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FINITE ENERGY SUM RULES AND THE CROSSOVER PROBLEM IN K-N SCATTERING

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We study the behaviour of the ω-residue functions in kaon-nucleon elastic scattering using the "finite energy sum rules" technique. We find that both spin flip and non spin flip ω-residues show a zero near $t \sim -0.1\,(\mathrm{GeV}/c)^2$: this is in agreement with the usual explanation of the crossover phenomena in KN and NN elastic scattering, but in contradiction with the behaviour of the ω-residue functions deduced from other reactions.

Recently the Regge pole model experienced some difficulties in connection with the so called "crossover phenomena" [1-3] in the elastic scattering $\pi^{\pm}$p, $K^{\pm}$p, pp and p$\bar{\mathrm{p}}$.

In fact these phenomena have been explained by Phillips [5,6] and Rarita assuming that the ω and ρ Regge pole residue functions vanish at the cross-over point $t = t_c$. However it has been shown that the amplitudes $O_{\{\lambda\}}$ characterized by Regge poles with odd change conjugation in the crossed channel [3], do not vanish at $t = t_c$ in the reactions $\pi + N \rightarrow \rho + N$ [7] and $\gamma + N \rightarrow \pi + N$ [4]. Moreover for the photoproduction reaction it has been shown that the ω exchange gives the only important contribution to $O_{\{\lambda\}}(I=0)$. This implies that, if the interpretation of the crossover phenomena is correct (i.e. vanishing of $O_{\{\lambda\}}$) and if the ω exchange gives the only important contribution to $O_{\{\lambda\}}$ in nucleon-nucleon scattering, the factorization breaks down and, with it, the predictivity of the Regge pole theory. In order to avoid this unpleasant situation, Barger and Durand [2] have speculated about the possible existence of additional $\bar{\omega}$ contributions.

In this letter we shall study the crossover problem in K-N scattering, using the "finite energy sum rule" approach [8-10]. This technique enables us to study the spin flip and non spin-flip residue functions as a function of t, relating them to low energy integrals.

The same technique has been used by the authors [3] in order to study the behaviour of the ω residue functions in π^0 photoproduction: these functions do not present any zeroes at $t = t_c$, contradicting, in a striking manner, the usual explanation of the crossover phenomena.

Only the ω exchange in the crossed t-channel contributes to the amplitude

$$M_{\omega}(\nu, t) = \tfrac{1}{4}[M(K^-p \rightarrow K^-p) + M(K^+p \rightarrow K^+p) - M(K^-n \rightarrow K^-n) - M(K^+n \rightarrow K^+n)]\,,$$

where $\nu = \omega + t/4m_N$, ω being the energy of the incident K meson in the laboratory system. At asymptotic energies we assume that ‡

$$\mathrm{Im}\, A_{\omega}(\nu, t) = (2\alpha_{\omega} + 1) S_A(\alpha_{\omega})\ (C(t)/m_K)\ (\nu/m_K)^{\alpha_{\omega}(t)}\,, \tag{1}$$

$$\mathrm{Im}\, B_{\omega}(\nu, t) = S_B(\alpha_{\omega})\ (D(t)/m_K^2)\ (\nu/m_K)^{\alpha_{\omega}(t)-1}\,. \tag{2}$$

‡ $A(\nu,t)$ is the non spinflip amplitude, which Singh call A' [11]. S_A and S_B depend on the behaviour of the ω trajectory near the nonsense point $\alpha_{\omega} = 0$: if the ω "chooses sense" $S_A(\alpha) \approx 1$ and $S_B(\alpha) \approx \alpha$, if it chooses nonsense, as recently suggested [3], $S_A(\alpha) \approx S_B(\alpha) \approx \alpha$.

Taking into account the crossing properties one has the following sum rules

$$\frac{1}{2\pi^2 m_N}\int_0^{\omega_A} \mathrm{Im}A_\omega(\nu,t)\mathrm{d}\omega = \frac{(2\alpha_\omega+1)S_A(\alpha_\omega)}{2\pi^2 m_N(\alpha_\omega+1)}\, C(t)\left(\frac{\omega_A+t/4m_N}{m_K}\right)^{\alpha_\omega(t)+1}, \tag{3}$$

$$\frac{m_N}{2\pi^2}\int_0^{\omega_A} \nu\,\mathrm{Im}B_\omega(\nu,t)\mathrm{d}\omega = \frac{m_N S_B(\alpha_\omega)}{2\pi^2(\alpha_\omega+1)}\, D(t)\left(\frac{\omega_A+t/4m_N}{m_K}\right)^{\alpha_\omega(t)+1}. \tag{4}$$

Extracting the pole terms and assuming that the amplitudes are dominated by the $Y_0^*(1405)$ and $Y_1^*(1385)$ from the $(\Lambda\pi)$ to the physical threshold, one has:

$$\frac{1}{4}\left(\frac{g_\Lambda^2}{4\pi}\right)\frac{1}{m_N^2}X_-(\Lambda,t)+\frac{3}{4}\left(\frac{g_\Sigma^2}{4\pi}\right)\frac{1}{m_N^2}X_-(\Sigma,t)+\frac{1}{4}\left(\frac{g_{Y_0^*}^2}{4\pi}\right)\frac{1}{m_N^2}X_+(Y_0^*,t)+\frac{3}{4}\left(\frac{g_{Y_1^*}^2}{4\pi}\right)\frac{1}{m_N^2}Z(Y_1^*,t)+$$
$$+\frac{1}{2\pi^2 m_N}\int_{m_K}^{\omega_A}\mathrm{Im}A_\omega(\nu,t)\mathrm{d}\omega = \frac{(2\alpha_\omega+1)S_A(\alpha)}{2\pi^2 m_N(\alpha_\omega+1)}\,C(t)\left(\frac{\omega_A+t/4m_N}{m_K}\right)^{\alpha_\omega(t)+1}, \tag{5}$$

and

$$\frac{1}{4}\left(\frac{g_\Lambda^2}{4\pi}\right)\nu_\Lambda(t)+\frac{3}{4}\left(\frac{g_\Sigma^2}{4\pi}\right)\nu_\Sigma(t)+\frac{1}{4}\left(\frac{g_{Y_0^*}^2}{4\pi}\right)\nu_{Y_0^*}(t)+$$
$$+\frac{3}{4}\left(\frac{g_{Y_1^*}^2}{4\pi}\right)\nu_{Y_1^*}(t)\left[\frac{\{(m_N+m_{Y_1^*})^2-m_K^2\}\{(m_N-m_{Y_1^*})^2-m_K^2-2m_N m_{Y_1^*}\}}{6m_{Y_1^*}^2}+\tfrac{1}{2}t\right]+ \tag{6}$$
$$+\frac{m_N}{2\pi^2}\int_{m_K}^{\omega_A}\nu\,\mathrm{Im}B_\omega(\nu,t)\mathrm{d}\omega = \frac{m_N S_B(\alpha)}{2\pi^2(\alpha_\omega+1)}\,D(t)\left(\frac{\omega_A+t/4m_N}{m_K}\right)^{\alpha_\omega(t)+1},$$

where

$$\nu_Y(t) = \frac{m_Y^2-m_N^2-m_K^2}{2m_N}+\frac{t}{4m_N} \qquad \text{and} \qquad X_\pm(Y,t) = \pm m_Y+m_N+\frac{\nu_Y}{1-t/4m_N^2},$$

and $Z(Y_1^*,t)$ can be found in ref. 12 where sum rules similar to ours have been used to investigate the behaviour of the A_2 trajectory near the nonsense right signature point $\alpha_{A_2}=0$. The integrals have been evaluated in the narrow width approximation, that gives

$$\frac{1}{2\pi^2 m_N}\int_{m_K}^{\omega_A}\mathrm{Im}A_\omega(\nu,t)\mathrm{d}\omega = \frac{1}{8m_N^2}\sum_{I,l_\pm}\pm C_I^{l_\pm}\frac{\Gamma_{l_\pm}^{\mathrm{el.}}}{q_{l_\pm}^3}\times$$
$$\times\{X_+(M_{l_\pm},t)[(M_{l_\pm}-m_N)^2-m_K^2]P'_{l_\pm\pm1}(\cos\theta_{l_\pm})-X_-(M_{l_\pm},t)[(M_{l_\pm}+m_N)^2-m_K^2]P'_{l_\pm}(\cos\theta_{l_\pm})\}, \tag{7}$$

$$\frac{N}{2\pi^2}\int_{m_K}^{\omega_A}\nu\,\mathrm{Im}B_\omega(\nu,t)\mathrm{d}\omega =$$
$$=\frac{1}{8}\sum_{I,l_\pm}\pm C^{l_\pm}\frac{\Gamma_{l_\pm}^{\mathrm{el.}}}{q_{l_\pm}^3}\left(\omega_{l_\pm}+\frac{t}{4m_N}\right)\{[(M_{l_\pm}-m_N)^2-m_K^2]P'_{l_\pm\pm1}(\cos\theta_{l_\pm})-[(M_{l_\pm}+m_N)^2-m_K^2]P'_{l_\pm}(\cos\theta_{l_\pm})\}, \tag{8}$$

where the usual notation has been used. The coefficient $C_I^{l\pm}$ is equal to $\frac{1}{2}$ for $I = 0$ and to $\frac{3}{2}$ for $I = 1$. The derivatives of the Legendre polynomials are to be evaluated at $\cos\theta_{l_\pm} = 1 + t/2q_{l_\pm}^2$, $q_{l_\pm}$ being the c.m. momentum at the resonance energy.

For convenience the choice $\omega_A = 2$ GeV has been made: some arguments which show that this can be a resonable value have been given in ref. 12.

Various ambiguities arise in the evaluation of the sum rules (5) and (6). Let us discuss them separately.

1) The coupling constant g_Λ and g_Σ. Although many investigations [13] have been devoted to them, they are still practically unknown. A recent analysis by Kim [14,15] appeared to clarify the situation, showing that both g_Λ and g_Σ are in agreement with the SU(3) predictions. However very recently [16] some arguments have been presented to show that both the constant scattering length [17] and the k-matrix approach [14] give values incompatible with SU(3) invariance. In view of this uncertainty we evaluated our sum rules in the two cases

a) Zovko values [13]: $g_\Lambda^2/4\pi = 6.9 \pm 2.9$ and $g_\Sigma^2/4\pi = 2.1 \pm 0.9$

b) Kim values [15]: $g_\Lambda^2/4\pi = 16.0 \pm 2.5$ and $g_\Sigma^2/4\pi = 0.3 \pm 0.5$.

2) The $Y_0^*(1405)$ coupling has been evaluated using the constant scattering length approximation, obtaining $g_{Y_0^*}^2/4\pi = 0.32$ [18]. Fortunately however the Y_0^* contribution is so small that a variation $\pm 100\%$ of its coupling constant would not produce significant variations of the results.

3) The $Y_1^*(1385)$ has been shown [14] to be mainly a $\pi\Lambda$ scattering resonance and its coupling to the $\bar{k}$N channel is extremely weak. In the following we shall take $g_{Y_1^*} = 0$.

4) The resonances parameters have been taken from the Rosenfeld tables [19]: only the firmly established resonances have been included: this point will be discussed below.

The results obtained with these data are shown in figs. 1 and 2 for both case a) (Zovko values) and b) (Kim values). Fortunately the qualitative features of the results are not too different in the two cases. In case a the left hand side of the sum rule (5) changes sign near $t \approx -0.09\,(\text{GeV}/c)^2$ and $t \approx -0.9\,(\text{GeV}/c)^2$ and of the sum rule (6) near $t \approx -0.11\,(\text{GeV}/c)^2$ and $t \approx -1.2\,(\text{GeV}/c)^2$. For case b these zeroes respectively at the points $t \approx -0.1\,(\text{GeV}/c)^2$, $t \approx -0.8\,(\text{GeV}/c)^2$ for the sum rule (5) and $t \approx -0.11\,(\text{GeV}/c)^2$, $t \approx -1.2\,(\text{GeV}/c)^2$ for the sum rule (6). Moreover from fig. 1 we see that the predictions obtained from the sum rule (5) are in qualitative agreement, for $t \gtrsim -0.7\,(\text{GeV}/c)^2$, with the results obtained using the nonflip parameters of the ω pole given by the high energy fit by Phillips and Rarita [5]. As general feature of these finite energy sum rules we wish to remark the excellent agreement between the high energy parameters fitted by high energy data and those obtained by the low energy structure. Therefore the finite energy sum rules seem to be a very powerful approach in order to study the high energy structure of the strong interactions when phase shift analysis in the 2 to 4 GeV/c region become available [e.g. 20].

From fig. 2 we see also that the ω exchange gives rise to a non negligible spinflip amplitude: up to now the high energy phenomenological fits were unable to establish this point [e.g. 20]: in fact the ω-exchange spinflip amplitude was neglected in ref. 5.

Let us at first consider the zeroes in the A_ω and B_ω amplitudes near $t = -0.1\,(\text{GeV}/c)^2$ under the usual assumption that the ω exchange gives the only significant $C = -1$ contribution. The zero in A_ω is in perfect agreement with the explanation of the crossover in kN and $\bar{k}$N scattering. A zero in B_ω is to be expected if the explanation of the crossover in pp and $\bar{p}$p is correct *and* the factorization theorem holds. However these results are in contradiction, via the factorization theorem, with the behaviour of the ω residue functions that has been found in other reactions [3,7].

We consider now the zeroes near $t \approx -1\,(\text{GeV}/c)^2$: such zeroes could be associated with the vanishing of the trajectory, provided that the ω is nonsense choosing [3] at the wrong signature point $\alpha = 0$. However these zeroes occur at values of t rather different for the A and B amplitudes and these values of $|t|$ are rather large to be associated with the vanishing of the ω-trajectory. If this displacement is not merely a third double spectral function effect [21], and if these zeroes at large values of $|t|$ can be taken seriously, they could suggest that, beside the ω exchange, other contributions, poles or branch cuts, are important.

Finally, we tried to check how these results depend upon the possible neglect of some still undiscovered direct-channel resonances. To this end we have included three more resonances ‡ whose param-

‡ We included two resonances $\frac{1}{2}^+$, respectively with $I = 0$ and $I = 1$, which are supposed to belong to an unitary octet and one resonace $\frac{1}{2}^-$ with $I = 1$ supposed to belong to a decuplet.

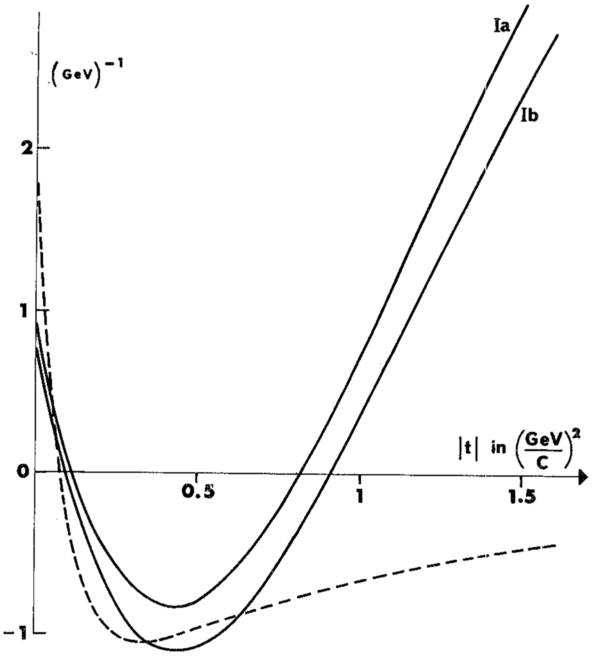

Fig. 1. The curves Ia and Ib are the plots of the left-hand side of eq. (5) using for g_Λ and g_Σ Zovko's and Kim's values respectively. Dashed curve represent the expression:

$$\frac{(2\alpha_\omega + 1)}{2\pi^2 m_N(\alpha_\omega + 1)} C(t) \left(\omega_A + \frac{t}{4m_N}\right)^{\alpha_\omega(t)+1} ,$$

fitted by Phillips and Rarita in ref. 5.

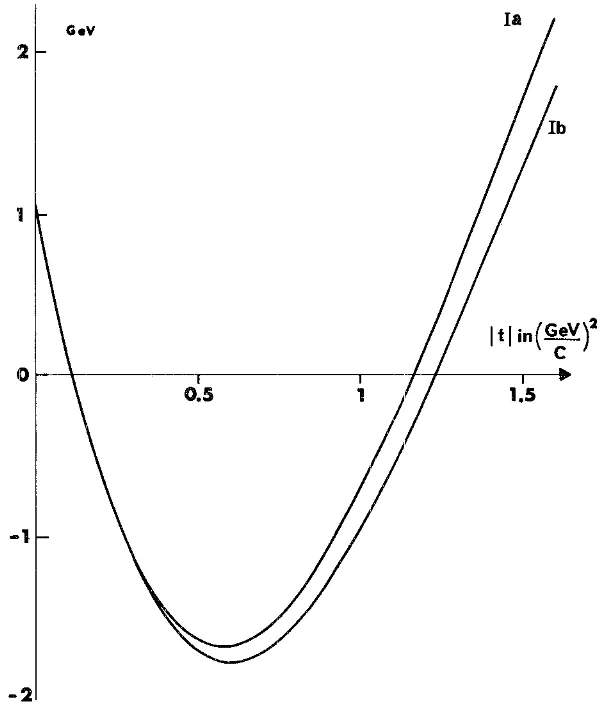

Fig. 2. The curves Ia and Ib are the plots of the left-hand side of eq. (6) using for g_Λ and g_Σ Zovko's and Kim's values respectively.

eters have been roughly extimated assuming SU(3) invariance. The qualitative features of our results do not change much: this seems to suggest that our results are of general validity, although the saturation procedure we used is admittenly rather rough.

The conclusions of this work are thus rather unpleasant. The various open possibilities appear to be the following:

1) additional singularities are present in the complex J-plane. Some new $\overline{\omega}$ [2] contributions, in the form of a new pole or of a cut, are needed at least in kaon-nucleon and nucleon-nucleon scattering.

2) the factorization theorem does not hold, perhaps because absorptive corrections are important.

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References

1. R.J.N.Phillips, in: Proc. of the Heidelberg Intern. Conf. on Elementary Particles (North-Holland, Amsterdam, 1968) p. 442.
2. V.Barger and L.Durand, Phys. Letters 19 (1967) 1295.
3. P.Di Vecchia, F.Drago and M.L.Paciello, University of Rome, preprint no. 150 (1968).
4. M.Braunschweig, W.Braunschweig, D.Husmann, K.Lübelsmeyer and D.Schmitz, Single photoproduction of neutral π-mesons on hydrogen at small angles between 4 and 5.8 GeV: preprint presented to: Heidelberg Intern. Conf. on Elementary particle physics, October 1967.
5. R.J.N.Phillips and W.Rarita, Phys. Rev. 139 (1965) B1336.

6. W. Rarita, R. J. Riddel, C. B. Chiu and R. J. N. Phillips, University of California Radiation Laboratory Report, UCRL-17523 (1967). Additional references are given in this paper.
7. A. P. Contogouris, H. J. Lubatti and J. Tran Thanh Van, Phys. Rev. Letters 19 (1967) 1352.
8. K. Igi and S. Matsuda, Phys. Rev. Letters 18 (1967) 625.
9. A. A. Logunov, L. D. Soloviev and A. N. Tavkhelidze, Phys. Letters 24B (1967) 181.
10. R. Dolen, D. Horn and C. Schmid, Phys. Rev. Letters 19 (1967) 402;
R. Dolen, D. Horn and C. Schmid, Preprint CALT-68-143 (1967).
11. V. Singh, Phys. Rev. 129 (1963) 1889.
12. S. Matsuda and K. Igi, Phys. Rev. Letters 19 (1967) 928.
13. M. Lusignoli, M. Restignoli, G. A. Snow and G. Violini, Phys. Letters 21 (1966) 229; Nuovo Cimento 45A (1966) 792;
N. Zovko, Phys. Letters 23 (1966) 143. And many other.
14. J. K. Kim, Phys. Rev. Letters 19 (1967) 1074.
15. J. K. Kim, Phys. Rev. Letters 19 (1967) 1079.
16. A. D. Martin and G. G. Ross, "A prediction for the ΛKN and Σ'KN coupling constants from a finite energy sum rule", University of Durham preprint.
17. J. K. Kim, Phys. Rev. Letters 14 (1965) 29;
M. Sakitt et al., Phys. Rev. 139 (1965) B719.
18. R. L. Warnock and G. Frye, Phys. Rev. 138 (1965) B947.
19. A. H. Rosenfeld et al., Rev. Mod. Phys. 39 (1967) 1.
20. For example see the V. Barger talk at the "Topical Conference on High Energy Collisions of Hadrons", CERN, January 1968.
21. S. Mandelstam and L. L. Wang, Phys. Rev. 160 (1967) 1490.

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