is to express $d\sigma/d\Omega$ for the reactions (1.1) and (1.2) for pion energies $E_\pi(80 \simeq 200)$ MeV by $1p$-shell nuclei in terms of the elementary interactions π-nucleon. The method we use is basically that of GLAUBER (3), the advantage of which lies in the fact that multiple-scattering effects are considered. Its great limitation is the small-angle approximation (for the energy E_π that we use the approximations are reasonable for minimum angles $\theta_0 \sim 70°$).

In Sect. **2** we give the basis of the Glauber method; in Sect. **3** we present the actual calculations and results for $\pi^+ + {}^{11}B \to \pi^0 + {}^{11}C$ and $\pi^+ + {}^7Li \to \pi^- + {}^7B$.

2. – The Glauber method.

Glauber's conditions are $KR \gg 1$ and $V_0/E(K) \gg 1$, where E is the kinetic energy of the incident particle upon a potential V of magnitude V_0 and range R.

Under these condtions we are justified in assuming that backward scattering will be very weak, that the wave function of the particle within the volume of the potential may to a good approximation be written in the form

$$\psi(\boldsymbol{r}) = \exp\left[i\boldsymbol{k}\cdot\boldsymbol{r} - (i/\hbar v)\right]\int_{-\infty}^{z} V(\boldsymbol{b} + \widehat{Kz'})\,dz',$$

where v is the velocity of the particle, b is its impact parameter.

Now, the scattering amplitude can be written as

$$(2.1) \qquad F(\boldsymbol{K}', \boldsymbol{K}) = \frac{-2m}{4\pi\hbar^2}\cdot \int \exp[-i\boldsymbol{k}'\cdot\boldsymbol{r}]\, V(\boldsymbol{r}) \exp\left[i\boldsymbol{k}\cdot\boldsymbol{r} - (i/\hbar v)\int_{-\infty}^{z} V(\boldsymbol{b} + \widehat{Kz'})\,dz'\right] dz\, d^2b\,.$$

For small scattering angles the vector $\boldsymbol{K} - \boldsymbol{K}'$ is nearly perpendicular to $\boldsymbol{K}$. Assuming it perpendicular, the z-integration is simply that of an exact dif-

(1) S. B. KAUFMAN and C. O. HOWER: *Phys. Rev.*, **140**, B 1272 (1965).

(2) F. BECKER and Z. MARIC: *Nuovo Cimento*, **41** B, 174 (1966); and references therein.

(3) R. J. GLAUBER: *Lectures in Theoretical Physics*, vol. **1** (New York, 1959), p. 315.

ferential and leads to

$$(2.2)\qquad F(\boldsymbol{K}', \boldsymbol{K}) = \frac{K}{2\pi i}\cdot \\ \cdot\int \exp[i(\boldsymbol{K}-\boldsymbol{K}')\cdot\boldsymbol{b}]\left\{\exp\left[-(i/\hbar v)\cdot\int\limits_{-\infty}^{+\infty} V(\boldsymbol{b}+\widehat{Kz'})\,\mathrm{d}z'\right]-1\right\}\mathrm{d}^2b\,.$$

The function

$$(2.3)\qquad -(i/\hbar v)\int\limits_{-\infty}^{+\infty} V(\boldsymbol{b}+\widehat{Kz'})\,\mathrm{d}z' = X(\boldsymbol{b})\,,$$

represents the total phase shift the wave suffers in traversing V.

In this approximation, the scattering of a particle by n centres in the fixed positions $\boldsymbol{r}_i$ is obtained by replacing $X(\boldsymbol{b})$ by $\sum X_i(\boldsymbol{b}-\boldsymbol{b}_i)$ and the single-particle wave function by the many-particle wave function $n(\boldsymbol{q}_1, \boldsymbol{q}_2, \dots, \boldsymbol{q}_n)$.

The amplitude is

$$(2.3')\qquad F(\boldsymbol{K}', \boldsymbol{K}) = \frac{k}{2\pi i}\int \exp[i(\boldsymbol{K}, \boldsymbol{K}')\cdot\boldsymbol{b}]|n\boldsymbol{q}_1, \dots, \boldsymbol{q}_n)|^2\cdot \\ \cdot\left[\exp\left[i\sum X_i(\boldsymbol{b}-\boldsymbol{b}_i)\right]-1\right]\mathrm{d}\boldsymbol{q}_1\dots\mathrm{d}\boldsymbol{q}_n\mathrm{d}^2b\,.$$

We assume, as a first approximation, that the nuclear ground-state wave function factorizes so that $|u_i(\boldsymbol{q}_1, \dots, \boldsymbol{q}_n)|^2$ may be written as

$$(2.4)\qquad |u(\boldsymbol{q}_1, \dots, \boldsymbol{q}_n)|^2 = \prod_{i=1}^{N} q_i(\boldsymbol{q}_i)\,,$$

where $q_i(\boldsymbol{q}_i)$ is the normalized density for the i-th particle.

With the definition $t_i(\boldsymbol{b}-\boldsymbol{b}_i) = -1+\exp[X_i(\boldsymbol{b}-\boldsymbol{b}_i)]$ eq. (2.3) becomes

$$(2.5)\qquad F(\boldsymbol{K}', \boldsymbol{K}) = \frac{K}{2\pi i}\int \exp[i(\boldsymbol{K}-\boldsymbol{K}')\cdot\boldsymbol{b}]\left[\sum_{i=1}^{N}\langle t_i(\boldsymbol{b})\rangle + \right. \\ \left. +\sum_{j=1}^{N}\sum_{i\neq j}^{N}\langle t_i(\boldsymbol{b})\rangle\langle t_j(\boldsymbol{b})\rangle\dots\right]\mathrm{d}^2b\,,$$

where

$$(2.6)\qquad \langle t_i(\boldsymbol{b})\rangle = \int t_i(\boldsymbol{b}-\boldsymbol{b}_i)q_i(\boldsymbol{b}_i)\,\mathrm{d}^2b_i\,,$$

and

$$(2.7)\qquad q_i(\boldsymbol{b}) = \int\limits_{-\infty}^{+\infty} q(\boldsymbol{b}_i, z_i)\,\mathrm{d}z_i\,.$$

For a single-scattering centre

$$F(\boldsymbol{K}', \boldsymbol{K}) = \frac{K}{2\pi i}\int \exp[-i(\boldsymbol{K}-\boldsymbol{K}')\cdot\boldsymbol{b}']t(\boldsymbol{b}')\,\mathrm{d}^2b' = \frac{K}{2\pi i}t(\Delta\boldsymbol{K})\,. \tag{2.8}$$

Equation (2.8) gives the connection between the two-body scattering amplitude $F(\boldsymbol{K}', \boldsymbol{K})$ and $t(\Delta\boldsymbol{K})$.

Now,

$$t_i(\boldsymbol{b}-\boldsymbol{b}_i) = \frac{1}{2\pi^2}\int \exp[i(\boldsymbol{b}-\boldsymbol{b}_i)(\boldsymbol{K}'-\boldsymbol{K})]t(\Delta\boldsymbol{K})\,\mathrm{d}^2\Delta K \tag{2.9}$$

and

$$\begin{aligned}\langle t_i(\boldsymbol{b})\rangle &= \int t_i(\boldsymbol{b}-\boldsymbol{b}_i)b(\boldsymbol{b}_i)\,\mathrm{d}^2b_i = \\ &= \frac{1}{(2\pi)^2}\int \exp[i(\boldsymbol{b}-\boldsymbol{b}_i)(\boldsymbol{K}'-\boldsymbol{K})]\; t(\Delta\boldsymbol{K})\,\mathrm{d}^2\Delta\boldsymbol{K} q(\boldsymbol{b}_i)\,\mathrm{d}^2b_i = \\ &= \frac{1}{(2\pi)^2}\int \exp[i\boldsymbol{b}\cdot\Delta\boldsymbol{K}]t(\Delta\boldsymbol{K})\,\mathrm{d}^2\Delta\boldsymbol{K}\int \exp[-i\boldsymbol{b}_i\cdot\Delta\boldsymbol{K}]q(\boldsymbol{b}_i)\,\mathrm{d}^2b_i = \\ &= \frac{1}{(2\pi)^2}\int \exp[i\boldsymbol{b}\cdot\Delta\boldsymbol{K}]t(\Delta\boldsymbol{K})q(\Delta\boldsymbol{K})\,\mathrm{d}^2\Delta\boldsymbol{K}\,.\end{aligned} \tag{2.10}$$

The single-scattering term $\sum_{i=1}^{N}\langle t_i(b)\rangle$ in eq. (2.5) is identical to that obtained from the impulse approximation, which is valid under less restrictive assumptions. The higher-order terms

$$\sum_i \sum_{j\neq i}, \qquad \sum_i \sum_{j\neq i} \sum_{K\neq j} \text{ etc}\,.$$

are double and triple scattering etc. A comparison of the expressions for double scattering given by GLAUBER and WATSON respectively similar to ours gives a difference of 15% or less for all angles ([4]). This is a good check for the Glauber method.

3. – Calculation and results.

3'1. *Single charge exchange.* – We consider the following reaction:

$$\pi^+ + {}^{11}\mathrm{B} \to \pi^0 + {}^{11}\mathrm{C}\,. \tag{3.1}$$

([4]) M. M. STERNHEIM: *Phys. Rev.*, **135**, B 912 (1964).

The ^{11}B is a $1p$-shell nucleus and is well described by a density function (5):

$$(3.2)\qquad q(r)=\frac{2}{\pi^{\frac{3}{2}}a_0^3(2+3\alpha)}\left(1+\frac{\alpha r^2}{a_0^2}\right)\exp\left[-\frac{r^2}{a_0^2}\right],$$

where $\alpha=(A-4)/6$.

The root-mean-square radius of this distribution is given by $\langle r^2\rangle^{\frac{1}{2}}=$ $=a_0(5/2-4/A)^{\frac{1}{2}}$. The nucleon-nucleon correlations we use are due to the Pauli exclusion principle. A check of the effects of hard correlations will require much more copious pion beams than are presently available.

The amplitudes $f(\boldsymbol{K}',\boldsymbol{K})$ of the elementary processes $\pi^+p\to\pi^+p$, $\pi^+n\to\pi^+n$, $\pi^+n\to\pi^0p$, $\pi^0n\to\pi^0n$, $\pi^0p\to\pi^0p$ (the last two are obtained from elementary considerations in isotopic-spin space) are well known.

Substituting into $f(\boldsymbol{K}',\boldsymbol{K})$ the S plus P-wave forms for pion-nucleon scattering

$$(3.3)\qquad f(\boldsymbol{K}',\boldsymbol{K})=\alpha+\beta\boldsymbol{K}'\cdot\boldsymbol{K}/k^2=\left[1-\frac{\beta}{2(\alpha+\beta)}\frac{\Delta\boldsymbol{k}^2}{K^2}\right](\alpha+\beta),$$

where for $\pi^+n\to\pi^+n$ scattering (neglecting the spin-flip term)

$$k\alpha=(a_3+2a_1)/3,$$
$$k\beta=(2a_{33}+a_{31}+4a_{13}+2a_{11})/3;$$

for $\pi^+p\to\pi^+p$

$$k\alpha=a_3,$$
$$k\beta=2a_{33}+a_{31};$$

for $\pi^0n\to\pi^0n=\pi^0p\to\pi^0p$

$$k\alpha=(2a_3+a_1)/3,$$
$$k\beta=(4a_{33}+2a_{13}+2a_{31}+a_{11})/3;$$

for $\pi^+n\to\pi^0p=\pi^0n\to\pi^-p$

$$k\alpha=(a_3-a_1)\sqrt{\tfrac{3}{2}},$$
$$k\beta=(2a_{33}-2a_{13}+a_{31}-a_{11})\sqrt{\tfrac{3}{2}}.$$

Here $a_i=\exp[i\delta_i]\sin\delta_i$, and the $\delta_{2T,2J}$ are the usual phase shifts (6).

(5) R. HOFSTADTER: *Nuclear and Nucleon Structure* (New York, 1963), p. 315.
(6) L. D. ROPER, R. WRIGHT and B. T. FELD: *Phys. Rev.*, **138**, B 190 (1965).

Extrapolating eq. (3.3) into an exponential form we have

$$f(\boldsymbol{K}', \boldsymbol{K}) = (\alpha + \beta) \exp[-\Delta \boldsymbol{K}^2] \frac{\beta}{2(\alpha+\beta)K^2} . \tag{3.4}$$

In multiple scattering some difficulties arise, as the elementary amplitude is affected by the virtual behaviour of intermediate pions. An expression for the $T = J = \frac{3}{2}$ off-shell π-$\mathcal{N}$ scattering amplitude, which is the relevant term in the energy range of intrest is [7]

$$f_{33}(u, -\mu^{*2}) \simeq K(-\mu^{*2})\, \psi(-\mu^{*2}) f_{33}(u, -\mu^2) , \tag{3.5}$$

where $K(-\mu^{*2})$ is the pionic form factor of the nucleon, where

$$\psi(-\mu^{*2}) = \left(1 + \frac{\mu^{*2}}{4m^2}\right) \frac{p_1 B_0(u_r, -\mu^{*2})}{q_1 B_0(u_r, -\mu^{*2})} , \tag{3.6}$$

where m and μ are the nucleon and pion masses; μ^* is the effective mass of the pion; $u = (\omega - m)/m$, ω being the total c.m. energy of the π-$\mathcal{N}$ system; $u_r = 0.314$ is the u-value of the $\frac{3}{2}$, $\frac{3}{2}$ resonance; $p_1 = (p_{10}^2 - m^2)^{\frac{1}{2}}$ and $q_1 = (q_{10}^2 - m^2)^{\frac{1}{2}}$ are the off-shell and on-shell c.m. momenta, where

$$p_{10} = (\omega^2 + m^2 + \mu^{*2})/2 , \qquad q_{10} = (\omega^2 + m^2 - \mu^2)/2 ,$$

where

$$B_0(u, -\mu^{*2}) = \frac{4}{3} \frac{f^2}{\mu^2} \frac{u}{(u+\delta)^2} \left[1 + \frac{2\delta}{u+\delta}\right] ,$$

with $f^2 = 0.08$ and $\delta = (\mu^{*2} + \mu^2)/2m^2$; $f_{33}(u, -\mu^2)$ is the real amplitude.

A way to determine μ^* is to consider the virtual particle as a wave packet $\psi(x, t)$ within the nucleus and to assume Gaussian distribution in momentum space, *i.e.*

$$a(K - Q) = (\Delta p \sqrt{2\pi})^{-1} \exp\left[-\frac{(K-Q)^2}{2(\Delta p)^2}\right] , \tag{3.6'}$$

with $\Delta p = (\Delta x)^{-1}$ and $\Delta x = \lambda$, the mean free path of the pion within the nucleus.

The effective distribution is then derived from the momentum distribution through energy-momentum conservation $\sum p_i = \sum p_f$.

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(7) E. FERRARI and F. SELLERI: *Phys. Rev. Lett.*, **7**, 387 (1961).

The mean free path λ is however long enough that the approximation $f_{3,3}(u, -\mu^{*2}) = f_{33}(u, -\mu^2)$ seems to be reasonably well satisfied except near $E_\pi = 200$ MeV.

Taking that into account the amplitude for the reaction $\pi^+ + {}^{11}B \to \pi^0 + {}^{11}C$ can be written as

$$(3.7) \qquad F(\boldsymbol{K}', \boldsymbol{K}) = \frac{K}{2\pi i}\Big[\int \exp[i(\boldsymbol{K}-\boldsymbol{K}')\cdot\boldsymbol{b}] \sum \langle t(\boldsymbol{b})\rangle \, d^2b + \int \exp[i(\boldsymbol{K}-\boldsymbol{K}')\cdot\boldsymbol{b}] \sum\sum \langle t(\boldsymbol{b})\rangle \langle t(\boldsymbol{b})\rangle + \int \exp[i(\boldsymbol{K}-\boldsymbol{K}')\cdot\boldsymbol{b}] \sum\sum\sum \langle t(\boldsymbol{b})\rangle\langle t(\boldsymbol{b})\rangle\langle t(\boldsymbol{b})\rangle + \dots \text{ etc.}\Big] .$$

In Fig. 1 we illustrate the two types of terms which can occur in double scattering.

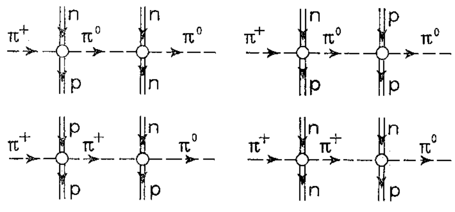

Fig. 1. – Possible contributions to double scattering for single-charge exchange.

The single-scattering term in eq. (3.7) is

$$(3.8) \qquad F_1(\boldsymbol{K}', \boldsymbol{K}) = \sum f(\boldsymbol{K}p', \boldsymbol{K})\, q(\Delta\boldsymbol{K}) .$$

The interpretation of term F_1 which is identical to that obtained from the impulse approximation, is simple. The interaction matrix element involves an integration over all nucleon co-ordinates as well as those of the meson. The

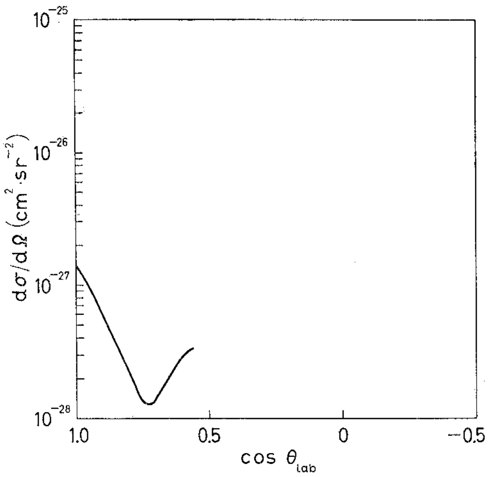

Fig. 2. – The angular distribution $(d\sigma/d\Omega)_{lab}$ for $\pi^+ + {}^{11}B \to \pi^0 + {}^{11}C$ and for $E_\pi = 80$ MeV. $a_0 = 1.56 \cdot 10^{-13}$ cm.

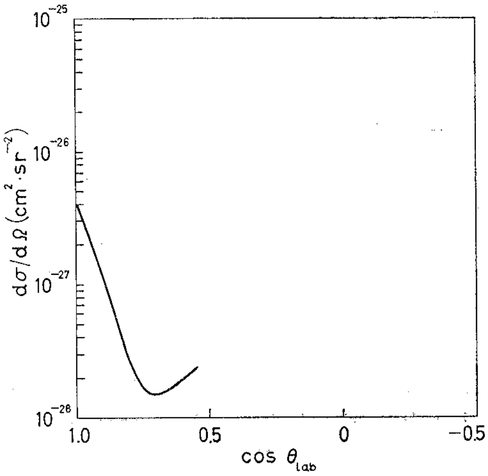

Fig. 3. – The angular distribution $(d\sigma/d\Omega)_{lab}$ for $\pi^+ + {}^{11}B \to \pi^0 + {}^{11}C$ and for $E_\pi = 100$ MeV. $a_0 = 1.56 \cdot 10^{-13}$ cm.

latter integration leads to the pion-single-nucleon scattering amplitude $F(\boldsymbol{K}', \boldsymbol{K})$, whereas the former becomes a weighting factor $q(\Delta \boldsymbol{K})$ (form factor) whose magnitude is determined by the probability that all nucleons remain in their unperturbed states following the collision.

The calculation of the eq. (3.7) is made for energy E_π of 80, 100 MeV. The angular distributions $(\mathrm{d}\sigma/\mathrm{d}\Omega)_{\mathrm{lab}}$ are given in Fig. 2, 3.

3·2. *Double charge exchange.* – The amplitude for double charge exchange (see Subsect. **3·1**) is

$$F(\boldsymbol{K}', \boldsymbol{K}) = \frac{K}{2\pi i}\left[\int \exp[i(\boldsymbol{K}-\boldsymbol{K}')\cdot \boldsymbol{b}]\left(\sum\sum \langle t(\boldsymbol{b})\rangle\langle t(\boldsymbol{b})\rangle\right) + \right.$$
$$+ \int \exp[i(\boldsymbol{K}-\boldsymbol{K}')\cdot \boldsymbol{b}]\left(\sum\sum\sum \langle t(\boldsymbol{b})\rangle\langle t(\boldsymbol{b})\rangle\langle t(\boldsymbol{b})\rangle\right) +$$
$$\left. + \sum\sum\sum\sum \langle t(\boldsymbol{b})\rangle\langle t(\boldsymbol{b})\rangle\langle t(\boldsymbol{b})\rangle\langle(t(\boldsymbol{b})\rangle + \dots \text{etc.}\right]. \tag{3.9}$$

The shell model says that ^{7}Li is composed of two protons and two neutrons in $1S$ state and one proton and two neutrons in $1P$ state. The elastic double charge exchange therefore occurs on the pair on $1p$ neutrons, so F_2 has a single term. The angular distributions $(\mathrm{d}\sigma/\mathrm{d}\Omega)_{\mathrm{lab}}$ are given in Fig. 4, 5, 6 for energy E_π of 80, 100, 195 MeV. It is very difficult to compare our results with experimental data because of the inability to deduce from experiments the bound states which take part in the reaction. GILLY *et al.* (8) give for ^{7}Li the forward cross-section $\left((0.9 \pm 0.1)\cdot 10^{-28}\,\mathrm{cm}^2\,\mathrm{sr}^{-1}\right)$ for incident mesons with kinetic energy of 195 MeV.

By means of the binding energy of the ^{7}B nucleus (16.6 MeV) and the π^--spectrum we can obtain a value for the experimental forward elastic scattering cross-section very near to ours.

Our calculation is also in accordance with data of BECHER

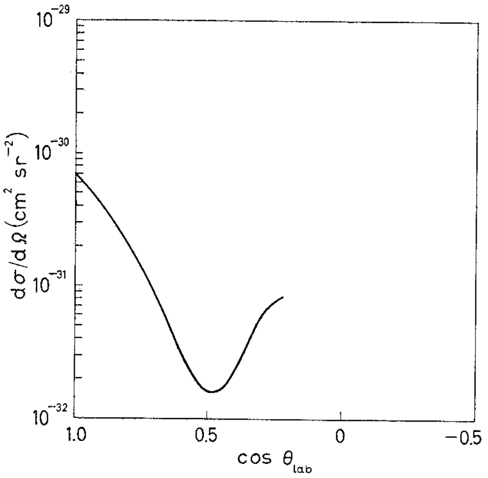

Fig. 4. – The angular distribution $(\mathrm{d}\sigma/\mathrm{d}\Omega)_{\mathrm{lab}}$ for $\pi^+ + {}^7\mathrm{Li} \to \pi^- + {}^7\mathrm{B}$ and for $E_\pi = 80$ MeV. $a_0 = 1.76 \cdot 10^{-13}$ cm.

(8) L. GILLY, M. J. MEUNIER, M. SPIGHEL, J. P. STROOT, P. DUTEIL and A. RODE: *Phys. Lett.*, **11**, 264 (1964).

and MARIC (2) when they assume that double charge exchange occurs on the $1p$ neutron pair and that the final nuclear wave function is the same as the initial one.

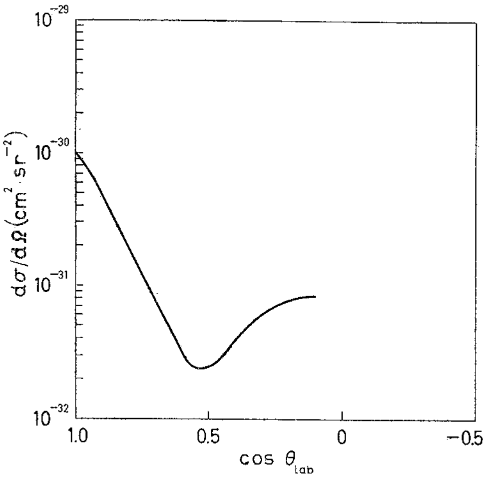

Fig. 5. – The angular distribution $(d\sigma/d\Omega)_{lab}$ for $\pi^+ + {}^7Li \to \pi^- + {}^7B$ and for $E_\pi = 100$ MeV. $a_0 = 1.76 \cdot 10^{-13}$ cm.

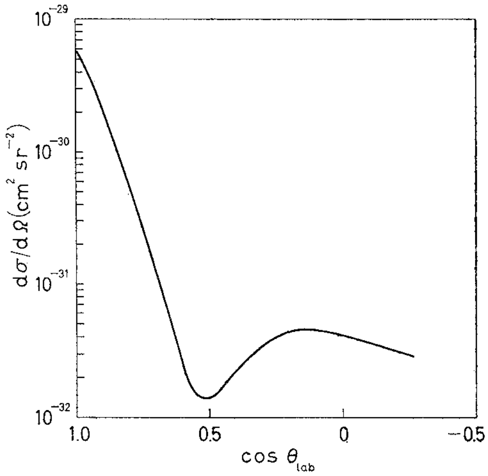

Fig. 6. – The angular distribution $(d\sigma/d\Omega)_{lab}$ for $\pi^+ + {}^7Li \to \pi^- + {}^7B$ and for $E_\pi = 195$ MeV. $a_0 = 1.76 \cdot 10^{-13}$ cm.

4. – Conclusion.

As one sees, for angles $\theta < \theta_0$, this simple theory can be used as a check to nuclear models and as a nuclear correlation study. The lack of experimental data does not allow this control.

* * *

It is a pleasure to thank Prof. B. TOUSCHEK and Dr. E. ETIM who red the manuscript and suggested many improvements.

RIASSUNTO

Si calcolano con il metodo di Glauber le sezioni d'urto per reazioni nucleari di semplice e doppio scambio di carica con pioni per nuclei dello strato $1p$. Si danno i risultati per pioni di energia cinetica $(80 \div 200)$ MeV per le seguenti reazioni: $\pi^+ + {}^{11}B \to \pi_0 + {}^{11}C$, $\pi^+ + {}^7Li \to \pi^- + {}^7B$.

Простые, но новые вычисления одно- и двух-зарядного обмена пионов на легких ядрах.

Резюме (*). — Используя метод Глаубера, вычисляется поперечное сечение для ядерных реакций пионов на «$1p$ оболочечных» ядрах с одно- и двух-зарядным обменом. Получаются результаты для реакций пионов $(80 \div 200)$ МэВ на боре $(\pi^+ + {}^{11}B \to \pi^0 + {}^{11}C)$ и литии $(\pi^+ + {}^7Li \to \pi^- + {}^7B)$.

(*) *Переведено редакцией.*