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# Minimal Solutions to the Conspiracy Problem and Classification of Regge-Pole Families.

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The most general approach to the conspiracy problem is, up to now, a group-theoretical one, essentially based on the expansion of the scattering amplitude in terms of the irreducible representation of the Lorentz group $O_{3,1}$ or of the group $O_4$. Since this formalism applies rigorously only at the point $t=0$ and for equal-mass scattering, more general formalisms have been developed recently in order to overcome these limitations. In particular very interesting results have been obtained recently by COSENZA, SCIARRINO and TOLLER [1,2]. Moreover the group-theoretical approach provides a classification of the Regge poles in families with well-defined quantum numbers [3].

In this note we will give some of the results of a nongroup-theoretical method, which will be described in full detail in a forthcoming paper.

We consider only the problems which arise in a neighbourhood of the point $t=0$. Our fundamental assumptions are: analiticity, crossing symmetry and the factorization principle for the residue functions of the Regge trajectories, which are supposed to be bosonic and with well-defined quantum numbers. In this note we will limit ourselves to the reactions $\mathrm{S}+\mathcal{N}\to \mathrm{J}+\mathcal{N}$ and the others related to these through factorization. Here $\mathcal{N}$ is a nucleon and J (S) is a spin $J$ ($S$) and mass $m_J$ ($m_S$) particle with

(1) G. COSENZA, A. SCIARRINO and M. TOLLER: Internal Report No. 158, Istituto di Fisica « G. Marconi », Roma (1968).

(2) G. COSENZA, A. SCIARRINO and M. TOLLER: Trieste preprint IC/68/24. In this paper a complete list of references about the group-theoretical work can be found.

(3) For a simple exposition of the relevant results of the group-theoretical approach to the scattering problem see L. BERTOCCHI: *Rapporteur's talk at the Heidelberg Conference on Elementary Particles* (1967).

$m_J \neq m_S \neq$ nucleon mass. We suppose in the following that $J \geqslant S$. Obviously these reactions are the most interesting from the physical point of view.

We will deduce the kinematical constraints at $t = 0$, without making use of transversity amplitudes ([4]), the « minimal » behaviour of the residue functions for the parent and daughter Regge trajectories and a classification of Regge trajectories in families equivalent to those deduced in the most general group-theoretical approach. In our approach the introduction of daughter trajectories is necessary, as in the spinless case ([5]), in order to ensure the analyticity of the full amplitude. The singularity structure of the daughter residue functions is however somewhat different than in the spinless case ([6]).

Let us quickly introduce some notations. The asymptotic contribution of a single Regge pole to the usual parity-conserving helicity amplitudes is given by

$$\tilde{f}^{(\pm)t}_{cd;ab} \underset{s \to \infty}{\simeq} \frac{2\alpha^\pm + 1}{\sin \pi\alpha^\pm} K^\pm_{cd,ab}(t)\, \gamma^\pm_{cd,ab}(t) \left(\frac{s}{s_0}\right)^{\alpha^\pm - N},$$

where $K^\pm$ is Wang's kinematical factor ([8]), $N = \max(|\lambda|, |\mu|)$, $\lambda = a - b$, $\mu = c - d$, and $\gamma^\pm_{cd,ab}(t)$ is the reduced residue free from kinematical singularities at $t = 0$. The objects which are assumed to factorize are the residue of the individual Regge poles: in our case the residue is

$$K^\pm_{cd,ab}(t)\, \gamma^\pm_{cd,ab}(t) \left(\frac{p_{cd}\, p_{ab}}{s_0}\right)^{\alpha^\pm(t) - N}.$$

We give now the kinematical constraints at $t = 0$ for the various cases in our discussion:

1) $EU$ case (*i.e.* $\mathrm{S} + \mathcal{N} \to \mathrm{J} + \mathcal{N}$). The constraints turn out to be

$$i\tilde{f}^{(+)\,t}_{cd,\frac{1}{2}-\frac{1}{2}} - \tilde{f}^{(-)t}_{cd,\frac{1}{2}\frac{1}{2}} = O(t)\,, \tag{1}$$

for any $c$ and $d$ satisfying the inequality $c \neq d$, and

$$\frac{i}{2}\, \tilde{f}^{(-)\,t}_{cc,\frac{1}{2}-\frac{1}{2}} - \tilde{f}^{(-)t}_{cc,\frac{1}{2}\frac{1}{2}} = O(t) \tag{2}$$

for any $c$.

2) $UU$ case (*i.e.* $\mathrm{S} + \mathrm{S} \to \mathrm{J} + \mathrm{J}$). The constraints are

$$\tilde{f}^{(+)t}_{cd,ab} + \tilde{f}^{(-)t}_{cd,ab} = O(t^m) \tag{3}$$

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([4]) G. COHEN-TANNOUDJI, A. MOREL and H. NAVELET: *Ann. of Phys.*, **46**, 239 (1968).

([5]) D. Z. FREEDMAN and J. M. WANG: *Phys. Rev.*, **153**, 1596 (1967).

([6]) This difference is mainly due to the fact that the so-called « parity-conserving » helicity amplitudes have definite parity only asymptotically, so that, when the parity-bounding phenomenon appears, a somewhat complicated cancellation between the members of the parity doublet is required ([7]). We are grateful to Prof. L. BERTOCCHI and Dr. A. SCIARRINO for enlightening discussions about this point.

([7]) P. DI VECCHIA, F. DRAGO and M. L. PACIELLO: to be published.

([8]) L. L. C. WANG: *Phys. Rev.*, **142**, 1187 (1966).

if $|\lambda-\mu|<|\lambda+\mu|$; and

$$\tilde{f}^{(+)t}_{cd,ab} - \tilde{f}^{(-)t}_{cd,ab} = O(t^m) \tag{4}$$

if $|\lambda-\mu|>|\lambda+\mu|$, where $m=\min(|\lambda|,|\mu|)$.

3) *EE* case: in the simplified treatment given here we have to consider only the nucleon-nucleon scattering. In this case the constraint is well known [9]:

$$f^t_{\frac{1}{2}\frac{1}{2},\frac{1}{2}\frac{1}{2}} + f^t_{\frac{1}{2}-\frac{1}{2},\frac{1}{2}-\frac{1}{2}} - f^t_{\frac{1}{2}\frac{1}{2},-\frac{1}{2}-\frac{1}{2}} - f^t_{\frac{1}{2}-\frac{1}{2},\frac{1}{2}-\frac{1}{2}} = O(t)\,. \tag{5}$$

The origin of the constraints (1), (2) and (5) are quite different: in fact while the latter follows from the forward conservation of the angular momentum in the direct $s$-channel, the former ones do not have so clear a physical origin. In fact they are a consequence of analyticity and crossing symmetry. They can be deduced following a method originally proposed by COHEN-TANNOUDJI, MOREL and NAVELET [4], writing down the crossing relations between the helicity amplitudes, *free from both s and t kinematical singularities*, in the $s$ and $t$ channels. The requirement that no spurious singularities are introduced leads to the constraints (1) and (2). The constraints (3) and (4) have essentially the same origin, although they can be deduced more simply without reference to the crossing matrix, following a method due to FRAUTSCHI and JONES [10]. The details of derivation of these constraints can be found in ref. [7,11].

It is well known that, in the Regge-pole model, these constraints can be satisfied in three different ways [12]: by evasion, by conspiracy between different poles and by a daughterlike conspiracy. In order to discuss this problem we first study the « minimal » solutions of the factorization conditions. In general, from a given solution to all the factorization requirements, one can obtain other solutions by increasing the number of $t$ powers of some of the Regge-pole residues in the original solution. If a given solution cannot be obtained from another in this way, we shall call it « minimal ». It is important to note that in order to decide if a solution is a minimal one, one must consider simoultaneously *all* the channels connected by the factorization.

In order to study the behaviour near $t=0$ of the residue functions one must write in compact form the factorization conditions, taking into account Wang's [8] kinematical factors. Once this is done, it is not too difficult to find the solutions given below [7,11]. For the reactions with unequal masses, of the type $S+S\to J+J$ or $S+S'\to J+J'$, one finds

$$\gamma^{\sigma}_{ab,ab} \sim t^{||\mu|-M|-n}\,. \tag{6}$$

When an equal-mass vertex is involved, the selection rules due to parity and $G$-parity invariance must be taken into account. This implies the identical vanishing of the residue if the following conditions are not satisfied: $\sigma\xi\tau(-1)^{S+1+n}=1$, $S=1$ if $\sigma=+1$.

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(9) D. V. VOLKOV and V. N. GRIBOV: *Sov. Phys. JETP*, **17**, 720 (1963).
(10) S. FRAUTSCHI and L. JONES: *Phys. Rev.*, **167**, 1335 (1968).
(11) P. DI VECCHIA, F. DRAGO and M. L. PACIELLO: Frascati Internal Report LNF-68/5, Frascati (1968) (unpublished).
(12) E. LEADER: *Phys. Rev.*, **166**, 1599 (1968).

Here $\sigma$ is the Regge-pole natural parity, $\tau$ is the signature of the parent Regge trajectory, the integer $n$ is the order of the daughter, and $S$ is the total spin of the $\mathcal{N}\bar{\mathcal{N}}$ system. The quantity $\xi$ is defined in terms of the internal quantum numbers $I$ and $G$ of the exchanged family by $\xi = G(-1)^I$. In the equal-mass reaction $\mathcal{N}+\mathcal{N} \to \mathcal{N}+\mathcal{N}$, for the residues which do not vanish we have

$$\gamma^{\sigma}_{ab,ab} \sim \begin{cases} t^M & \text{if} \quad (-1)^{\lambda+n} = \sigma\,, \\ t^{|M-1|} & \text{if} \quad (-1)^{\lambda+n} = -\sigma\,. \end{cases} \tag{7}$$

For the families with $\sigma = +1$ and $\tau = -\xi$, whose parent (and even daughter) trajectories do not couple to the $\mathcal{N}\bar{\mathcal{N}}$ system ([13]), we have for the odd daughters

$$\gamma^{(+)}_{ab,ab} \sim \begin{cases} t^{M+1} & \text{if} \quad \lambda = 0\,, \\ t^{|M-1|+1} & \text{if} \quad |\lambda| = 1\,. \end{cases} \tag{8}$$

(In this case however there are two other evasive solutions not present in the group-theoretical approach and that we do not discuss here.) The solutions for the equal-unequal mass case can be obtained through the factorization and can be found explicitly in ref. ([7]). $M$ is a number that we introduce at this point in order to label the minimal solutions. It can assume all the integer values between zero and infinity. Every family of Regge poles is characterized by a value of $M$. An interesting feature of our results is that, for all the values of the masses, a family with a given $M$ contributes asymptotically only to the forward $s$-channel helicity amplitudes with helicity flip equal to $\pm M$. $M$ is then the Toller quantum number introduced in the group-theoretical approach. This conclusion will be enforced by the following considerations.

The Regge-pole families can be classified according to the values of $M$, $\sigma$, $\xi$. These families and the residue behaviour are coincident with those found in the general group-theoretical approach ([1,2]).

*Class I*: $M = 0$, $\sigma = +1$, $\tau = -\xi$.

A parent trajectory requires the existence of a family of trajectories with « angular momentum » $\alpha_n(0) = \alpha(0) - n$, where $\alpha(0)$ is the intercept of the parent trajectory. For $n$ even we have $\tau_n = P_n = \xi = \tau$ and for $n$ odd $\tau_n = P_n = -\xi = -\tau$. Only the trajectory with $n$ even can couple to the $\mathcal{N}\bar{\mathcal{N}}$ system. This is class I of Freedman and Wang ([14]). Poles of this class *never conspire*, as can be seen from the behaviour of the residues listed above.

*Class Ia*: $M = 0$, $\sigma = +1$, $\tau = -\xi$.

Poles with $n$ even have $\tau_n = P_n = -\xi = \tau$ and with $n$ odd $\tau_n = P_n = \xi = -\tau$. The parent and the even daughters of this class do not couple to the $\mathcal{N}\bar{\mathcal{N}}$ system. This

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([13]) We thank Prof. M. TOLLER for calling our attention to these families.
([14]) D. Z. FREEDMAN and J. M. WANG: *Phys. Rev.*, 160, 1560 (1967).

explains why this class is not contained in the Freedman and Wang classification. Also these poles never conspire.

*Class II*: $M = 0$, $\sigma = -1$, $\tau = -\xi$.

The poles of this class satisfy the constraints (2) and (5) by a daughterlike conspiracy. All the other constraints are satisfied by evasion. For $n$ even we have $\tau_n = -P_n = -\xi = \tau$ and for $n$ odd $\tau_n = -P_n = \xi = -\tau$.

*Class IIa*: $M = 0$, $\sigma = -1$, $\tau = \xi$.

Poles with $n$ even have $\tau_n = -P_n = \xi = \tau$ and with $n$ odd $\tau_n = -P_n = -\xi = -\tau$. The parent trajectories of this class, which are decoupled from the $\mathcal{N}\overline{\mathcal{N}}$ system at $t = 0$, satisfy the constraints by evasion. Conspiracy between daughters is allowed.

*Class III*: $M = 1$, $\tau = \xi$.

In this class we find the well-known parity-doubling phenomenon. The parity doublet structure not only allows us to satisfy by conspiracy the constraints (1), (3), (4) and (5), but is also imposed by other general analyticity requirements. One has the following quantum numbers:

$$\sigma = +1 \begin{cases} n \text{ even}: & \tau_n = P_n = \xi = \tau\,, \\ n \text{ odd}: & \tau_n = P_n = -\xi = -\tau \end{cases}$$

(only trajectories with $n$ even can couple to the $\mathcal{N}\overline{\mathcal{N}}$ system);

$$\sigma = -1 \begin{cases} n \text{ even}: & \tau_n = -P_n = \xi = \tau\,, \\ n \text{ odd}: & \tau_n = -P_n = -\xi = -\tau\,. \end{cases}$$

*Class IIIa*: $M = 1$, $\tau = -\xi$.

The poles of this class have the following quantum numbers:

$$\sigma = +1 \begin{cases} n \text{ even}: & \tau_n = P_n = -\xi = \tau\,, \\ n \text{ odd}: & \tau_n = P_n = \xi = -\tau \end{cases}$$

(only trajectories with $n$ odd can couple to the $\mathcal{N}\overline{\mathcal{N}}$ system);

$$\sigma = -1 \begin{cases} n \text{ even}: & \tau_n = -P_n = -\xi = \tau\,, \\ n \text{ odd}: & \tau_n = -P_n = \xi = -\tau\,. \end{cases}$$

The parent trajectories satisfy the constraints by evasion. Conspiracy between daughter trajectories is allowed. Poles with $M > 1$ are decoupled, at $t = 0$, from the $\mathcal{N}\overline{\mathcal{N}}$ system, in agreement with the group-theoretical results.

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