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PARITY DOUBLET CONSPIRACY OF THE PION FROM FINITE ENERGY SUM RULES IN π^+ PHOTOPRODUCTION

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We test the essential features of the conspiracy of the pion Regge pole with a positive parity pole in π^+ photoproduction, using finite energy sum rules. In particular we find that the pion residue function varies rapidly with the four momentum transfer t vanishing at $t = -0.03$ GeV^2 and the conspirator trajectory chooses nonsense at $\alpha = 0$.

One of the most interesting features of the high energy π^+ photoproduction is the peak shown by the differential cross section in the forward direction [1].

Under the assumption of a Regge behaviour for the amplitudes, this fact has been recently interpreted [2-5] in terms of a conspiracy mechanism.

Amongst others, a very simple model of conspiracy has been proposed by Ball et al. [5]. They assume that the pion Regge pole conspires with another pole which has the same quantum numbers but possitive parity (parity doublet or class III conspiracy).

Such a model was already considered by Phillips [6] and Arbab and Dash [7] to fit the pn charge exchange data. In order to get a good fit for the differential cross-section Ball et al. assume that the pion residue function varies rapidly with t, whereas they keep the conspirator residue function constant, i.e. equal to the value given by the conspiracy relation at $t = 0$. Furthermore they take (for small values of t) the consprator and the pion trajectories degenerate, assuming therefore that the conspirator trajectory "chooses nonsense" at $\alpha = 0$.

In this note we will show how, using finite energy sum rules [8] for π^+ photoproduction, we can confirm essentially all the principal features of the model of Ball et al.

We find thus that the pion residue function for small t has the rapidly varying behaviour suggested by Ball et al., and in particular that it vanishes in the neighbourhood of $t = 0$ (more precisely at $t = -0.03$ GeV^2). This behaviour is expected from the group theoretical approach if the pion belongs to an $M = 1$ representation of O(4), i.e. conspires in class III [5,9].

Moreover, we show explicitely that the conspirator trajectory "chooses nonsense" at $\alpha = 0$.

Following Ball [5,10], we can write the four conventional [11] amplitudes for pion photoproduction as

$$A_1 = -\frac{tF_1 + 2mF_3}{t-4m^2}$$
$$A_2 = \frac{F_1}{t-4m^2} + \frac{1}{t}\left[\frac{F_2}{t-\mu^2} + \frac{2mF_3}{t-4m^2}\right] \qquad (1)$$
$$A_3 = -F_4$$
$$A_4 = -\frac{2mF_3 + F_3}{t-4m^2},$$

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where μ is the pion mass, m is the nucleon mass, and the F_i are the parity conserving helicity amplitudes free from any kinematical singularities. The A_i are defined to be regular at $t = 0$ [10], so that we must have a relation between F_2 and F_3 at $t = 0$, namely

$$F_2(0) = -\frac{\mu^2}{2m} F_3(0) . \tag{2}$$

Assuming at high energies Regge behaviour for the amplitudes, this relation can be satisfied trivially by evasion, or else by conspiracy.

Looking at the behaviour of the differential cross section in π^+ photoproduction, the evasive solution (i.e. the vanishing of the residues of the Regge poles at $t = 0$) can be excluded. Since the π^+ photoproduction amplitudes can be written as

$$A_i^{\pi+} = -\sqrt{2}\, A_i^{(-)} - \sqrt{2}\, A_i^{(0)} ,$$

where (-) stands for an isovector photon and (0) for an isoscalar, we can have a conspiracy phenomenon in the (0) amplitudes, or in the (-) amplitudes or in both of them. However, looking at the "dip" of the differential cross-section of π^0 photoproduction [e.g. 12] (where the (0) amplitudes contribute) in the neighbourhood of $t = 0$, and also from the results of a finite energy sum rules calculation for the (0) amplitudes [13], one is led to consider the conspiracy mechanism only for the amplitudes in the (-) configuration.

If we keep only the leading Regge pole for every helicity amplitude, we see that F_2 (where the pion Regge pole contributes) and F_3 receive contributions by poles of unnatural (singlet $N\bar{N}$) and natural (triplet $N\bar{N}$) parity respectively. The simplest hypothesis is then to assume [5-7] a conspiracy of the pion with a Regge pole of the same signature (and G-parity of course) but different parity (parity doublet conspiracy).

Writing then the Regge pole contribution to $F_2^{(-)}$ and $F_3^{(-)}$

$$F_2^{(-)} = \beta_\pi(t)\alpha_\pi(t) \left(\frac{1+\exp(-i\pi\alpha_\pi)}{\sin\pi\alpha_\pi}\right)\left(\frac{2m\nu}{s_0}\right)^{\alpha_\pi-1}$$
$$F_3^{(-)} = \beta_c(t)\alpha_c(t) \left(\frac{1+\exp(-i\pi\alpha_c)}{\sin\pi\alpha_c}\right)\left(\frac{2m\nu}{s_0}\right)^{\alpha_c-1} , \tag{3}$$

where $2m\nu = s - m^2 + \frac{1}{2}(t-\mu^2) \approx s$ for high energies, and $s_0 = 1$ GeV, eq. (2) implies

$$\alpha_\pi(0) = \alpha_c(0) \quad \text{and} \quad \beta_\pi(0) = -\frac{\mu^2}{2m}\beta_c(0) . \tag{4}$$

Ball et al. then fit the SLAC data [1] of forward π^+ photoproduction, assuming, like Phillips, $\beta_c(t) = \text{const} = \beta_c(0)$ and the trajectories of the pion and of the conspirator degenerate (with slope 1 GeV^{-2}).

They assume moreover a linear variation of the pion residue function

$$\beta_\pi(t) = \beta(\mu^2)\left[1 + \frac{\lambda(t-\mu^2)}{\mu^2}\right] , \tag{5}$$

with $\beta(\mu^2) = -\pi e g \mu^2 \approx -0.25$.

Phillips, fitting np charge exchange scattering, uses a similar form, with $\beta(\mu^2) = g^2$ and $\lambda = 0.54$.

Taking $\lambda = 0.4$ Ball et al. give a rather good fit, especially to the higher energy data (dashed curve in fig. 2).

Eq. (5) shows that the pion residue function vanishes for $t = -\mu^2(1-\lambda)/\lambda \approx -0.03$ GeV^2 for $\lambda = 0.4$. This zero is predicted around $t = 0$ from the group theoretical approach if the pion belongs to an $M = 1$ representation of O(4) [9].

In order to check this interesting feature of the pion residue function, we have used the finite energy sum rules technique. Taking into account the crossing properties of the invariant aplitudes of Chew et al. [11] we have the following sum rules

$$\tfrac{1}{2}ef(t+\mu^2) + \frac{\mu}{\pi}\int_{\nu_0}^{\nu_{max}} \text{Im}\, F_2^{(-)}(\nu,t)\, d\nu = \frac{-\mu}{2\pi m}\beta_\pi(t)\,(2m\nu_{max})^{\alpha_\pi} \tag{6}$$

$$\frac{ef}{4m}(t-4m^2) - \frac{tef}{4m}(1+\mu_p' - \mu_n) + \frac{\mu}{\pi}\int_{\nu_0}^{\nu_{max}} \text{Im}\, F_3^{(-)}(\nu,t)\, d\nu = \frac{-\mu}{2\pi m}\beta_c(t)\,(2m\nu_{max})^{\alpha_c} \tag{7}$$

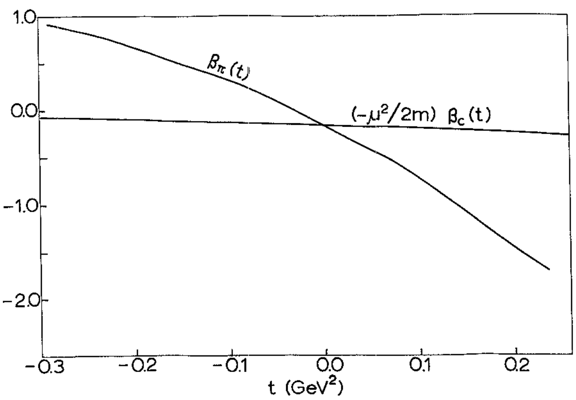

Fig. 1. The residue function of the pion and of the conspirator versus t.

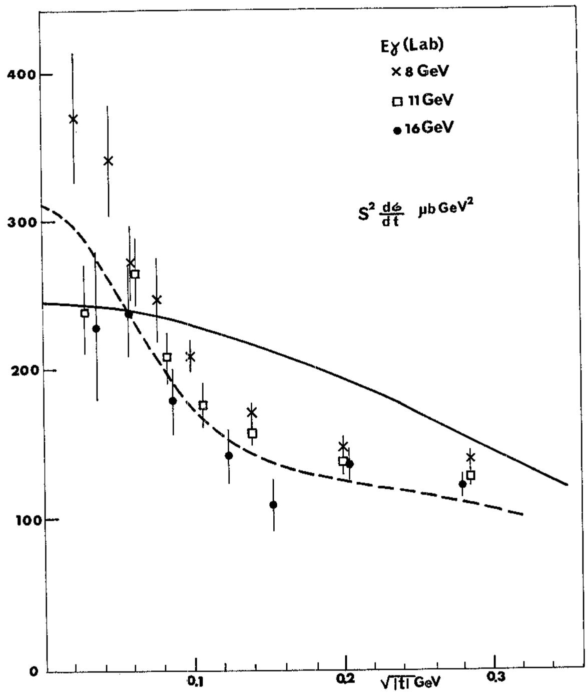

Fig. 2. The differential cross section $s^2 d\sigma/dt$. The experimental points are taken from ref. 1. Dashed curve shows the fit of Ball et al. [5]. Full curve is the obtained from the sum rules for $E_\gamma = 11$ GeV.

where $e^2/4\pi = 1/137$, $f^2/4\pi = 0.08$ and μ_p', μ_n are the anomalous magnetic moment of the proton and neutron. We can thus determine the pion and the conspirator residue functions in terms of low energy pion photoproduction data.

We use a low energy fit done by R. L. Walker at Cal-Tech. It covers the energy range from threshold up to $W = \sqrt{s} = 1.8$ GeV. The essential features of this fit have been described elsewhere [14].

We use for ν_{max} a value corresponding to $W = 1.8$ GeV. The first result that we have obtained from our sum rules (6) and (7) is that there is a conspiracy phenomenon: we have evaluated the left hand side of eqs. (6) and (7) for different values of t and we have seen that they *do not* vanish at $t = 0$. Their relative value at this point is then fixed from eq. (2). (In the sum rules F_2 and F_3 are computed in terms of the invariant amplitudes).

We then proceed, in order to evaluate $\beta_\pi(t)$ and $\beta_c(t)$ to determine $\alpha_\pi(t)$ and $\alpha_c(t)$. This is done calculating the integrals in the left hand side at a given t, for ν_{max} and for another nearby value, say ν'_{max}. Taking then the ratio we can determine the trajectories at a given t. Varying t we have obtained

$$\alpha_\pi = -0.002 + 0.2t\;; \quad \alpha_c = -0.002 + 0.3t\,. \qquad (8)$$

These trajectories are rather flat, especially α_π, in comparison with other values quoted in the literature [15] (Ball et al. for instance use a slope of 1 GeV^{-2}). One should consider, however, that the method we have used for the determination of α_π and α_c is not very accurate, due to the very smooth dependence of the integrals in eqs. (6), (7) from ν_{max}*.

Anyway in our case the precise value of the trajectories has a very little influence on the determination of the residue functions.

We then evaluate $\beta_c(t)$ and $\beta_\pi(t)$. The results are shown in fig. 1. Around $t = 0$ $\beta_\pi(t)$ can be expressed as a linear function of the form

$$\beta_\pi(t) = 0.25\left(1 + \frac{\lambda(t-\mu^2)}{\mu^2}\right),$$

with $\lambda = 0.39$. The agreement with the results of Ball et al. is quite remarkable ($\beta(\mu^2)$ is determined from the Born term in eq. (6)).

We emphasize again the importance of the zero in the pion residue function at $t \approx -0.03$ GeV^2, because, as we said before, it gives a strong support to the hypothesis that the pion belongs to an $M = 1$ representation of O(4) [9]. It is interesting to note that Phillips' value $\lambda = 0.54$ gives a very poor fit to the differential cross section in π^+ photoproduction [5]. In fact it would be worthwhile to investigate if the np data can be fitted by an expression for the pion residue function of the type $\beta_\pi(t) = g^2[1 + \lambda(t-\mu^2)/\mu^2]^2$ with $\lambda \approx 0.4$.

Such expression is suggested by analiticity and factorization, in order to avoid the vanishing of all the pion residue functions in any reaction.

In fig. 1 we have also plotted $-(\mu^2/2m)\beta_c(t)$. One can see that it has a much slower variation with t than $\beta_\pi(t)$.

Another interesting fact is that we can effectively show that the conspirator trajectory "chooses nonsense" at $\alpha_c = 0$. In fact, being F_3 a "nonsense-nonsense" amplitude, we can write the conspirator residue function as

* The method proposed by R. Dolen et al. [8], i.e. considering also the sum rule for $\int \nu^2 \,\mathrm{Im}\, F(\nu,t)\, d\nu$, and then taking the ratio, can not be applied to our case, since, our ν_{max} is not very high, so that the "second moment" integral is critically dependent from the high energy tail of the fit. See the discussion about this point in ref. 14.

$$\beta_c(t) = f(\alpha_c)\gamma_c(t) ,$$

where $f(\alpha_c)$, the "sense-nonsense" factor, is equal to 1 for the "Gell-Mann or nonsense choosing" mechanism and to α_c or α_c^2 for the "sense choosing", the "non compensating" and the "Chew" mechanism*. We see from fig. 1 that $\beta(t)$ *does not vanish* for $\alpha_c = 0$ (which is somewhere around $t \approx 0.02$) so that $f(\alpha_c) = 1$ and the trajectory "chooses nonsense" in agreement with the fact that $\alpha_\pi(t)$ and $\alpha_c(t)$ are very similar.

We have also evaluated, using our data, the asymptotic differential cross section

$$\frac{d\sigma}{dt} = \frac{1}{16\pi} [|A_1^{(-)} + (t-\mu^2)A_2^{(-)}|^2 + |A_1^{(-)}|^2 + - (t-\mu^2)(|A_3^{(-)}|^2 + |A_4^{(-)}|^2)] .$$

Taking into account only the pion and the conspirator contribution (a very reasonable approximation for small t), the result is given in fig. 2 (full curve) for $E_\gamma = 11$ GeV. The disagreement of our curve with the experimental data, in the region between $\sqrt{|t|} = 0.1$ and $\sqrt{|t|} = 0.3$, is probably due to the flatness of the trajectories used (eq. (8)). If we consider for α_c and α_π slopes of the order of 0.7 or 1 GeV^{-2} our curve in that region shifts down by 10 or 15%.

Anyway in order to get a good fit one should also consider other contributions, such as the A_2 Regge pole, the B and the ρ poles in the (0) amplitudes, that we have neglected in this approximation. Work in this direction is in progress.

Summarizing, our results strongly suggest, as it has been shown, that the pion conspires in class III, at least in π^+ photoproduction. I should however be noted that the hypothesis of a pure class III conspiracy for the pion, does not seem able to explain some features of other reactions [17].

We hope that the results we have obtained show once mor how powerful can be the application of the finite energy sum rules, provided there is a good knowledge of the low and intermediate energy data. We think therefore that experiments and analysis of the data in that energy region are still very important and can give extremely interesting predictions for the high energy parameters.

* About sense and nonsense factor and related problems see e.g. ref. 16.

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