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Finite-Energy Sum Rules for Pion Photoproduction and the ω Regge Pole Residue Functions.

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Summary. — We have studied the behaviour of the ω residue functions in pion photoproduction. We show that they have no zeros at $t \simeq -0.15\ (\text{GeV}/c)^2$; this is in striking contradiction with the usual explanation of the cross-over phenomena. The behaviour of the ω trajectory near the wrong signature nonsense point $\alpha = 0$ has also been investigated. The usual theory of the dip seems to work well and suggests that unlike the ρ, the ω trajectory is nonsense-choosing.

1. – Introduction and statement of the problem.

During the last year the Regge-pole theory has been applied successfully to the description of high-energy elementary particle interactions. However, the large number of free parameters present in the theory would probably make all the high-energy fits meaningless if one did not have the possibility of connecting the various processes, extracting in this way common features of the various Regge poles. Such a possibility is provided, for the Regge-pole residues, by the factorization theorem [1,2], which is assumed to hold in general as a direct consequence of the unitarity condition.

Although the factorization hypothesis gives generally self-consistent results,

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(1) M. GELL-MANN: *Phys. Rev. Lett.*, **8**, 263 (1962).
(2) V. N. GRIBOV and I. Y. POMERANCHUK: *Phys. Rev. Lett.*, **8**, 343 (1962).

it has recently led to serious difficulties about the behaviour of the ω-pole residue functions [3,4]. For the sake of clarity let us state the problem from the beginning.

The problem arises in connection with the so-called cross-over phenomenon (*). In fact it is an experimental fact, known for a long time [5], that if one considers elastic scattering processes π^+p, K^+p, and pp, and the processes in which the incident particle has been changed into the corresponding antiparticle, namely π^-p, K^-p, $\bar{p}p$, one finds that the process which has the biggest cross-section at $t=0$ has also the biggest slope. Therefore the momentum transfer distributions cross each other, irrespectively of the value of the energy, near a common value of momentum transfer: $t=t_c\simeq-0.15\ (\text{GeV}/c)^2$. More precisely stated, if we define the quantity

$$\Delta=\frac{d\sigma(\bar{A}B)}{dt}-\frac{d\sigma(AB)}{dt},$$

we find that it changes sign near $t=t_c$ at all the explored energies (**). This problem has been discussed, from the point of view of the Regge-pole theory, particularly in ref. [6,7]. In Regge theory the quantity Δ has the following expression:

$$\Delta=2\,\text{Re}\sum_{\{\lambda\}}E^*_{\{\lambda\}}O_{\{\lambda\}},$$

where $\{\lambda\}$ denotes the helicities in the crossed t-channel, $E_{\{\lambda\}}$ is the sum of the contributions of all Regge-pole exchanges with even charge conjugation, and $O_{\{\lambda\}}$ is the sum of those with odd charge conjugation.

There are now several experimental facts which indicate that in all the processes under consideration, the helicity nonflip part of $E_{\{\lambda\}}$ is essentially imaginary and large at $t=0$, while the helicity flip part of $E_{\{\lambda\}}$ is much smaller. Since the point where the cross-over occurs is not too far from $t=0$, the imaginary part of the nonhelicity flip $E_{\{\lambda\}}$ will still give the dominant contribution to the full scattering amplitude, and will still be essentially imag-

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(3) R. J. N. PHILLIPS: private communication and *Heidelberg Conference on Elementary Particles Physics, October* 1967.

(4) V. BARGER and L. DURAND III: *Phys. Rev. Lett.*, **19**, 1295 (1967).

(*) We are indebted to Dr. R. J. N. PHILLIPS for pointing this out to us.

(5) K. J. FOLEY, S. J. LINDENBAUM, W. A. LOVE, S. OZAKI, J. J. RUSELL and L. C. L. YUAN: *Phys. Rev. Lett.*, **10**, 376 (1963); **11**, 425, 503 (1963); **15**, 45 (1965).

(**) We are always interested in the high-energy range.

(6) R. J. N. PHILLIPS and W. RARITA: *Phys. Rev.*, **139**, B 1336 (1965).

(7) W. RARITA, R. J. RIDDELL, C. B. CHIU and R. J. N. PHILLIPS: University of California Radiation Laboratory Report, UCRL-17523 (1967). Additional references are given in this paper.

inary. We are thus led to the following approximate expression:

$$\Delta \simeq 2 \sum_{\text{nonflip}} \operatorname{Im} E^*_{\{\lambda\}} \operatorname{Im} O_{\{\lambda\}} .$$

We see from this that the quantity Δ can change sign if and only if the imaginary parts of the helicity nonflip amplitudes change sign. Let us see how this argument works in various cases. Consider first the $\pi^{\pm}$-p scattering; in this process $O_{\{\lambda\}}$ receives a contribution only from ρ-exchange (*): a consistent picture is obtained by requiring that the factorized helicity nonflip $\mathcal{N}\overline{\mathcal{N}}$ residue change sign at $t = t_c$, while the helicity flip residue must stay finite in order to explain the maximum of the momentum transfer distribution in the reaction $\pi^- + p \to \pi^0 + n$ near $t = t_c$.

Let us now discuss the much more involved problem of the p-p and $\bar{p}$-p cross-over. Empirically $O_{\{\lambda\}}$ is found to receive the largest contribution from ω-exchange. In fact the coupling of the ρ-trajectory to the nucleon-antinucleon system is much weaker than the ω coupling, as can be deduced from the smallness of the cross-section for the p-n charge-exchange reaction compared to the elastic scattering. The φ exchange is usually neglected since its coupling to the nucleon-antinucleon system is apparently very weak. If this is the real situation, the cross-over in the p-p and $\bar{p}$-p scattering requires that the helicity nonflip-nonflip residue function much change sign at $t = t_c$. This apparently reasonable requirement gives rise to one of the most serious difficulties for the Regge-pole theory. In fact it is an easy matter to combine the factorization theorem with the analyticity and reality requirements for the amplitudes to show that not only all the $\omega\mathcal{N}\overline{\mathcal{N}}$ factorized residue functions must vanish at $t = t_c$, but that all the ω residue functions must vanish in any reaction at that point. This possibility, although it explains nicely the cross-over in the $K^{\pm}p$ scattering, is clearly unpleasant and in contradiction with the information that can be deduced from other processes, as will be discussed below. We note at this point that the behaviour assumed for the ρ residue is completely different from the one assumed for the ω residue.

We emphasize that the foregoing conclusions are a direct consequence of the assumption that the ω Regge pole gives the only significant $C = -1$ contribution to the $\bar{p}p$ and pp elastic scattering amplitudes and that the factorization theorem holds.

The difficulty arises when one looks at the high-energy data on the reaction $\gamma + p \to \pi^0 + p$. During the last year this reaction has been studied by

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(*) The ρ'-exchange is neglected since it is not important for this kind of considerations.

several authors (8-10) in the framework of the Regge-pole theory. The only quantum numbers that can be exchanged are those of the ω, ρ and B mesons. The ρ-exchange contribution is generally neglected because of the relatively weak $\rho\mathcal{N}\bar{\mathcal{N}}$ and $\rho\pi\gamma$ coupling. It can be shown (9) however that the inclusion of the ρ exchange does not change the results drastically. The B contribution however seems to be important (10), at least at moderate energies, despite the fact that its trajectory is supposed to be low: in fact it is needed in order to explain the absence of a zero at the nonsense point where the ω trajectory goes to zero. Therefore if one looks only at the photoproduction reaction, disregarding completely any other information from other processes, one is naturally led to the following picture:

1) ω and B exchange are both important in π^0 photoproduction;

2) the kinematical constraint at $t=0$ is satisfied by evasion (9-12) (see below);

3) the ω trajectory is similar to the well-known ρ trajectory; all the residue functions can be approximated with constants.

In this way one can explain very well the experimental dip at $t=0$, and the nearly forward maximum in the momentum transfer distribution at $t \simeq -0.1$ $(\text{GeV}/c)^2$.

This model is however inconsistent with the previous interpretation of the cross-over effect: in fact one has the problem of explaining a maximum exactly where one would expect a striking minimum. In order to explain this situation, BARGER and DURAND (4) assumed that extra ω-type contributions must be present and denoted them by $\bar{\omega}$: they suggested that the $\bar{\omega}$ could be a conspiring pole or a conspiring cut. Obviously in this case the cross-over in $\bar{p}p$ and pp scattering can be reproduced provided that the helicity nonflip component of $\text{Im}(\omega+\bar{\omega})$ changes sign at $t=t_c$. This does not give any restriction on the ω-exchange residue. However, if this is the case, a fit to the high-energy photoproduction data would require a very rapid and unusual behaviour of the residue or of the discontinuities across the cut in the small $|t|$ region $(|t|\leqslant 1\ (\text{GeV}/c)^2)$ and a distinction between the various possibilities is by no means possible with the available high-energy data.

In order to obtain further insight into this problem we have used the

(8) M. P. LOCHER and H. ROLLNIK: *Phys. Lett.*, **22**, 696 (1966).
(9) P. DI VECCHIA and F. DRAGO: *Phys. Lett.*, **24** B, 405 (1967).
(10) F. P. ADER, M. CAPDEVILLE and PH. SALIN: CERN preprint, TH. 803 (1967).
(11) M. B. HALPERN: *Phys. Rev.*, **160**, 1441 (1967).
(12) S. FRAUTSCHI and L. JONES: *Phys. Rev.*, **163**, 1820 (1967).

« finite-energy sum rule » ([13-15]) techniques by means of which the high-energy parameters can be expressed as integrals over the low-energy amplitudes. This analysis is in essential agreement with the statements 1), 2) and 3) given above and with an analysis of the reactions ([16]) $\pi^{\pm}+p\to\rho^{\pm}+p$ and $\pi^{-}+p\to\rho^{0}+n$ from which the ω contribution can be extracted easily.

In Sect. 2 we give the photoproduction amplitudes; in Sect. 3 we deduce the sum rules; in Sect. 4 we discuss the general behaviour of the ω residue functions; in Sect. 5 we discuss the problem of the dips for the ω and ρ trajectories using the sum-rule approach; the related problem of the saturation of the Schwarz superconvergence relations is also discussed.

2. – The photoproduction amplitudes.

We give in this Section the essential machinery of the photoproduction reactions. Following the classical paper by CHEW, GOLDBERGER, LOW and NAMBU ([17]) the matrix element for single-pion photoproduction can be written as

$$H = \sum_{j} H^{(j)} g^{(j)}, \tag{2.1}$$

$$H^{(j)} = \sum_{i} M_i A_i^{(j)}(\nu, t), \tag{2.2}$$

with

$$M_1 = i\gamma_5(\gamma\cdot\varepsilon)(\gamma\cdot k),$$

$$M_2 = 2i\gamma_5[(P\cdot\varepsilon)(q\cdot k)-(P\cdot k)(q\cdot\varepsilon)],$$

$$M_3 = \gamma_5[(\gamma\cdot\varepsilon)(q\cdot k)-(\gamma\cdot k)(q\cdot\varepsilon)],$$

$$M_4 = 2\gamma_5[(\gamma\cdot\varepsilon)(P\cdot k)-(\gamma\cdot k)(P\cdot\varepsilon)-iM(\gamma\cdot\varepsilon)(\gamma\cdot k)],$$

where k and q are respectively the photon and pion four-momenta, $P=\frac{1}{2}(p_1+p_2)$, p_1 and p_2 being the initial and final nucleon four-momenta, and ε is the photon

(13) K. IGI and S. MATSUDA: *Phys. Rev. Lett.*, **18**, 625 (1967).

(14) A. A. LOGUNOV, L. D. SOLOVIEV and A. N. TAVKHELIDZE: *Phys. Lett.*, **24** B, 181 (1967); V. A. MESHCHERIAKOV, K. V. RERIKH, A. N. TAVKHLIDZE and V. I. ZHURAREEV: *Phys. Lett.*, **25** B, 341 (1967).

(15) R. DOLEN, D. HORN and C. SCHMID: *Phys. Rev. Lett.*, **19**, 402 (1967); preprint CALT-68-143 (1967).

(16) A. P. CONTOGOURIS, J. TRAN THANH VAN and H. F. LUBATTI: *Phys. Rev. Lett.*, **19**, 1352 (1967).

(17) G. F. CHEW, M. L. GOLDBERGER, F. E. LOW and Y. NAMBU: *Phys. Rev.*, **106**, 1345 (1957).

polarization. We define, furthermore

$$\nu_1 = -\frac{(qk)}{2M}, \qquad \nu = -\frac{(Pk)}{M} = \frac{W^2 - M^2}{2M} - \nu_1, \qquad t = \mu^2 - 4M\nu_1, \qquad s = W^2.$$

μ and M are the pion and nucleon masses, and W is the c.m. energy.

The superscript (j) refers as usual to the values $(\pm, 0)$, and

$$g^{(+)} = \delta_{\alpha 3}, \qquad g^{(-)} = \tfrac{1}{2}[\tau_\alpha, \tau_3], \qquad g^{(0)} = \tau_\alpha,$$

where α is the isospin index of the outgoing pion.

Using standard techniques, the invariant functions may be projected on the helicity amplitudes in the crossed t-channel. One obtains, suppressing the isospin index,

$$\bar{f}^{+}_{\frac{1}{2}\frac{1}{2},10} = \bar{f}^{+}_{01} = -\frac{k_t\sqrt{t}}{2}(A_1 - 2MA_4), \tag{2.3}$$

$$\bar{f}^{-}_{\frac{1}{2}\frac{1}{2},10} = \bar{f}^{-}_{01} = p_t k_t (A_1 + tA_2), \tag{2.4}$$

$$\bar{f}^{+}_{\frac{1}{2}-\frac{1}{2},10} = \bar{f}^{+}_{11} = \frac{k_t}{2}(2MA_1 - tA_4), \tag{2.5}$$

$$\bar{f}^{-}_{\frac{1}{2}-\frac{1}{2},10} = \bar{f}^{-}_{11} = -p_t k_t \sqrt{t} A_3, \tag{2.6}$$

where

$$\bar{f}_{\lambda\mu} = \left(\cos\frac{\theta_t}{2}\right)^{-|\lambda+\mu|}\left(\sin\frac{\theta_t}{2}\right)^{-|\lambda-\mu|} f_{\lambda\mu},$$

$$\bar{f}^{\pm}_{\lambda\mu} = \bar{f}_{\lambda\mu} \pm \bar{f}_{-\lambda,\mu}$$

and k_t, p_t, θ_t are respectively the boson momentum, nucleon momentum and scattering angle in the c.m. system of the t-channel. The $\bar{f}_{\lambda\mu}$ are free from kinematical singularities in s ([18]) and satisfy an unsubtracted dispersion relation at fixed t, as do the invariant functions ([17]). Moreover one can show ([18]) that only natural (unnatural) parity poles contribute to the asymptotic behaviour of the $\bar{f}^{+}_{\lambda\mu}$ ($\bar{f}^{-}_{\lambda\mu}$).

The π^0-photoproduction amplitudes are given by

$$A_i^{(\pi_0)} = A_i^{(+)} + A_i^{(0)},$$

the $A_i^{(+)}$ amplitudes receive contributions from $I = 0^-$ poles (*i.e.* the ω), and

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([18]) M. GELL-MANN, M. L. GOLDBERGER, F. E. LOW, E. MARX and F. ZACHARIASEN: *Phys. Rev.*, **133**, B 145 (1964).

the $A_i^{(0)}$ from $I=1^+$ poles (*i.e.* ρ and B). In the following we shall be mostly interested in the isospin (+) amplitudes so that we shall drop the superscript (+) where this omission cannot give rise to ambiguity.

We remind the reader also of the kinematical constraint that has to be satisfied at $t=0$ [11,12] (*)

$$\sqrt{t}\, \bar{f}_{01}^{-} = i\sqrt{t}\, \bar{f}_{11}^{+}. \tag{2.7}$$

It is usually assumed that the ω Regge pole satisfies this constraint by evasion; that means that the ω helicity flip residue function must vanish at $t=0$. A similar evasive behaviour is assumed to hold also for the ρ and B trajectories.

This property of the residue function will play an important role in the following discussion.

3. – The sum rules.

Finite-energy sum rules are consistency conditions imposed by analyticity [13-15]: if a function $F(\nu, t)$ satisfies a dispersion relation for fixed t and can be expanded at high energy ($\nu \gg A$) as a sum of Regge poles

$$\frac{\pm 1 - \exp[-i\pi\alpha_i]}{\sin \pi\alpha_i}\beta_i(t)\left(\frac{\nu}{\nu_0}\right)^{\alpha_i(t)}$$

then the following relations are equally valid (**) (***):

$$\frac{1}{A^{n+1}}\int_0^A \nu^n \operatorname{Im} F(\nu, t)\, d\nu = \sum_i \frac{\beta_i(t)}{\alpha_i + n + 1} A^{\alpha_i(t)}, \tag{3.1}$$

where n is an even or an odd positive integer when $F(\nu, t)$ is an odd or even function of ν. It should be noted that it is irrelevant in the sum rule whether a singularity is above or below a certain point, say $\alpha = -1$. In fact when $(\alpha_i + n) < -1$ one can obtain the relation (3.1) directly from the superconvergence relation for the amplitude $\nu^n F(\nu, t)$, while for $(\alpha_i + n) > -1$ one starts by writing down a superconvergence relation for the amplitude $\nu^n[F(\nu, t) -$

(*) Note that both $\bar{f}_{01}^{-}$ and $\bar{f}_{11}^{+}$ have a singular behaviour $\sim (t)^{-\frac{1}{2}}$ for $t \sim 0$.

(**) The integration is over the right-hand cut in s and is always defined to include the Born term even if the latter occurs at negative values of ν.

(***) For simplicity of writing here and in the following we take $\nu_0 = 1$ GeV, all the energies being expressed in GeV.

$-F_R(\nu, t)]$, where $F_R(\nu, t)$ contains the contributions of all the Regge poles such that $(\alpha_i+n)>-1$; from this superconvergence relation one again obtains the relation (3.1).

Sum rules of the kind (3.1) provide a relation between the high-energy Regge parameters, trajectories and residue functions, and the low-energy parameters, masses and widths, that characterize the s-channel resonance spectrum.

By means of relations of the type (3.1) we shall test the compatibility of the hypothesis made in the description of high-energy π^0 photoproduction with the low-energy data, and we shall investigate the structure of the residue functions $\beta_\omega(t)$.

Starting from the dispersion relations for the amplitudes A given in ref. (17), the following sum rules are easily obtained for the isospin (+) amplitudes:

$$S_i^n = R(A_i)\nu_B^n + \frac{\mu}{\pi}\int_{\nu_0}^{A}\nu^n \operatorname{Im} A_i(\nu, t)\,d\nu = \frac{\mu}{\pi}\frac{\sin\pi\alpha_\omega}{\alpha_\omega+n} g_i(t) A^{\alpha_\omega(t)+n}, \tag{3.2}$$

where

$$\nu_B = \frac{t-\mu^2}{4M}, \qquad \nu_0 = \mu + \frac{t-\mu^2}{4M},$$

$$R(A_1^+) = -\frac{1}{2} ef,$$

$$R(A_2^+) = -\left(\frac{1}{2M\nu_1}\right)\frac{1}{2} ef,$$

$$R(A_3^+) = R(A_4^+) = \frac{ef}{2M}(\mu_p^a - \mu_n),$$

where μ_p^a and μ_n are the rationalized anomalous nucleon magnetic moments and e and f are the rationalized and renormalized electronic charge and pion nucleon coupling constant: $e^2/4\pi = 1/137$ and $f^2/4\pi \simeq 0.08$. Taking into account the crossing properties of the A_i^+ amplitudes (17) n must be an odd positive integer for $i = 1, 2, 4$ and an even positive integer for $i = 3$. The functions $g_i(t)$ are explicitly given in the Appendix: they are proportional to some combinations of the helicity flip and helicity nonflip residue functions of the ω Regge trajectory.

Relations (3.2) hold under the hypothesis that no other singularities, in the $I = 0^-$ channel beside the ω Regge pole, are present in the complex J-plane to the right of $J = -n$. In the following, n will be equal to one or three for $i = 1, 2, 4$ and to zero or two for $i = 3$. For $A \to \infty$, S_3^0 turns into a well-known superconvergence relation (19).

(19) A. BIETTI, P. DI VECCHIA and F. DRAGO: *Nuovo Cimento*, **49** A, 511 (1967). References to previous works can be found in this paper.

We saturate the sum rules (3.2) using a phenomenological multipole analysis of pion photoproduction up to $W = 1.8$ GeV done by WALKER at Cal-Tech [20]. In this analysis the experimental data are fitted by six resonances (the $\mathcal{N}^*$ $J = \frac{3}{2}^+$ at 1.236 GeV, the $J = \frac{3}{2}^-$ at 1.519 GeV, the $J = \frac{5}{2}^+$ at 1.672 GeV, the $J = \frac{5}{2}^-$ at 1.652 GeV, the $J = \frac{1}{2}^+$ at 1.471 GeV and the $J = \frac{1}{2}^-$ at 1.561 GeV, respectively with multipoles M_{1+} and E_{1+}, E_{2-} and M_{2-}, E_{3-} and M_{3-}, E_{2+} and M_{2+}, M_{1-} and E_{0+}), the Born amplitudes and a « background » introduced in order to have a better fit at certain energies. The resonant multipoles are given in Breit-Wigner form with energy-dependent width. Using the relations between the invariant amplitudes $A_i(\nu, t)$ and the multipoles $M_{l\pm}$ and $E_{l\pm}$ given by CGLN we can thus directly saturate the sum rules.

4. – Discussion.

We give in Table I the values of the various S_i^n obtained by numerical integration. In Table II we give the combinations $R_{01}^n = [S_1^n - 2MS_4^n]$ and

TABLE I.

t (GeV/c)^2	0.1	0.0	−0.1	−0.2	−0.3	−0.4	−0.5	−0.6	−0.7	−0.8	−0.9	−1.0
MS_1^1 (GeV)	0.02	0.03	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.03	0.03	0.02
MS_1^3 (GeV)^3	0.00	0.01	0.02	0.02	0.02	0.02	0.02	0.02	0.01	0.00	0.00	−0.01
MS_2^1 (GeV)$^{-1}$	0.14	0.13	0.12	0.11	0.10	0.09	0.08	0.07	0.06	0.06	0.05	0.05
MS_2^3 (GeV)	0.18	0.15	0.12	0.10	0.08	0.06	0.04	0.03	0.02	0.01	0.00	−0.01
MS_4^1	0.34	0.30	0.25	0.21	0.17	0.13	0.10	0.06	0.03	0.00	−0.03	−0.06
MS_4^3 (GeV)^2	0.17	0.13	0.10	0.08	0.06	0.04	0.03	0.01	0.01	0.00	−0.01	−0.01

$R_{11}^n = [2MS_1^n - tS_4^n]$ which are proportional respectively to the helicity-nonflip and helicity-flip ω residue functions (see (2.3), (2.5) and Appendix). The discussion of this data is complicated by the simultaneous presence of various aspects of the problem, as we shall see.

As a first step we must be sure that only the ω pole contributes to the isospin (+) amplitudes. To this end we try to determine the ω trajectory using the method proposed in ref. [15]; in fact it is easy to show that, if one

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([20]) R. L. WALKER: private communication. This phenomenological fit has already been used in ref. ([18]).

TABLE II.

t (GeV/c)^2	0.1	0.0	−0.1	−0.2	−0.3	−0.4
α_ω^1	−0.01	−0.55	−0.29	−0.17	−0.13	−0.16
α_ω^2	1.73	1.45	1.11	0.79	0.49	0.19
α_ω^4	−0.14	−0.45	−0.49	−0.53	−0.56	−0.59
t (GeV/c)^2	−0.5	−0.6	−0.7	−0.8	−0.9	−1.0
α_ω^1	−0.26	−0.40	−0.60	−0.84	−1.13	−1.46
α_ω^2	−0.10	−0.37	−0.62	−0.84	−1.04	−1.21
α_ω^4	−0.63	−0.68	−0.77	−3.36	−0.51	−0.59

defines $X_i(t) = S_i^3/A^2 S_i^1$ for $i = 1, 2, 4$ (*), the trajectory that one obtains from the various sum rules is given by

$$\alpha_\omega^i(t) = \frac{3X_i(t) - 1}{1 - X_i(t)} . \tag{4.1}$$

Using this formula one obtains very bad results; for reference they are given in Table III. However it is very easy to explain the complete failure of rela-

TABLE III.

t (GeV/c)^2	0.2	0.1	0.0	−0.1	−0.2	−0.3	−0.4
$(S_1^1 - 2MS_4^1)M$ (GeV)	−0.73	−0.63	−0.53	−0.44	−0.36	−0.28	−0.21
$(S_1^3 - 2MS_4^3)M$ (GeV)^3	−0.41	−0.32	−0.24	−0.18	−0.13	−0.09	−0.05
$(2MS_1^1 - tS_4^1)M$ (GeV)^2	−0.02	0.01	0.01	0.10	0.12	0.13	0.14
$(2MS_1^3 - tS_4^3)M$ (GeV)^4	−0.07	−0.02	0.02	0.05	0.06	0.06	0.06

t (GeV/c)^2	−0.5	−0.6	−0.7	−0.8	−0.9	−1.0
$(S_1^1 - 2MS_4^1)M$ (GeV)	−0.14	−0.08	−0.02	+0.03	0.08	0.13
$(S_1^3 - 2MS_4^3)M$ (GeV)^3	−0.03	−0.01	0.00	0.01	0.01	0.01
$(2MS_1^1 - tS_4^1)M$ (GeV)^2	0.13	0.11	0.09	0.06	0.02	−0.02
$(2MS_1^3 - tS_4^3)M$ (GeV)^4	0.05	0.04	0.02	0.01	−0.01	−0.03

(*) We disregard in the following the sum rules obtained from the amplitude A_3. This is because to ω-pole enters in this amplitude only in nonasymptotic form (*i.e.* $\sim \nu^{\alpha_\omega - 2}$) and with a residue function singular at $t = 0$, so that daughter trajectories are necessary in order to restore the analyticity of the full amplitude. The relation of the daughter to the parent trajectory, although known at $t = 0$, is generally unknown, so that the one-pole approximation for the A_3 amplitude seems unlikely.

tion (4.1) (*). In fact the value of $\alpha(t)$ determined from (4.1) is extremely sensitive to the value of X_i (for $X_i = 0.3$, $\alpha = -0.14$, for $X_i = 0.4$ $\alpha = 0.33$, and for $X_i = 0.5$, $\alpha = 1$!) which in turns depends on the third momentum sum rule. Now at $W = 1.8$ GeV the resonance spectrum is certainly not exhausted and it is clear that the contribution of the higher resonances will affect much more strongly the third than the first momentum sum rule. This is confirmed from the study of the residue functions: in fact we find that the residues determined from the third momentum sum rule are different from those determined from the first momentum sum rule. They are smaller or bigger in such a way that the failure of relation (4.1) can be completely understood. The previous discussion leads to the conclusion that relations of the type (4.1) are generally not reliable, so that, for example, the need of additional trajectories derived from the failure of this kind of relations is not to be taken too seriously in our opinion.

In order to find the trajectory we have therefore used the following method. We have evaluated the first momentum sum rules at various values of t $\left(-0.5(\text{GeV}/c)^2 \leqslant t \leqslant 0\right)$ with different upper limits A, and then we have fitted the right-hand side of eq. (3.2) to the values so obtained. The best fit is obtained with constant residue functions (once the evasion of the spin flip residue has been taken in account, see below) and a linear trajectory given by

$$\alpha_\omega(t) = 0.61 + 0.75t\,. \tag{4.2}$$

The ω trajectory is not so well known from other processes, different values being given by the various authors. In our opinion this trajectory is perfectly acceptable for the ω exchange(**), so that no other contribution besides the ω pole is necessary in order to saturate the sum rules (3.2). This conclusion will be strengthened by the following discussion: in fact we shall assume from now on that only the ω-exchange contribution is present and we shall see that in this way the behaviour of R^n_{01} and R^n_{11} can be understood in a satisfactory way.

Let us first look at the sign change of R^1_{11} and R^3_{11} near $t = 0.1$ $(\text{GeV}/c)^2$. This zero is expected from the evasive behaviour at $t = 0$ of the pole; of course one cannot expect that the sum rules reproduce it exactly at $t = 0$. One may ask if it is not possible that the ω helicity flip residue satisfies the constraint (2.7) going to zero at $t = 0$ like an even power of t, say like t^2, so that the sign change in R^1_{11} and R^3_{11} would be caused by the sign change of the residue at $t = t_c \simeq -0.13$ $(\text{GeV}/c)^2$. This possibility does not appear very

(*) We are indebted to Prof. A. Bietti for a discussion about this.

(**) It is interesting to note that the value $\alpha_\omega(t) = 0.60 + 0.75t$ has been obtained independently from a fit to the high-energy π^0 photoproduction (3).

likely; moreover if one believes in the factorization theorem it can easily be shown that the behaviour $\bar{f}^+_{11} \sim t^2$ near $t = 0$ would imply that the amplitude $\bar{f}^+_{01} \sim t$ and the ω contribution to the nucleon-nucleon scattering amplitude $f^+_{\frac{1}{2}\frac{1}{2},\frac{1}{2}\frac{1}{2}}$ goes like t^2 or that $f^+_{\frac{1}{2}-\frac{1}{2},\frac{1}{2}-\frac{1}{2}} \sim t^3$; both these possibilities appear to be very unlikely. Moreover in the isospin (0) amplitudes the same sign change around $t = 0$ is reproduced by the low-energy data, via finite-energy sum rules, in the ρ residue function of the amplitude $\bar{f}^+_{11}$ and in the B residue function of the amplitude $\bar{f}^-_{01}$.

We conclude that almost certainly the ω helicity flip residue functions do not change sign at $t = t_c$ and that certainly the helicity nonflip residue does not change sign at t_c. In fact if the helicity nonflip residue changes sign at t_c the only possible way not to have an overall sign change is that it has another zero near $t = 0$. However, if this is the case we should expect that R^1_{01} and R^3_{01} are strongly depressed in the small t region, and this does not happen, as can be seen from Table II.

5. – The problem of the dips.

As a by-product of our investigation we can study the behaviour of the ω exchange at the wrong signature nonsense point $\alpha = 0$. The usual theory of the dips, neglecting for the moment the third double spectral function effects, gives, according to the various mechanisms, the following behaviour near $\alpha = 0$ for the amplitudes $\bar{f}^+_{0.1}$ (sense-nonsense transition) and $\bar{f}^+_{11}$ (nonsense-nonsense transition):

1) Sense choosing: $\bar{f}^+_{01} \sim \alpha$, $\bar{f}^+_{11} \sim \alpha^2$.

2) Nonsense choosing: $\bar{f}^+_{01} \sim \alpha$, $f^+_{11} \sim \alpha$.

3) Chew mechanism: $\bar{f}^+_{01} \sim \alpha^2$, $\bar{f}^+_{11} \sim \alpha^3$.

4) Noncompensating mechanism: $f^+_{01} \sim \alpha^2$, $\bar{f}^+_{11} \sim \alpha^2$.

We see from Table II that $\bar{f}^+_{01}$ changes sign near $t = -0.7$ (GeV/c)^2 and $\bar{f}^+_{11}$ between -0.8 and -1 (GeV/c)^2.

A similar investigation has been carried out for the ρ-exchange contribution; we found in this case that $\bar{f}^+_{01}$ changes sign near $t = -0.4$ (GeV/c)^2 and $\bar{f}^+_{11}$ does not change sign; nor does it go to zero. Moreover the ρ residue functions are found, as expected, to be smaller than that for ω by something more than an order of magnitude. If these data can be taken seriously, various interpretations are possible. The fact that for the ρ-exchange contribution $\bar{f}^+_{01}$ changes sign and $\bar{f}^+_{11}$ does not can be understood assuming that the ρ-trajectory

« chooses sense »: this is the expected behaviour for this pole ([21]). The displacement of the zero near $t = -0.4\ (\text{GeV}/c)^2$ can probably be understood, since of course we cannot expect that the low-energy fit reproduces, via sum rules, a zero exactly where it is expected. The displacement also may be due to the third double spectral function effects. The nonvanishing of $\bar{f}_{11}^{+}$ at $\alpha_\rho = 0$ can probably be understood since this amplitude has to vanish at $t = 0$, since it is assumed to satisfy the constraints (2.7) by evasion, and again at $\alpha_\rho = 0$, where it is expected to have a very narrow dip owing to the α_ρ^2 behaviour. We feel that it is unlikely that the low-energy data, at least in the form in which we know them now, could reproduce such a narrow dip.

The behaviour of the ω trajectory however does not seem to agree with the usual naïve symmetry arguments: in fact in terms of the possibilities 1) ... 4) given above, we see that one is forced to assume that, unlike that of the ρ, the ω trajectory is nonsense-choosing.

The previous discussion has ignored the third spectral function effects, or in other words, the possible presence of fixed poles at nonsense-wrong signature points. In fact it has been shown by MANDELSTAM and WANG ([22]) that, if the third spectral function effects are important, a precise distinction between trajectories which choose sense and those which choose nonsense at an integer of the wrong signature is no longer possible and all the usual dip-analysis breaks down. Since in our case this analysis seems to work well if one assumes that the small displacement of the zero is due to the fit, one is led to the conclusion that the third double spectral function effects are negligible. However if this is the case, one can derive an observable consequence of the absence of fixed poles, namely a Schwarz ([23]) superconvergence relation. SCHWARZ has shown that, if no fixed poles are present at the nonsense points of the wrong signature, superconvergence relations can be derived also for the amplitudes which have the wrong crossing properties (the condition that no fixed poles are present at the right signature-nonsense points gives the ordinary superconvergence relations).

In the photoproduction case we can derive Schwarz superconvergence relations for the amplitudes $A_3^{(-)}$, $[A_1^{(+)}+tA_2^{(+)}]$, $[A_1^{(0)}+tA_2^{(0)}]$. The amplitude $[A_1^{(0)}+tA_2^{(0)}]$ receives contributions from the B exchange and the $A_3^{(-)}$ eventually only from the A_1 trajectory, which does not seem to play an important role in pion photoproduction. The amplitude $[A_1^{(+)}+tA_1^{(+)}]$ can receive a contribution only from the exchange of unnatural parity $I^G = 0^-$ trajectories: none of the known trajectories have these quantum numbers, so that the super-

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([21]) A. ARBAB and C. B. CHIU: *Phys. Rev.*, **147**, 1045 (1966); G. HÖLER, J. BAACKE, H. SCHLAILE and P. SONDEREGGER: *Phys. Lett.*, **20**, 79 (1966).

([22]) S. MANDELSTAM and L. L. WANG: *Phys. Rev.*, **160**, 1490 (1967).

([23]) J. H. SCHWARZ: *Phys. Rev.*, **159**, 1269 (1967).

convergence relation for this amplitude is expected to converge very rapidly, if it converges at all. In fact the ordinary superconvergence relation for $\nu[A_1^+ + tA_2^+]$ converges very rapidly: from Table I we can see the cancellation which is effective between S_1 and tS_2.

The Schwarz relations for the amplitudes in discussion are

$$\{R[A_1^{(\overset{+}{0})}] + tR[A_1^{(\overset{+}{0})}]\} + \frac{\mu}{\pi}\int_{\nu_0}^{\infty}\mathrm{Im}\,\{A_1^{(\overset{+}{0})}(\nu, t) + tA_2^{(\overset{+}{0})}(\nu, t)\}\,\mathrm{d}\nu = 0\;, \tag{5.1}$$

$$R[A_3^{(-)}] + \frac{\pi}{\mu}\int_{\nu_0}^{\infty}\mathrm{Im}\,A_3^{(-)}(\nu, t)\,\mathrm{d}\nu = 0\;, \tag{5.2}$$

where

$$R[A_1^{(0)}] = R[A_1^{(+)}]\,,$$
$$R[A_2^{(0)}] = R[A_2^{(+)}]\,,$$
$$R[A_3^{(-)}] = R[A_4^{(+)}]\,.$$

All these relations do not seem to converge, as can be seen from Table IV. This fact can be understood, particularly for the isospin (+) amplitudes, only if one assumes that fixed poles with nonnegligible residues are present at $J = 0$ in the wrong signature partial-wave amplitudes.

The previous analysis seems to show that a simple direct connection be-

TABLE IV *a*.

t (GeV/c)^2	M (Born term)	W_m	$[\{R[A_1^0] + tR[A_2^0]\} + (\mu/\pi)\int_{\nu_0}^{\nu_m}\mathrm{Im}[A_1^0 + tA_2^0]\,\mathrm{d}\nu]M$	$\{[R(A_1^+) + tR(A_2^+)] + (\mu/\pi)\int_{\nu_0}^{\nu_m}\mathrm{Im}[A_1^+ + tA_2^+]\,\mathrm{d}\nu\}M$
0.0	−0.142	1.60	−0.15	−0.08
		1.80	−0.15	−0.07
−0.1	0.098	1.60	0.09	0.18
		1.80	0.09	0.18
−0.2	0.117	1.60	0.12	0.21
		1.80	0.11	0.21
−0.3	0.126	1.60	0.12	0.23
		1.80	0.12	0.23
−0.4	0.130	1.60	0.13	0.24
		1.80	0.13	0.24

W_m is the c.m. energy up to which the integral has been evaluated. All the numbers given, except t, are in GeV.

TABLE IV *b*.

t (GeV/c)^2	W_m	$\mu\{R(A_3^-)+(\mu/\pi)\int_{\nu_0}^{\nu_m}\operatorname{Im}A_3^-(\nu,t)\,\mathrm{d}\nu\}$
0	1.60	0.04
	1.80	0.03
−0.1	1.60	0.05
	1.80	0.03
−0.2	1.60	0.05
	1.80	0.04
−0.3	1.60	0.06
	1.80	0.05
−0.4	1.60	0.07
	1.80	0.06

W_m is the c.m. energy (in GeV) up to which the integral has been evaluated. The Born term $MR(A_3^-) = -0.0098$.

tween the nonconvergence of the Schwarz relations and the breakdown of the dip analysis cannot generally be inferred. A similar situation has been met in ref. (15) for the π-$\mathcal{N}$ system.

6. – Conclusions.

Using finite-energy sum rules we have shown that the photoproduction isospin (+) amplitudes are dominated by the ω-exchange and that there is no need of additional ϖ contributions, in the form of a new pole or of a cut; moreover the ω residue functions do not change sign in contradiction with the usual interpretation of the cross-over phenomenon. If one cannot show with some certainty that additional ϖ contributions are present in $K^{\pm}p$, $\bar{p}$-p and p-p scattering (*) and if no other explanation of the cross-over phenomena can be found in the framework of the Regge-pole theory, one is forced to admit that the factorization theorem is not of general validity.

(*) Finite-energy sum rules at $t=0$ have been applied to the K-$\mathcal{N}$ system (24). The sum rules have been saturated with the experimental cross-section, via the optical theorem, and quite good agreement has been found with the high-energy model of PHILLIPS and RARITA (6). This result however cannot be considered conclusive since one can certainly conceive of a model in which, for example, a conspiring pole does not contribute at $t=0$ (a model of this kind has been discussed for the ρ' in ref. (25)).

(24) A. BORGESE, M. COLOCCI, M. LUSIGNOLI, M. RESTIGNOLI and G. VIOLINI: University of Rome preprint (1967).

(25) L. SERTORIO and M. TOLLER: *Phys. Rev. Lett.*, **19**, 1146 (1967).

We have also found some evidence that the ω trajectory is nonsense-choosing at $\alpha_\omega = 0$, while it is generally assumed on symmetry grounds that it is sense-choosing, like that of the ρ.

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APPENDIX

We give here some details about the Reggeization of the photoproduction amplitudes.

The relation between the invariant amplitudes and the parity-conserving helicity amplitudes free from s and t kinematical singularities [26] are given by (*)

$$\begin{aligned}\tilde{f}^{+}_{01} &= A_1 - 2MA_4\,,\\ \tilde{f}^{-}_{01} &= (t-\mu^2)[A_1 + tA_2]\,,\\ \tilde{f}^{+}_{11} &= 2MA_1 - tA_4\,,\\ \tilde{f}^{-}_{11} &= (t-\mu^2)A_3\,.\end{aligned}$$

We are only interested in the asymptotic ω-exchange contribution to these amplitudes. Using the standard Reggeization rules [18] we find

$$\begin{aligned}\tilde{f}^{+}_{01} &= \quad 2h(t)\left[\frac{\alpha}{\alpha+1}\right]^{\frac{1}{2}}\beta^{+}_{01}(t)(-2Z)^{\alpha-1}\,,\\ \tilde{f}^{+}_{11} &= -2h(t)\left[\frac{\alpha}{\alpha+1}\right]\beta^{+}_{11}(t)(-2Z)^{\alpha-1}\,,\\ \tilde{f}^{-}_{01} &= \quad 0\,,\\ \tilde{f}^{-}_{11} &= \quad o(Z^{\alpha-1})\,,\end{aligned}$$

where

$$h(t) = \pi^{\frac{1}{2}}\frac{2\alpha+1}{\sin\pi\alpha}\,\frac{\Gamma(\alpha+\frac{1}{2})}{\Gamma(\alpha+1)}\cdot\frac{1-\exp[-i\pi\alpha]}{2} \qquad \text{and} \qquad Z = \cos\theta_t\,,$$

θ_t being the scattering angle in the c.m. system of the t-channel.

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[26] L. L. WANG: *Phys. Rev.*, **142**, 1187 (1966); **153**, 1664 (1967).

(*) The relation between the $\tilde{f}^{\pm}_{\lambda\mu}$ and the $\bar{f}^{\pm}_{\lambda\mu}$ defined in Sect. **2** is given by $\bar{f}^{\pm}_{\lambda\mu} = K^{\pm}(t)\tilde{f}^{\pm}_{\lambda\mu}$, where $K^{\pm}(t)$ contains all the kinematical singularities in t of the amplitude, and can be evaluated using the rules given in ref. [26].

From these expressions we obtain

$$\tilde{f}_{01}^{+} = 2h(t)F_{01}(\alpha)\gamma_{01}^{+}(t)\left(\frac{\nu}{\nu_0}\right)^{\alpha-1},$$

$$\tilde{f}_{11}^{+} = -2h(t)F_{11}[\alpha]\gamma_{11}^{+}(t)\left(\frac{\nu}{\nu_0}\right)^{\alpha-1},$$

where the reduced residues $\gamma(t)$ are assumed to be analytic functions of t for $t\leqslant 0$ and $F_{01}(\alpha)$ and $F_{11}(\alpha)$ are some functions of $\alpha(t)$ which depend on the particular mechanism that the trajectory chooses at the nonsense points. The behaviour near $\alpha=0$ of $\tilde{f}_{10}^{+}$ and $\tilde{f}_{11}^{+}$ has been given in Sect. **5**. The t in $\tilde{f}_{11}^{+}$ comes from the assumption that the kinematical constraint (2.7) is satisfied by evasion.

From the relations

$$A_1 = \frac{1}{4M^2-t}[2M\tilde{f}_{11}^{+} - t\tilde{f}_{01}^{+}],$$

$$A_2 = -\frac{1}{t}A_1,$$

$$A_3 = O(\nu^{\alpha-2}),$$

$$A_4 = \frac{1}{4M^2-t}[\tilde{f}_{11}^{+} - 2M\tilde{f}_{01}^{+}]$$

the functions $g_i(t)$ defined in Sect. **3** are easily obtained:

$$g_1(t) = -\frac{\pi^{\frac{1}{2}}}{4M^2-t}\,\frac{2\alpha+1}{\sin\pi\alpha}\,\frac{\Gamma(\alpha+\frac{1}{2})}{\Gamma(\alpha+1)}\,\alpha t[2MF_1(\alpha)\gamma_{11}^{+} + F_3(\alpha)\gamma_{01}^{+}],$$

$$g_2(t) = -\frac{1}{t}g_1(t),$$

$$g_4(t) = -\frac{\pi^{\frac{1}{2}}}{4M^2-t}\,\frac{2\alpha+1}{\sin\pi\alpha}\,\frac{\Gamma(\alpha+\frac{1}{2})}{\Gamma(\alpha+1)}\,\alpha[\alpha tF_1(\alpha)\gamma_{11}^{+} + 2MF_3(\alpha)\gamma_{01}^{+}].$$

RIASSUNTO

Si è studiato il comportamento dei residui dell'omega nella fotoproduzione di pioni. Si mostra che questi non hanno zeri a $t\simeq -0.15\ (\text{GeV}/c)^2$; ciò è in contraddizione con l'usuale interpretazione del fenomeno del cross-over. Si è inoltre studiato il comportamento della traiettoria dell'ω intorno al punto $\alpha=0$. Viene suggerito che, a differenza di quella del ϱ, la traiettoria dell'ω sceglie nonsenso ad $\alpha=0$.

Правила сумм при конечных энергиях для фоторождения пионов и ω остаточные функции для полюсов Редже.

Резюме (*). — Мы изучили поведение ω остаточных функций при фоторождении пионов. Мы показываем, что они не имеют нулей при $t \simeq -0.15\,(\text{ГэВ}/c)^2$: это находится в поразительном противоречии с обычным об'яснением кроссинг-явлений. Также было исследовано поведение ω-траектории вблизи бессмысленной точки с неправильной сигнатурой $\alpha=0$. Кажется, что обычная теория «наклона» хорошо работает, и предполагается, что в противоположность ρ, траектория ω выбирается без физического смысла.

(*) *Переведено редакцией.*