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Conspiracy and Factorization in Vector-Meson Photoproduction.

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Summary. — We deduce the conspiracy relations for reactions of the type $\gamma+\mathcal{N}\to V+\mathcal{N}$. We then study the residua factorization that, in principle, could give some information about conspiracy or evasion. We show that, in this case, the group theoretical results about the conspiracy or the evasion of the involved Regge trajectories can be deduced from crossing symmetry, analyticity, factorization and some very general experimental information.

1. – Introduction.

Recently the importance of conspiracy relations among scattering amplitudes at $t=0$ has been discussed (1). The first example of conspiracy relations has been discovered by Goldberger, Grisaru, Mac Dowell and Wong (2) in their classical paper on nucleon-nucleon scattering; the same example has been extensively discussed by Gribov and Volkov (3). In these early papers constraints between the helicity amplitudes were found by writing down the relations which connect the invariant scalar amplitudes to the helicity ampli-

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(*) Sponsored in part by the Air Force Office of Scientific Research through the European Office of Aerospace Research, OAR, United States Air Force, under contract F61052 67 C 0084.

(1) M. Gell-Mann and E. Leader: *XIII International Conference on High Energy Physics at Berkeley*, 1966.

(2) M. L. Goldberger, M. T. Grisaru, S. W. Mac Dowell and D. Y. Wong: *Phys. Rev.*, **120**, 2250 (1960).

(3) D. V. Volkov and V. N. Gribov: *Sov. Phys. JETP*, **17**, 720 (1963).

tudes: in order that spurious singularities are not introduced in the invariant amplitudes, some suitable combinations of kinematic singularity-free helicity amplitudes must vanish in particular kinematical configurations.

In recent papers [4-6] the problem of conspiracy relations has been discussed in a more general framework starting from the crossing relations [7,8], between the helicity amplitudes. However a uniform discussion, able to cover in a satisfactory way all the possible kinematical situations, has not yet been given. In fact, in the equal-mass case the deduction of conspiracy relations at $t=0$ does not give rise to any difficulty and their physical meaning is very clear being linked in a very simple way to the forward angular momentum conservation in the crossed channel. On the contrary the situation is not so clear in the unequal-mass case.

In the present paper we discuss some aspects of conspiracy and Regge behaviour, studying in detail the factorization requirements, for reactions of the type $\gamma+\mathcal{N}\to V+\mathcal{N}$.

In Sect. **2** we derive the conspiracy relations at $t=0$ following essentially the method due to COHEN-TANNOUDJI *et al.* [6] (see however also ref. [5]). In this way conspiracy relations have been derived for the reactions $\gamma+\mathcal{N}\to\pi+\mathcal{N}$ [9] and $\pi+\mathcal{N}\to V+\mathcal{N}$ [10].

In Sect. **3** we study the Regge representation (*i.e.* asymptotic behaviour of the Sommerfeld-Watson transformed partial-wave expansion) for our reactions.

In Sect. **4** we study the consequences of the factorization of the residues on the sets of reactions $\overline{\mathcal{N}}+\mathcal{N}\to V+\gamma$, $\overline{\mathcal{N}}+\mathcal{N}\to\overline{\mathcal{N}}+\mathcal{N}$, $V+\gamma\to V+\gamma$. In Appendix A we give kinematical factors involved in the crossing matrix.

2. – Conspiracy relations in $\gamma+\mathcal{N}\to V+\mathcal{N}$.

We shall use the customary notation $f^t_{cd;ab}$ to denote a helicity [11] amplitude for the t-channel reaction $a+b\to c+d$. Helicity amplitudes free from

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([4]) E. ABERS and V. L. TEPLITZ: *Phys. Rev.*, **158**, 1365 (1967).

([5]) H. HOGAASEN and PH. SALIN: *Nucl. Phys.*, B **2**, 657 (1967).

([6]) G. COHEN-TANNOUDJI, A. MOREL and H. NAVELET: *Kinematical singularities, crossing matrix and kinematical constraints for two body helicity amplitudes*, Saclay preprint (1967).

([7]) T. L. TRUEMAN and G. C. WICK: *Ann. Phys.*, **26**, 322 (1964).

([8]) I. MUZINICH: *Journ. Math. Phys.*, **5**, 1481 (1964).

([9]) S. FRAUTSCHI and L. JONES: *Small angle photoproduction and conspiracy*, Caltech preprint CALT-68-132 (1967).

([10]) S. FRAUTSCHI and L. JONES: *Conspiracy relations in vector meson production*, Caltech preprint CALT-68-142 (1967).

([11]) M. JACOB and G. C. WICK: *Ann. Phys.*, **7**, 404 (1959).

kinematical singularities in s and t must be used in the derivation of conspiracy relations. The first step is to define amplitudes [12,13] free from kinematical singularities in s:

$$\bar{f}^t_{cd;ab} = \left(\sin\frac{\theta_t}{2}\right)^{-|\lambda-\mu|}\left(\cos\frac{\theta_t}{2}\right)^{-|\lambda+\mu|} f^t_{cd;ab}\,, \tag{2.1}$$

where $\lambda = a - b$, $\mu = c - d$.

The works by HARA [14] and WANG [15] show how one can then remove the t kinematical singularities from the amplitudes formed into « parity conserving » combinations. The Wang result can be written

$$\bar{f}^t_{cd;ab} \pm \bar{f}^t_{-c-d;ab} = K^{\pm}_{cd;ab}\bar{\bar{f}}^t_{cd;ab}\,, \tag{2.2}$$

where $K^{\pm}$ is a known factor containing the kinematical singularities: the behaviour at $t = 0$ of the relevant $K^{\pm}$ is listed in Table I.

The amplitudes $\bar{\bar{f}}_{cd;ab}(s, t)$, being free from kinematical singularities in s and t, contain only the dynamics and can be Reggeized [12].

The conspiracy relations provide additional kinematic zeros at $t = 0$ in certain linear combinations of the parity-conserving amplitudes.

The derivation of the conspiracy relations is given in some detail in Appendix B. The relations obtained at $t = 0$ are

$$it^{\frac{1}{2}}[\bar{\bar{f}}^t_{-11;\frac{1}{2}-\frac{1}{2}} - \bar{\bar{f}}^t_{1-1;\frac{1}{2}-\frac{1}{2}}] = t^{\frac{1}{2}}[\bar{\bar{f}}^t_{-11;\frac{1}{2}\frac{1}{2}} - \bar{\bar{f}}^t_{1-1;\frac{1}{2}\frac{1}{2}}]\,, \tag{2.3}$$

$$it^{\frac{1}{2}}[\bar{\bar{f}}^t_{01;\frac{1}{2}-\frac{1}{2}} + \bar{\bar{f}}^t_{0-1;\frac{1}{2}-\frac{1}{2}}] = t^{\frac{1}{2}}[\bar{\bar{f}}^t_{01;\frac{1}{2}\frac{1}{2}} + \bar{\bar{f}}^t_{0-1;\frac{1}{2}\frac{1}{2}}]\,, \tag{2.4}$$

$$\frac{i}{2}\,t^{\frac{1}{2}}[\bar{\bar{f}}^t_{11;\frac{1}{2}-\frac{1}{2}} - \bar{\bar{f}}^t_{-1-1;\frac{1}{2}-\frac{1}{2}}] = t^{\frac{1}{2}}[\bar{\bar{f}}^t_{11;\frac{1}{2}\frac{1}{2}} - \bar{\bar{f}}^t_{-1-1;\frac{1}{2}\frac{1}{2}}]\,. \tag{2.5}$$

For all the parity-conserving amplitudes involved here, the Wang kinematical factor K allows at $t^{-\frac{1}{2}}$ behaviour at $t = 0$ (Table I).

If only Regge poles contribute to the scattering amplitudes, these relations can be satisfied in two different ways:

1) All the residues of the $\bar{\bar{f}}^t$ involved contain a factor of t and therefore they vanish individually at $t = 0$ (evasion).

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(12) M. GELL-MANN, M. L. GOLDBERGER, F. E. LOW, E. MARX and F. ZACHARIASEN: *Phys. Rev.*, **133**, B 145 (1964).

(13) F. CALOGERO and J. CHARAP: *Ann. Phys.*, **26**, 44 (1964); F. CALOGERO, J. CHARAP and E. SQUIRES: *Ann. Phys.*, **25**, 325 (1963).

(14) Y. HARA: *Phys. Rev.*, **136**, B 507 (1964).

(15) L. L. WANG: *Phys. Rev.*, **142**, 1187 (1966); **153**, 1664 (1967).

TABLE I.

	Amplitude	$\lvert\lambda\rvert$	$\lvert\mu\rvert$	Behaviour near $t=0$ $K^{\pm}(t)$	Dominant parity
a	$\bar{f}^t_{11;\frac{1}{2}\frac{1}{2}} + \bar{f}^t_{-1-1;\frac{1}{2}\frac{1}{2}}$	0	0	1	$(-1)^J$
b	$\bar{f}^t_{11;\frac{1}{2}\frac{1}{2}} - \bar{f}^t_{-1-1;\frac{1}{2}\frac{1}{2}}$	0	0	$t^{-\frac{1}{2}}$	$(-1)^{J+1}$
c	$\bar{f}^t_{11;\frac{1}{2}-\frac{1}{2}} + \bar{f}^t_{-1-1;\frac{1}{2}-\frac{1}{2}}$	1	0	1	$(-1)^J$
d	$\bar{f}^t_{11;\frac{1}{2}-\frac{1}{2}} - \bar{f}^t_{-1-1;\frac{1}{2}-\frac{1}{2}}$	1	0	$t^{-\frac{1}{2}}$	$(-1)^{J+1}$
e	$\bar{f}^t_{01;\frac{1}{2}\frac{1}{2}} + \bar{f}^t_{0-1;\frac{1}{2}\frac{1}{2}}$	0	1	$t^{-\frac{1}{2}}$	$(-1)^{J+1}$
f	$\bar{f}^t_{01;\frac{1}{2}\frac{1}{2}} - \bar{f}^t_{0-1;\frac{1}{2}\frac{1}{2}}$	0	1	1	$(-1)^J$
g	$\bar{f}^t_{01;\frac{1}{2}-\frac{1}{2}} + \bar{f}^t_{0-1;\frac{1}{2}-\frac{1}{2}}$	1	1	$t^{-\frac{1}{2}}$	$(-1)^J$
h	$\bar{f}^t_{01;\frac{1}{2}-\frac{1}{2}} - \bar{f}^t_{0-1;\frac{1}{2}-\frac{1}{2}}$	1	1	1	$(-1)^{J+1}$
i	$\bar{f}^t_{-11;\frac{1}{2}\frac{1}{2}} + \bar{f}^t_{1-1;\frac{1}{2}\frac{1}{2}}$	0	2	1	$(-1)^J$
l	$\bar{f}^t_{-11;\frac{1}{2}\frac{1}{2}} - \bar{f}^t_{1-1;\frac{1}{2}\frac{1}{2}}$	0	2	$t^{-\frac{1}{2}}$	$(-1)^{J+1}$
m	$\bar{f}^t_{-11;\frac{1}{2}-\frac{1}{2}} + \bar{f}^t_{1-1;\frac{1}{2}-\frac{1}{2}}$	1	2	1	$(-1)^{J+1}$
n	$\bar{f}^t_{-11;\frac{1}{2}-\frac{1}{2}} - \bar{f}^t_{1-1;\frac{1}{2}-\frac{1}{2}}$	1	2	$t^{-\frac{1}{2}}$	$(-1)^J$

2) Every amplitude $\bar{f}^t$ retains its singular behaviour at $t=0$: in this case both sides of the equation must approach to the same constant (conspiracy).

If only one trajectory contributes to the amplitudes it is impossible to have conspiracy: in fact amplitudes with different dominant parity appear in the left- and right-hand sides of eqs. (2.3) and (2.4); on the other hand in eq. (2.5) enter amplitudes with the same dominant parity but different behaviour in s. Therefore one can think of two mechanisms for the conspiracy:

a) Conspiracy between different trajectories: this implies constraints on the intercepts $\alpha(0)$ of the various trajectories.

b) Daughter trajectories come into play and combine with main ones.

Similar possibilities are open for the contribution of other singularities in the angular momentum complex plane (cuts and fixed poles).

The two possibilities are experimentally distinguishable because in the first case (evasion) all the helicity flip amplitudes are suppressed, in the s-channel, for small t near the forward direction, while in the second case (conspiracy) only those amplitudes which do not conserve the angular momentum are suppressed for small t. This feature is not peculiar to the reactions $\gamma+\mathcal{N}\to V+\mathcal{N}$, but is also present in other cases [9,10] and can be discussed with the method presented in Sect. **3** of ref. [10].

A more detailed discussion of these conspiracy relations will be given in connection with the study of the constraints imposed by the factorization. We note here only that, in the group theoretical language, relations (2.3) and (2.4) require a « Class III conspiracy », while relation (2.5) requires a « Class II » conspiracy [16-19].

In order to have in Sect. **4** all the possible kinematical constraints at $t=0$ concerning the reactions related to our process through the factorization theorem, we give the conspiracy relations in $\mathcal{N}+\mathcal{N}\to\mathcal{N}+\mathcal{N}$:

$$f^t_{\frac{1}{2}\frac{1}{2};\frac{1}{2}\frac{1}{2}}+f^t_{\frac{1}{2}-\frac{1}{2};-\frac{1}{2}\frac{1}{2}}-f^t_{\frac{1}{2}\frac{1}{2};-\frac{1}{2}-\frac{1}{2}}-f^t_{\frac{1}{2}-\frac{1}{2};\frac{1}{2}-\frac{1}{2}}=0 \tag{2.6}$$

and in $\gamma+\gamma\to V+V$:

$$\bar{\bar{f}}^{+}_{-11;1-1}-\bar{\bar{f}}^{-}_{-11;1-1}=0\,, \tag{2.7}$$

$$\bar{\bar{f}}^{+}_{01;01}+\bar{\bar{f}}^{-}_{01;01}=0\,. \tag{2.8}$$

The relation (2.6) can be found in ref. [2]. The relations (2.7) and (2.8) have been obtained following the method discussed in Appendix B.

3. – The Regge representation and the factorization of the residues at $t=0$.

The partial-wave expansion for the parity-conserving amplitudes, free from s kinematical singularities, is given by [12]

$$\bar{f}^t_{cd;ab}\pm\eta_c\eta_d(-1)^{\lambda+M+s_c+s_d-\nu}\bar{f}^t_{-c-d;ab}=\sum_J(2J+1)[e^{J+}_{\lambda\mu}F^{J\pm}_{cd;ab}+e^{J-}_{\lambda\mu}F^{J\mp}_{cd;ab}]\,, \tag{3.1}$$

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[16] G. DOMOKOS and P. SURANYI: *Nucl. Phys.*, **54**, 529 (1964).

[17] M. TOLLER: Internal Reports No. 76 and 84, Istituto di Fisica « G. Marconi », Roma (1965); M. TOLLER: CERN preprint TH 780, Geneva, 1967.

[18] D. Z. FREEDMAN and J. M. WANG: Berkeley preprint 1967.

[19] L. BERTOCCHI: rapporteur's talk at the 1967 *Heidelberg Conference*, CERN Report TH 835, Geneva, 1967. A complete list of references and a simple discussion of these topics can be found in this paper.

here η means intrinsic parity, S spin, $M = \max(|\lambda|; |\mu|)$, v is $\frac{1}{2}$ for half-integral S_c+S_d and 0 for integral S_c+S_d and the $e^{J\pm}_{\lambda\mu}$ functions are defined in ref. ([12]). We perform the Sommerfeld-Watson transformation on the previous expansion (3.1) and, assuming that only moving poles are present in the complex angular momentum plane, we find that, if $|\cos\theta_t| \gg 1$, the asymptotic contribution of a single Regge pole $\alpha^\pm(t)$ to (3.1) is given by (*)

$$\pi\frac{2\alpha^\pm(t)+1}{\sin\pi\alpha^\pm(t)}E^{\alpha^\pm}_{\lambda\mu}(\cos\theta_t)\bar{K}^\pm_{cd;ab}\left(\frac{p_{ab}p_{cd}}{s_0}\right)^{\alpha^\pm-M}\gamma^\pm_{cd;ab}\sim\bar{K}^\pm_{cd;ab}(t)\gamma^\pm_{cd;ab}(t)\left(\frac{s}{s_0}\right)^{\alpha^\pm-M}, \tag{3.2}$$

where $\bar{K}^\pm_{cd;ab}$ is the Wang kinematic factor and $\gamma_{cd;ab}$ is the dynamical part of the residue.

Here and in the following we omit the signature factor not essential for our considerations.

The reason for studying the factorization requirements is that, as shown by several authors ([10,21,22]), the factorization requirements definitely imply additional t-dependence in some cases, which could give some information about conspiracy or evasion.

It is generally assumed that the residua of Regge poles obey the factorization condition $[\beta^{(a+b\to c+d)}]^2 = \beta^{(a+b\to a+b)}\cdot\beta^{(c+d\to c+d)}$ all along the trajectory ([23,24]).

The quantities which factorize are the residues of the individual poles in the $F^{J\pm}_{cd;ab}$ (**): then in our case the residue is

$$\bar{K}^\pm_{cd;ab}(t)\gamma^\pm_{cd;ab}(t)(p_{ab}p_{cd})^{\alpha^\pm-M}. \tag{3.3}$$

(*) We note that the Regge representation breaks down at $t=0$: in fact at $t=0$, $\cos\theta_t=0$ (see Appendix A) for any s no matter how large and so the expansion (3.2) is not valid in the limit $t=0$.

In the spinless case this difficulty was overcome by FREEDMAN and WANG ([20]) by introducing the daughter trajectories: using the Khuri representation these authors showed that the $s^{\alpha(t)}$ behaviour survives at $t=0$ provided that one introduces as required by the analyticity, for any trajectory $\alpha(t)=\alpha_0(t)$ a family of trajectories of alternating signature satisfying $\alpha_n(0)=\alpha_0(0)-n$. The extension of the Freedman and Wang work to the case of the scattering of particles with spin, described by the helicity formalism, is not trivial and should be investigated in detail. In the following we will assume the (3.2) to hold for any t, regardless of the specific mechanism by which the $t=0$ difficulty is overcome.

([20]) D. Z. FREEDMAN and J. M. WANG: *Phys. Rev.*, **153**, 1597 (1967).

([21]) L. L. WANG: *Phys. Rev.*, **153**, 1664 (1967).

([22]) E. LEADER: *Conspiracy and evasion: a property of Regge poles*, Cambridge University preprint (1967).

([23]) V. N. GRIBOV and I. YA. POMERANCHUK: *Phys. Rev. Lett.*, **8**, 343 (1962).

([24]) M. GELL-MANN: *Phys. Rev. Lett.*, **8**, 263 (1962).

(**) We factorize the residua of the partial-wave parity-conserving amplitudes, since in the hypothesis of parity conservation, a given pole will only contribute to one of the two F.

For the conspiracy relations we are interested in the factorization at $t=0$: however we can study the factorization at $t\neq 0$, the requirements so obtained being valid up to $t=0$ under the hypothesis of the validity at $t=0$ of the representation (3.2).

Hence if reaction 1 is $a+b\to a+b$, reaction 2 is $a+b\to c+d$ and reaction 3 is $c+d\to c+d$, the factorization condition can be stated as

$$[\gamma_2(t)K_2(t)(p_{ab}p_{cd})^{\alpha-M_2}]^2=[\gamma_1(t)K_1(t)(p_{ab})^{2(\alpha-M_1)}][\gamma_3(t)K_3(t)(p_{cd})^{2(\alpha-M_3)}]\,. \tag{3.4}$$

The behaviour near $t=0$ of both sides of eq. (3.4) is given in Table II*a*).

TABLE II *a*.

	Factorization conditions in $\mathcal{N}\mathcal{N}\to V\gamma$	Factorization satisfied?
a	$(\gamma^{+}_{11;\frac12\frac12})^2 = (\gamma^{+}_{11;11})(\gamma^{+}_{\frac12\frac12;\frac12\frac12})$	yes
b	$(\gamma^{-}_{11;\frac12\frac12})^2 = t\gamma^{-}_{\frac12\frac12;\frac12\frac12}\gamma^{-}_{11;11}$	no
c	$t\,(\gamma^{+}_{11;\frac12-\frac12})^2 = \gamma^{+}_{\frac12-\frac12;\frac12-\frac12}\gamma^{+}_{11;11}$	no
d	$(\gamma^{-}_{11;\frac12-\frac12})^2 = \gamma^{-}_{\frac12-\frac12;\frac12-\frac12}\gamma^{-}_{11;11}$	yes
e	$(\gamma^{-}_{01;\frac12\frac12})^2 = \gamma^{-}_{\frac12\frac12;\frac12\frac12}\gamma^{-}_{01;01}$	yes
f	$t\,(\gamma^{+}_{01;\frac12\frac12})^2 = \gamma^{+}_{01;01}\gamma^{+}_{\frac12\frac12;\frac12\frac12}$	no
g	$(\gamma^{+}_{01;\frac12-\frac12})^2 = (\gamma^{+}_{01;01})(\gamma^{+}_{\frac12-\frac12;\frac12-\frac12})$	yes
h	$t\,(\gamma^{-}_{01;\frac12-\frac12})^2 = \gamma^{-}_{\frac12-\frac12;\frac12-\frac12}\gamma^{-}_{01;01}$	no
i	$t^2(\gamma^{+}_{-11;\frac12\frac12})^2 = \gamma^{+}_{-11;-11}\gamma^{+}_{\frac12\frac12;\frac12\frac12}$	no
l	$t\,(\gamma^{-}_{-11;\frac12\frac12})^2 = \gamma^{-}_{\frac12\frac12;\frac12\frac12}\gamma^{-}_{-11;-11}$	no
m	$t^2(\gamma^{-}_{-11;\frac12-\frac12})^2 = \gamma^{-}_{\frac12-\frac12;\frac12-\frac12}\gamma^{-}_{-11;-11}$	no
n	$t\,(\gamma^{+}_{-11;\frac12-\frac12})^2 = \gamma^{+}_{\frac12-\frac12;\frac12-\frac12}\gamma^{+}_{-11;-11}$	no

That part of the kinematical factor $K^{\pm}(t)$ analytic at $t=0$ has been incorporated into the $\gamma^{\pm}$ functions.

We see that the factorization is satisfied by the kinematical factors for the amplitudes *a*), *d*), *f*), *h*), while the introduction of extra t factors ([10,21,22]) is

TABLE II *b*.

Factorization conditions in $V\gamma \to V\gamma$	Factorization satisfied?
$t^2(\gamma^{+}_{11;1-1})^2 = \gamma^{+}_{11;11}\gamma^{+}_{1-1;1-1}$	no
$t^2(\gamma^{-}_{11;1-1})^2 = \gamma^{-}_{11;11}\gamma^{-}_{1-1;1-1}$	no
$t\,(\gamma^{+}_{11;01})^2 = \gamma^{+}_{11;11}\gamma^{+}_{01;01}$	no
$t\,(\gamma^{-}_{11;01})^2 = \gamma^{-}_{11;11}\gamma^{-}_{01;01}$	no
$t\,(\gamma^{+}_{-11;01})^2 = \gamma^{+}_{-11;-11}\gamma^{+}_{01;01}$	no
$t\,(\gamma^{-}_{-11;01})^2 = \gamma^{-}_{-11;-11}\gamma^{-}_{01;01}$	no
Factorization condition in $\overline{\mathcal{N}}\mathcal{N} \to \overline{\mathcal{N}}\mathcal{N}$	
$t\,(\gamma^{+}_{\frac{1}{2}\frac{1}{2};\frac{1}{2}-\frac{1}{2}})^2 = \gamma^{+}_{\frac{1}{2}\frac{1}{2};\frac{1}{2}\frac{1}{2}}\gamma^{+}_{\frac{1}{2}-\frac{1}{2};\frac{1}{2}-\frac{1}{2}}$	no

necessary in order to satisfy the factorization requirements for the other amplitudes. In the following discussion we will consider only the asymptotic terms in s in (3.2).

4. – Discussion and conclusions.

We have shown that if the kinematical constraints in the photoproduction of vector mesons are to be satisfied by conspiracy, a Class III conspiracy is needed for relations (2.3) and (2.4), while a Class II conspiracy is needed for relation (2.5) (*).

We see (Table II*a*)) that, as a consequence of the factorization, relation (2.5) is satisfied by evasion, so that the only possible conspiracy left is Class III. The factorization conditions given in Tables II*a*), II*b*) imply evasion also for the conspiracy relation (2.7).

We show now that the results of the group-theoretical analysis of the relativistic scattering about the conspiracy or the evasion of the various Regge

(*) A conspiracy between trajectories of different spin-parity is called Class III, and between trajectories of the same spin-parity $P=(-1)^{J+1}$ Class II. Class I concontains only nonconspiring $P=(-1)^J$ trajectories.

trajectories can be deduced from crossing symmetry, analyticity (Wang's kinematical factors), factorization, and some very general experimental information.

It is well known that the simplest solutions of factorization equations in the $\mathcal{N}$-$\mathcal{N}$ scattering require that $\gamma^{+}_{\frac12-\frac12;\frac12-\frac12}$ or $\gamma^{+}_{\frac12\frac12;\frac12\frac12}$ vanish like t for $t\to 0$: the two solutions correspond to the trajectories α and β of GRIBOV and VOLKOV.

Let us study the implications of these two possibilities on our reactions. If $\gamma^{+}_{\frac12\frac12;\frac12\frac12}\sim t$, and $\gamma^{+}_{\frac12-\frac12;\frac12-\frac12}\sim \text{const}$, one can easily prove that the factorization conditions admit solutions consistent with conspiracy. We give in Table III the solution of the factorization conditions which requires the least power of t near $t=0$.

TABLE III.

$V\gamma\to V\gamma$	$\mathcal{N}\bar{\mathcal{N}}\to\mathcal{N}\bar{\mathcal{N}}$	$\bar{\mathcal{N}}\mathcal{N}\to V\gamma$
$\gamma^{+}_{11;11}\sim t$	$\gamma^{+}_{\frac12\frac12;\frac12\frac12}\sim t$	$\gamma^{+}_{11;\frac12\frac12}\sim t$
$\gamma^{-}_{11;11}\sim t$	$\gamma^{-}_{\frac12\frac12;\frac12\frac12}\sim \text{const}$	$\gamma^{-}_{11;\frac12\frac12}\sim t$
$\gamma^{-}_{01;01}\sim \text{const}$	$\gamma^{+}_{\frac12-\frac12;\frac12-\frac12}\sim \text{const}$	$\gamma^{+}_{11;\frac12-\frac12}\sim \text{const}$
$\gamma^{+}_{01;01}\sim \text{const}$	$\gamma^{-}_{\frac12-\frac12;\frac12-\frac12}\sim t$	$\gamma^{-}_{11;\frac12-\frac12}\sim t$
$\gamma^{+}_{-11;-11}\sim t$		$\gamma^{-}_{01;\frac12\frac12}\sim \text{const}$
$\gamma^{-}_{-11;-11}\sim t$		$\gamma^{+}_{01;\frac12\frac12}\sim \text{const}$
		$\gamma^{+}_{01;\frac12-\frac12}\sim \text{const}$
		$\gamma^{-}_{01;\frac12-\frac12}\sim \text{const}$
		$\gamma^{+}_{-11;\frac12\frac12}\sim \text{const}$
		$\gamma^{-}_{-11;\frac12\frac12}\sim \text{const}$
		$\gamma^{-}_{-11;\frac12-\frac12}\sim \text{const}$
		$\gamma^{+}_{-11;\frac12-\frac12}\sim \text{const}$

However this is not an acceptable solution for trajectories like the P and P'. Indeed it is known that the total cross-section for nucleon-nucleon scattering is proportional, via the optical theorem, to the immaginary part of the amplitude

$f^{+t}_{\frac{1}{2}\frac{1}{2};\frac{1}{2}\frac{1}{2}}$ at $t=0$. This requires that, for such trajectories, $\gamma^{+}_{\frac{1}{2}\frac{1}{2};\frac{1}{2}\frac{1}{2}} \sim$ const at $t=0$. If however we assume $\gamma^{+}_{\frac{1}{2}\frac{1}{2};\frac{1}{2}\frac{1}{2}} \sim$ const and $\gamma^{+}_{\frac{1}{2}-\frac{1}{2};\frac{1}{2}-\frac{1}{2}} \sim t$ *no conspiratorial solutions can be found and the kinematical constraints are satisfied by evasion*, as can be seem from Table IV where the solution involving the least power of t near $t=0$ is given.

TABLE IV.

$V+\gamma \to V+\gamma$	$\mathcal{N}\mathcal{N} \to \mathcal{N}\mathcal{N}$	$\mathcal{N}\mathcal{N} \to V+\gamma$
$\gamma^{+}_{11;11} \sim$ const	$\gamma^{+}_{\frac{1}{2}\frac{1}{2};\frac{1}{2}\frac{1}{2}} \sim$ const	$\gamma^{+}_{11;\frac{1}{2}\frac{1}{2}} \sim$ const
$\gamma^{-}_{11;11} \sim$ const	$\gamma^{-}_{\frac{1}{2}\frac{1}{2};\frac{1}{2}\frac{1}{2}} \sim t$	$\gamma^{-}_{11;\frac{1}{2}\frac{1}{2}} \sim t$
$\gamma^{+}_{01;01} \sim t$	$\gamma^{+}_{\frac{1}{2}-\frac{1}{2};\frac{1}{2}-\frac{1}{2}} \sim t$	$\gamma^{+}_{11;\frac{1}{2}-\frac{1}{2}} \sim$ const
$\gamma^{-}_{01;01} \sim t$	$\gamma^{-}_{\frac{1}{2}-\frac{1}{2};\frac{1}{2}-\frac{1}{2}} \sim t^2$	$\gamma^{-}_{11;\frac{1}{2}-\frac{1}{2}} \sim t$
$\gamma^{+}_{-11;-11} \sim t^2$		$\gamma^{-}_{01;\frac{1}{2}\frac{1}{2}} \sim t$
$\gamma^{-}_{-11;-11} \sim t^2$		$\gamma^{+}_{01;\frac{1}{2}+\frac{1}{2}} \sim$ const
		$\gamma^{+}_{01;\frac{1}{2}-\frac{1}{2}} \sim t$
		$\gamma^{-}_{01;\frac{1}{2}-\frac{1}{2}} \sim t$
		$\gamma^{+}_{-11;\frac{1}{2}\frac{1}{2}} \sim$ const
		$\gamma^{-}_{-11;\frac{1}{2}\frac{1}{2}} \sim t$
		$\gamma^{-}_{-11;\frac{1}{2}-\frac{1}{2}} \sim t$
		$\gamma^{+}_{-11;\frac{1}{2}-\frac{1}{2}} \sim t$

The results of the previous discussion are thus the following:

i) The factorization conditions imply evasion for the conspiracy relations (2.5) and (2.7); the other relations (2.3), (2.4), (2.6), (2.8) are related in such a way that conspiracy in $\gamma+\mathcal{N} \to V+\mathcal{N}$ requires conspiracy in $\mathcal{N}+\mathcal{N} \to \mathcal{N}+\mathcal{N}$ and $\gamma+V \to \gamma+V$: the constant behaviour for $\gamma^{+}_{\frac{1}{2}-\frac{1}{2};\frac{1}{2}-\frac{1}{2}}$ obtained by conspiracy in $\mathcal{N}\mathcal{N}$ is by now widely known.

ii) Trajectories like P and P' do not conspire.

We show now that these results are in agreement with those derived from the Lorentz or O_4 symmetry of the amplitude at $t=0$ ([16-19]). Using the notation of ref. ([17]) the P and P' would belong to a family of trajectories with the Lorentz quantum numbers $M = 0$, $T = 1$, $\sigma = 1$: the trajectories of this class never conspire in the group theoretical approach. The behaviour of the residue given in Table IV is consistent with the Cosenza, Sciarrino, Toller results for the contribution of Class I Regge poles ([25]).

The only possibility open is a Class III conspiracy, *i.e.* a conspiracy between parity doublet, and the conspiring behaviour of the residue given in Table III is consistent with the behaviour assumed in the group-theoretical analysis for conspiring $M = 1$ Regge poles.

The previous discussion heavily rests on the assumption that only moving poles whose residua factorize are present in the complex J-plane.

In principle, however, there can be a number of possible contributions not associated with single Regge poles and thus not required to factorize. In fact when conspiracy occurs different trajectories may become degenerate $t = 0$ and if two such trajectories contribute to the same helicity amplitude, the total residue in general does not factorize. On the other hand if other singularities in the complex J-plane such as branch cuts, whose contribution does not factorize, are important, different possibilities are open: the possibility that branch cuts « conspiring with itself » may be important in nucleon-nucleon and nucleon-antinucleon charge exchange has been discussed by HUANG and MUZINICH ([26]).

The study of the phenomenological implications of these various possibilities in the reaction $\gamma + \mathcal{N} \to \mathrm{V} + \mathcal{N}$ is in progress.

* * *

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APPENDIX A

We collect in this Appendix some useful kinematical formulae. We consider the t-channel reaction $\bar{\mathrm{p}} + \mathrm{p} \to \mathrm{V} + \gamma$ $(a + b \to c + d)$ and the s-channel reaction $\gamma + \mathrm{p} \to \mathrm{V} + \mathrm{p}$ $(D + b \to c + A)$; the crossing matrix ([7,8]) is given by

$$f^{s}_{cA;Db} = \sum_{a'b'c'd'} d^{J_a}_{a'A}(\chi_a)\, d^{J_b}_{b'b}(\chi_b)\, d^{J_c}_{c'c}(\chi_c)\, d^{J_a}_{a'D}(\chi_a)\, f^{t}_{c'a';a'b'} \,. \tag{A.1}$$

([25]) G. COSENZA, A. SCIARRINO and M. TOLLER: private communication. We are indebted to Prof. M. TOLLER for a discussion about these topics.

([26]) K. HUANG and I. J. MUZINICH: *Nucleon-nucleon and nucleon-antinucleon charge exchange scattering and conspiracy of singularities in the complex angular-momentum plane*, M.I.T. preprint.

In our case $J_a = J_c = \frac{1}{2}$, $J_c = J = 1$. Moreover, owing to the zero mass of the photon, it may easily be shown ([27]) that $\cos\chi_d = -1$, so that $d^1_{d'D}(\pi) = \delta_{d',-D}$ and the crossing reduces to

$$f^s_{cA;Db} = \sum_{a'b'c'} d^{\frac{1}{2}}_{a'A}(\chi_a)\, d^{\frac{1}{2}}_{b'b}(\chi_b)\, d^1_{c'c}(\chi_c)\, f_{C'-D;a'b'}\,. \tag{A.2}$$

The crossing angles are explicitly given by

$$\cos\chi_a = \frac{-t(s+M^2-\mu^2)-2\mu^2M^2}{\sqrt{[s-(M-\mu)^2][s-(M+\mu)^2]}\sqrt{t(t-4M^2)}}\,, \tag{A.3}$$

$$\cos\chi_b = \frac{t(s+M^2)-2\mu^2M^2}{(s-M^2)\sqrt{t(t-4M^2)}}\,, \tag{A.4}$$

$$\cos\chi_c = \frac{(t+\mu^2)(s+\mu^2-M^2)-2\mu^4}{\sqrt{[s-(M-\mu)^2][s-(M+\mu)^2]}\,(t-\mu^2)}\,, \tag{A.5}$$

where M = nucleon mass, μ = vector meson mass.

Another useful formula is the following:

$$\cos\theta_t = \frac{2st+t^2-t(2M^2+\mu^2)}{(t-\mu^2)\sqrt{t(t-4M^2)}}\,. \tag{A.6}$$

Appendix B

The method used for deriving the conspiracy relations given in Sect. 2 is due to Cohen-Tannoudji, Morel and Navelet ([6]). If $\bar{\bar{f}}^t$ is a parity-conserving helicity amplitude, free from s and t kinematical singularities, the crossing matrix may be written in the form

$$\bar{f}^s_i = \sum_j \bar{\bar{M}}_{ij}\bar{\bar{f}}^t_j\,, \tag{B.1}$$

where $\bar{f}^s$ is free from t-kinematical singularities.

If all the masses involved are not unequal, near $t=0$ the matrix elements of M have the form

$$\bar{\bar{M}}_{ij} = \frac{c_{ij}}{(t)^x} + (\text{terms regular at } t=0)\,. \tag{B.2}$$

([27]) H. K. Shepard: *Phys. Rev.*, **159**, 1331 (1967); H. D. I. Abarbanel and S. Nussinov: *Phys. Rev.*, **158**, 1462 (1967).

Thus the relations

$$\sum_j c_{ij} f_j^t = 0 \tag{B.3}$$

must generally hold at $t=0$, since neither the $\bar{f}_i^s$ nor the $\bar{\bar{f}}_j^t$ have kinematical singularities at $t=0$. The eqs. (B.3) are the desidered conspiracy relations.

In the case of the reaction $\gamma+\mathcal{N}\to V+\mathcal{N}$ the singularity of the elements of the crossing matrix M at $t=0$ is of the type of $1/t$; $1/t^{\frac{1}{2}}$ comes from the $d^{\frac{1}{2}}_{a'A}(\chi_a)$ and $d^{\frac{1}{2}}_{b'b}(\chi_b)$ (see Appendix A) and $1/t^{\frac{1}{2}}$ comes from the kinematical K factor (see Table I). Writing down explicitly the relations (B.3) we find the following equations:

$$\left\{\begin{aligned} &-\frac{1}{4}\cos^2\frac{\chi_c}{2}A+\frac{\sin\chi_c}{2\sqrt{2}}B-\sin^2\frac{\chi_c}{2}C=0\,,\\ &\frac{1}{4}\sin^2\frac{\chi_c}{2}A+\frac{\sin\chi_c}{2\sqrt{2}}B+\cos^2\frac{\chi_c}{2}C=0\,,\\ &\frac{\sin\chi_c}{4\sqrt{2}}A+\frac{1}{2}\cos\chi_c B-\frac{\sin\chi_c}{\sqrt{2}}C=0\,,\end{aligned}\right. \tag{B.4}$$

where

$$\left\{\begin{aligned} &A=\{i[\bar{f}^t_{-11;\frac{1}{2}-\frac{1}{2}}-\bar{f}^t_{1-1;\frac{1}{2}-\frac{1}{2}}]-[\bar{f}^t_{-11;\frac{1}{2}\frac{1}{2}}-\bar{f}^t_{1-1;\frac{1}{2}\frac{1}{2}}]\}\,t^{\frac{1}{2}}\,,\\ &B=\{i[\bar{f}^t_{01;\frac{1}{2}-\frac{1}{2}}+\bar{f}^t_{0-1;\frac{1}{2}-\frac{1}{2}}]-[\bar{f}^t_{01;\frac{1}{2}\frac{1}{2}}+\bar{f}^t_{0-1;\frac{1}{2}\frac{1}{2}}]\}\,t^{\frac{1}{2}}\,,\\ &C=\left\{\frac{i}{2}[\bar{f}^t_{11;\frac{1}{2}-\frac{1}{2}}-\bar{f}^t_{-1-1;\frac{1}{2}-\frac{1}{2}}]-[\bar{f}^t_{11;\frac{1}{2}\frac{1}{2}}-\bar{f}^t_{-1-1;\frac{1}{2}\frac{1}{2}}]\right\}t^{\frac{1}{2}}\,.\end{aligned}\right. \tag{B.5}$$

One can easily show that the unique solution of the system (B.4) is

$$A=0\,,\qquad B=0\,,\qquad C=0 \tag{B.6}$$

and these are precisely the conspiracy relations given in Sect. 2.

RIASSUNTO

Si deducono le relazioni di cospirazione per reazioni del tipo $\gamma+\mathcal{N}\to V+\mathcal{N}$. Viene poi studiata la fattorizzazione dei residui che, in linea di principio, può dare informazioni sulla cospirazione o l'evasione. Si mostra che, in questo caso, i risultati, dovuti alla teoria dei gruppi, sulle proprietà di cospirazione o di evasione delle traiettorie di Regge coinvolte possono essere dedotti dalla simmetria di crossing, analiticità, fattorizzazione e qualche informazione sperimentale di carattere generale.

Конспирация и факторизация в фоторождении векторных мезонов.

Резюме (*). — Мы выводим соотношения конспирации для реакций типа $\gamma+\mathcal{N}\to V+\mathcal{N}$. Затем мы исследуем остаточную факторизацию, которая, в принципе, может дать некоторую информацию о конспирации или неуловимости. Мы показываем, что в этом случае результаты теории групп о конспирации или неуловимости затронутых траекторий Редже могут быть выведены из аналитичности кроссинг-симметрии, факторизации и некоторой очень общей экспериментальной информации.

(*) *Переведено редакцией.*