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PROPOSAL FOR A NEW ORGANIZATION OF DECISION ELECTRONICS IN MULTICOUNTER EXPERIMENTS

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In view of the introduction of integrated logic elements into prompt electronic modules, a new "correlation unit" is studied.

By examining typical experimental requirements, it is shown that for optimum efficiency the unit should: 1. produce any logical function of n binary variables, in a time not much longer than that required for usual coincidence-anticoincidence operations; 2. switch easily, on external program from one function to another of the same variables. Practical block diagrams for such correlation modules are then analyzed. It is seen that, up to 4–5 variables, cost and complexity are not much larger than for a conventional electronic module.

1. Introduction

Fast, reliable and cheap integrated circuit elements, suitable for building counter experiments electronics, are available. They make it possible to obtain complex logical functions in a short time, and therefore to introduce more involved and sophisticated experimental techniques.

The aim of this paper is to study the possibility of taking advantage of this situation.

There are at least two ways of facing the problem of finding the logical organization of electronic circuits that guarantees the maximum flexibility and efficiency for the widest possible class of experiments.

One can systematically study some typical experiments, and find which general requirements the electronics must satisfy; the second way, of course, is to analyze the kind of circuits that can be built, and then discuss their utilization in experiments.

Since the class of possible experiments is much wider than we are familiar with, and our skill in figuring out new experiments is influenced by the instruments at our disposal, we chose to follow the second point of view, that is to study the electronic problem; its connections with the physics are given as an introductory step.

We arrive to an unexpected result: a quite new circuit module.

2. Statement of the problem

Let us assume that in a counter experiment the "answer" of the apparatus to each event be given by the actual values of n binary variables $x_1,\ldots,x_n$ (discriminator outputs, etc.).

An arbitary Boolean function $F(x_1,\ldots,x_n)$ of these variables can be written, according to a well known theorem[1]):

$$F(x_1,\cdots,x_n) = \sum_{i=1}^{k} a_i N_i(x_1,\cdots,x_n), \tag{1}$$

where $k = 2^n$, the a_i's are binary numbers, and $N_i(x_i\ldots,x_n)$ are the normal products or minterms* of $x_1,\ldots,x_n$.

We define a word:

$$w_i = (X_{i1},\cdots,X_{in}), \tag{2}$$

as the set of values of the variables for which N_i is "true"; we define also an s-word phrase or program

$$\Phi_s = (a_1,\cdots,a_k), \tag{3}$$

as the set of k bits that completely specify $F(x_1,\ldots,x_n)$, being s the number of non vanishing a_i in Φ.

A minterm will then identify a simple AND type function, while many-word programs are needed to identify AND-OR type functions.

In order to investigate on the form of function F, we may distinguish between two main types of experiments.

1. Experiments where many kinds of particles are detected, each kind giving a well defined word as "answer" of the apparatus.

The problem is to change a general decision signal ("master trigger") from one kind of particles to another.

An example is sketched in fig. 1. Three scintillation counters A, B, C, in the focus of an analyzing magnet,

* I.e. products of the variables or their complements. The minterms of three binary variables, for instance, are: $x_1x_2x_3$, $x_1x_2\bar{x}_3$, $x_1\bar{x}_2x_3$, $x_1\bar{x}_2\bar{x}_3$, $\bar{x}_1x_2x_3$, $\bar{x}_1x_2\bar{x}_3$, $\bar{x}_1\bar{x}_2x_3$, $\bar{x}_1\bar{x}_2\bar{x}_3$.

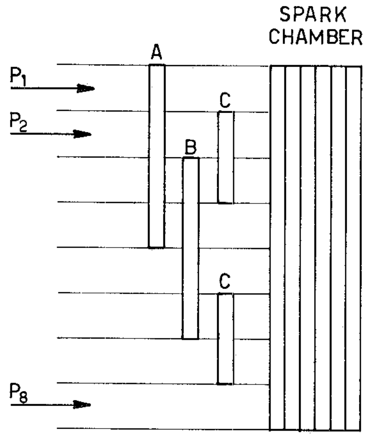

Fig. 1. Particles of different momenta correspond to different minterms of the variables A,B,C. For instance, the "word" for the 4^{th} channel of momenta, $\vec{p}_4$, is $AB\bar{C}$ = "1".

divide a particle beam in eight momentum bands, all of which are analyzed in a spark chamber SC.

The behaviour of the SC for different momenta can be systematically studied by changing the SC trigger from function $F_1 = A\bar{B}\bar{C}$ to $F_2 = A\bar{B}C$, $F_3 = ABC$, etc.

2. Experiments where different kinds of particles are detected, and each kind can give different words as an answer.

Many-word programs (phrases) are needed to separate different particles, usually by multiparametric correlation analysis.

Each particle is identified by applying independent selection criteria to some or all of the binary variables x_i. As these variables are associated to basically stochastic processes, the answer to each type of event will show statistical fluctuations.

Let $p_{j,\beta}, (j = 1,\ldots,n)$ be the probability that a particle of kind β gives an answer $x_j =$ "1", then the (composite) probability that the answer of the apparatus to a particle β be the word w_i is

$$p_\beta(w_i) = \prod_{j=1}^{n} X_{ij}\rho_{j,\beta}, \tag{4}$$

where

$$\rho_{j,\beta} = p_{j,\beta} \text{ if } X_{ij} = 1, \qquad \rho_{j,\beta} = (1-p_{j,\beta}) \text{ if } X_{ij} = 0.$$

Similarly, the probability that the answer of the apparatus be the word w_i, for particles of all kinds $\gamma \neq \beta$ (background), is

$$p_\gamma(w_i) = \prod_{j=1}^{n} X_{ij}(1-\rho_{j,\beta}) = \prod_{j=1}^{n} \bar{X}_{ij}\rho_{j,\beta}. \tag{5}$$

Note that, by definition, the minterms N_i, when $i = 1,\ldots,2^n$, are the set of all possible answers, therefore

$$\sum_{i=1}^{k} p_\beta(w_i) = \sum_{i=1}^{k} p_\gamma(w_i) = 1. \tag{6}$$

Finally, the probability of having $F_\beta = 1$ for a selection function whose phrase is

$$\Phi_\beta = (a_{1\beta}, \cdots, a_{k\beta}), \tag{7}$$

from a particle β is

$$P_\beta(F_\beta) = \sum_{i=1}^{k} a_{i\beta} p_\beta(w_i), \tag{8}$$

while for all particles $\gamma \neq \beta$,

$$P_\gamma(F_\beta) = \sum_{i=1}^{k} a_{i\beta} p_\gamma(w_i). \tag{9}$$

The electronic problem, if we want to select particles of kind β, is that of building the logic function F_β of variables $x_1,\ldots,x_n$ so that

$$\varepsilon_\beta = P_\beta \simeq 1 \text{ and } C_\gamma = P_\gamma/P_\beta \ll 1. \tag{10}$$

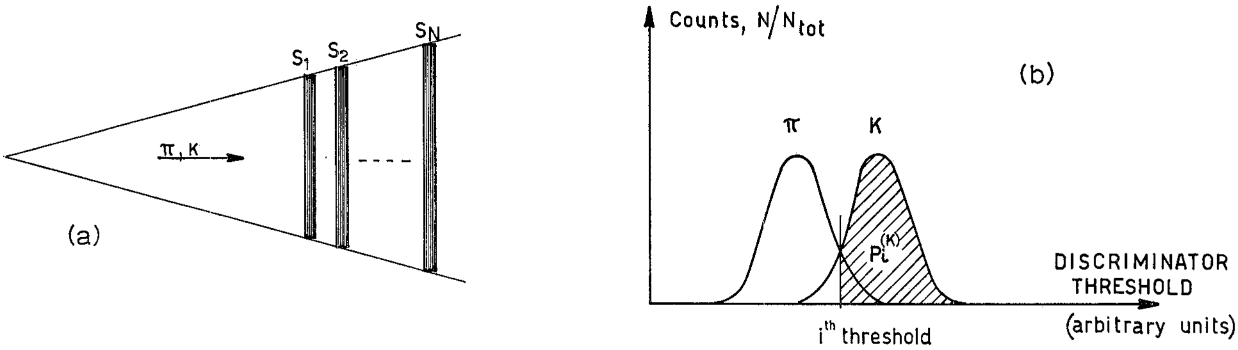

Fig. 2. a. In a counter telescope, $S_1 \ldots, S_N$, some particles can give many-word answers. For instance, if $N \geq 4$, a good choice for K-mesons would be $F_K = S_1S_2S_3 + S_1S_2\bar{S}_3S_4$, b. For simplicity the amplitude spectra in the i^{th} counter ($i = 1,\ldots, N$) are assumed to be similar for two different kinds of particles (π's and K's in the text). The threshold of the counter discriminator coincides with the symmetry axes of the spectra.

Here ε_β is the detecting efficiency of function F_β for β particles, while C_γ is a contamination factor from background.

Clearly, optimization of efficiency and rejection, often contradictory goals, may require an involved selection function.

Changing from one kind of particles to another, or even modifying the selection criteria for the same kind, involves manipulation of the phrase Φ_β pertaining to the decision function F_β; this should be done easily, during the experimental live time.

We can consider, as an example, a telescope of N counters, in which pions are to be separated from K-mesons through correlated amplitude analysis (fig. 2).

We assume, for simplicity, that the probability p_i that a K gives "1" in the i^{th} discriminator equals the probability that a π gives "0". Then we can perform step by step the following systematic procedure:

1. evaluate the probabilities $p_\pi(w_i)$ and $p_K(w_i)$ of all possible answers;
2. order all words w_i according to decreasing values of p_K/p_π for the selection function F_K (and of p_π/p_K for function F_π);
3. take $a_i = 1$ in eq. (7) up to an $i = s$ such that

$$\sum_{i=1}^{s} a_{i,K} p_K(w_i) \geqq \varepsilon_K, \tag{11}$$

where ε_K is an assigned efficiency for K's detection. The procedure for π's is analogous.

Note that these steps for optimization of F_K or F_π can easily be accomplished by a small computer; if the computer is on-line, it can also perform real time measurements of integral efficiencies needed to evaluate the terms $p(w_i)$, or modify F_K or F_π as some counter efficiency drifts, etc.

We examine two numerical cases.

TABLE 1

Probabilities of different "words" ($S_1S_2S_3$); the detection efficiencies of counters S_1, S_2, S_3 have been assumed to be: $p_1 = p_2 = 0.9$, $p_3 = 0.8$ for K's, and $q_1 = q_2 = 0.1$, $q_3 = 0.2$ for π's.

s	w_i	$p_K(w_i)$	$p_\pi(w_i)$
1	$S_1S_2S_3$	0.648	0.002
2	$S_1S_2\bar{S}_3$	0.162	0.008
3	$S_1\bar{S}_2S_3$	0.072	0.018
4	$\bar{S}_1S_2S_3$	0.072	0.018
5	$S_1\bar{S}_2\bar{S}_3$	0.018	0.072
6	$\bar{S}_1S_2\bar{S}_3$	0.018	0.072
7	$\bar{S}_1\bar{S}_2S_3$	0.008	0.162
8	$\bar{S}_1\bar{S}_2\bar{S}_3$	0.002	0.648

TABLE 2

Overall efficiencies when minterms of table 1 with $i \leqq s$ are included in the selection function.

s	ε_K	ε_π	$R_\pi = \varepsilon_K/\varepsilon_\pi$
3	0.882	0.028	31.5
4	0.954	0.046	20.7
5	0.972	0.112	8.68

Example 1

Assume $N = 3$, $p_1 = p_2 = 0.9$, $p_3 = 0.8$. The detection efficiencies for words w_i are given in table 1. Overall efficiencies for different choices of s are shown in table 2. Suppose that K's are to be selected.

One can have a low efficiency, high rejection function by taking $s = 3$:

$$F_K^{(1)} = S_1S_2S_3 + S_1S_2\bar{S}_3 + S_1\bar{S}_2S_3 = S_1S_3 + S_1S_2, \tag{12}$$

or a high-efficiency, low rejection one, by taking $s = 5$:

$$F_K^{(2)} = F_K^{(1)} + \bar{S}_1S_2S_3 + S_1\bar{S}_2\bar{S}_3 = S_1 + S_2S_3. \tag{13}$$

$p_\pi(w_i)$:

	S_1	S_1		
S_2	0.008	0.002	0.018	0.072
	0.072	0.018	0.162	0.648
		S_3	S_3	

$p_K(w_i)$:

	S_1	S_1		
S_2	0.162	0.648	0.072	0.018
	0.018	0.072	0.008	0.002
		S_3	S_3	

$F_K^{(1)}$:

	S_1	S_1		
S_2	1	1		
		1		
		S_3	S_3	

$F_K^{(2)}$:

	S_1	S_1		
S_2	1	1	1	
	1	1		
		S_3	S_3	

Fig. 3. Veitch diagrams of the detection efficiencies of different words w_i and of the two selection functions for K's of example 1.

TABLE 3

Probabilities of words ($S_1S_2S_3S_4$). The detection efficiencies of counters S_1, S_2, S_3, S_4 have been assumed to be: $p_1 = p_2 = 0.9$, $p_3 = p_4 = 0.8$ for K's, and $q_1 = q_2 = 0.1$, $q_3 = q_4 = 0.2$ for π's.

s	w_i	$p_K(w_i)$	$p_\pi(w_i)$
1	$S_1S_2S_3S_4$	0.5184	0.0004
2	$S_1S_2S_3\bar{S}_4$	0.1296	0.0016
3	$S_1S_2\bar{S}_3S_4$	0.129·	0.0016
4	$S_1\bar{S}_2S_3S_4$	0.0576	0.0036
5	$\bar{S}_1S_2S_3S_4$	0.0576	0.0036
6	$S_1S_2\bar{S}_3\bar{S}_4$	0.0224	0.0064
7	$S_1\bar{S}_2S_3\bar{S}_4$	0.0144	0.0144
8	$S_1\bar{S}_2\bar{S}_3S_4$	0.0144	0.0144
9	$\bar{S}_1S_2S_3\bar{S}_4$	0.0144	0.0144
10	$\bar{S}_1S_2\bar{S}_3S_4$	0.0144	0.0144
11	$\bar{S}_1\bar{S}_2S_3S_4$	0.0064	0.0324
12	$S_1\bar{S}_2\bar{S}_3\bar{S}_4$	0.0036	0.0576
13	$\bar{S}_1S_2\bar{S}_3\bar{S}_4$	0.0036	0.0576
14	$\bar{S}_1\bar{S}_2S_3\bar{S}_4$	0.0016	0.1296
15	$\bar{S}_1\bar{S}_2\bar{S}_3S_4$	0.0016	0.1296
16	$\bar{S}_1\bar{S}_2\bar{S}_3\bar{S}_4$	0.0004	0.5184
		1.000	1.0000

Both functions may be useful, and are advantageous when compared with the unidimensional efficiency ($\varepsilon_K = 90\%$) and rejection ($R_\pi = 9$).

The situation can be represented in a more functional way by plotting the selection functions $F_K^{(1)}$, $F_K^{(2)}$ into Veitch diagrams (fig. 3), from which their simplified expressions, eqs. (12), (13), can be easily deduced.

Example 2

Assume $N = 4$, $p_1 = p_2 = 0.9$, $p_3 = p_4 = 0.8$ (table 3). The various choices for s (table 4) can all be interesting, from $s = 2$ which gives low efficiency and extremely high rejection, to $s = 10$ which gives almost 100% efficiency and still 13:1 rejection.

The expression of F_{10}, however, shows that even with $N = 4$ counters the selection function can be so involved as to be hardly realizable with conventional electronics.

The examples and discussion above are only a way of showing that considerable improvement in counter experiments data handling can be achieved by a new circuit module.

Besides giving all the logic functions $F(x_1, \ldots, x_n)$ of a set of n binary variables (the outputs of counter discriminators and other selection circuits), this module should be easily switchable, either manually or by computer control, from one function to another.

This latter feature is essential to make easy the checks, periodical changes of master, etc. that one usually has to make in a large experimental apparatus.

We will see that, for reasonable devices, the maximum number n of handled variables cannot be very large, even if using integrated circuitry.

However, it is useful to keep a general point of view, as long as possible.

3. Basic decision circuits: General principles and block diagrams

Formula (1) suggest a straightforward way of obtaining the function F. It is sufficient to obtain all possible minterms N_i of variables $x_1, \ldots, x_n$ and then add them after a line of AND gates controlled by a register, in which the phrase Φ is written.

A block diagram for $n = 4$ variables is shown in fig. 4. We will call this circuit P-type ("parallel" processing").

Other ways of obtaining the general Boolean function of n variables may be found. For instance by dividing the set of variables $x_1, \ldots, x_n$ in two sets

$$Y = (x_1, \cdots, x_m), \qquad Z = (x_{m+1}, \cdots, x_n). \tag{14}$$

Eq. (1) can be written as

$$F(x_1, \cdots, x_n) = \sum_{r,s} \lambda_{rs} N_r(Y) N_s(Z), \tag{15}$$

TABLE 4

Overall efficiencies when minterms of table 3 with $i \leqq s$ are included in the selection function.

s	ε_K	ε_π	$R_\pi = \varepsilon_K/\varepsilon_\pi$	F_K(minimized)
2	0.648	0.0020	324	$F_2 = S_1S_2S_3$
3	0.7776	0.0036	216	$F_3 = F_2 + S_1S_2S_4$
4	0.8352	0.0072	116	$F_4 = F_3 + S_1S_2S_4$
5	0.8928	0.0108	82.7	$F_5 = F_4 + S_2S_3S_4$
6	0.9252	0.0172	53.8	$F_6 = S_1S_2 + S_1S_3S_4 + S_2S_3S_4$
7	0.9396	0.0316	29.7	$F_7 = S_1S_2 + S_1S_3 + S_2S_3S_4$
8	0.9540	0.0460	20.7	$F_8 = F_7 + S_1S_4$
9	0.9684	0.0604	16.0	$F_9 = F_8 + S_2S_3$
10	0.9828	0.0748	13.1	$F_{10} = S_1S_2 + S_1S_3 + S_1S_4 + S_2S_3 + S_2S_4 + S_3S_4$

where $N_r(Y)$, $N_s(Z)$ are minterms of the sets $(x_1, \ldots, x_m)$, $(x_{m+1}, \ldots, x_n)$ respectively.

Taking $m = \frac{1}{2}n$ (n even), or $m = \frac{1}{2}(n-1)$ (n odd), diagrams like that of fig. 5 are obtained; we call them PS-type ("parallel-series" processing).

It is readily seen that the circuit in fig. 5 is simpler

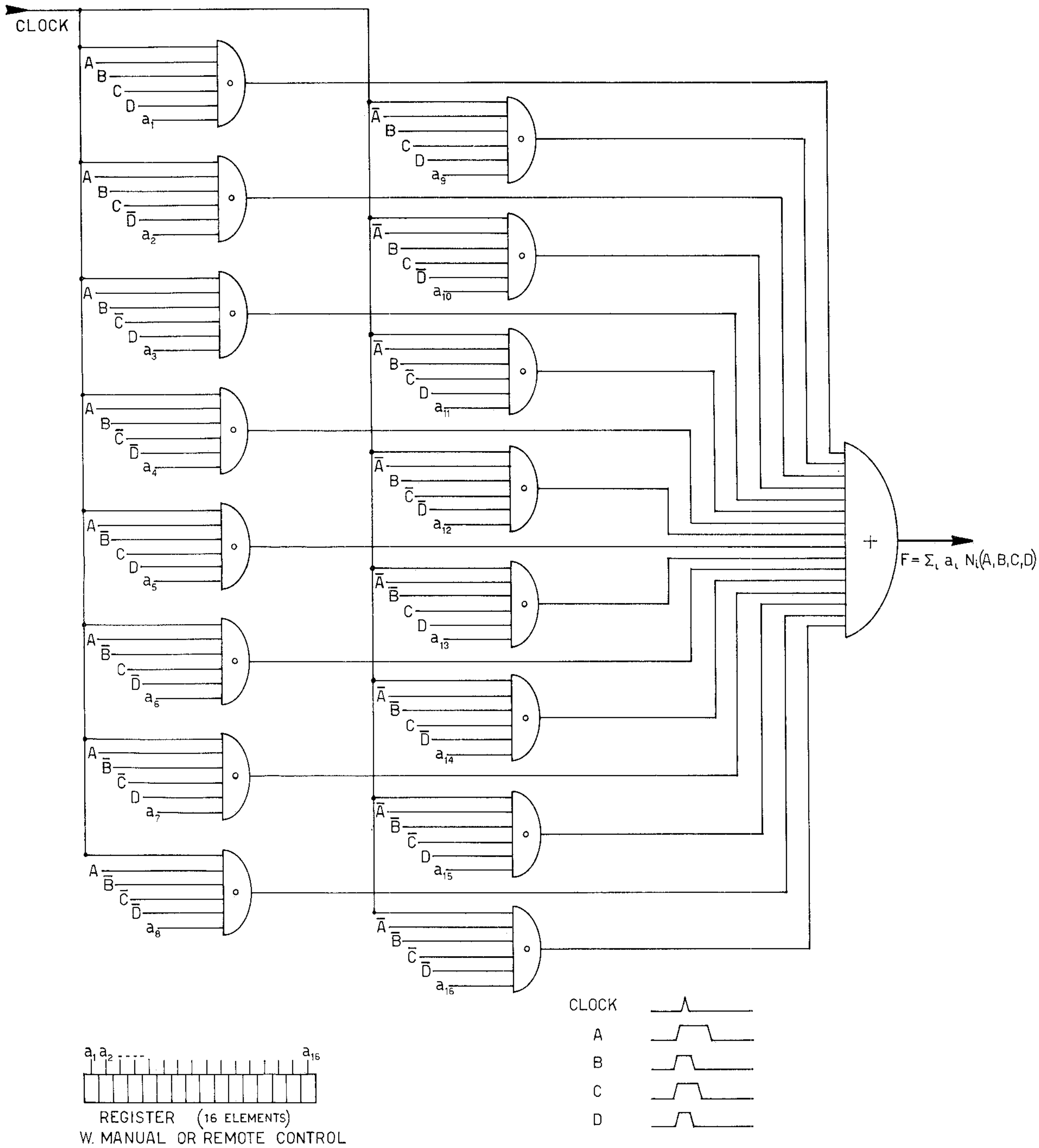

Fig. 4. P-type block diagram to obtain Boolean functions of 4 variables, $F(A,B,C,D)$. The remotely controlled a_i coefficients determine the particular function. The clock signal is just a pulse synchronous with beam pulses (storage rings), or can be obtained from a coincidence between low threshold counter discriminators. All variables, the clock and the coefficients are processed in parallel by the AND-gates; the a_iN_i terms are added in the OR-gate, and may also generate partial outputs.

than the one in fig. 4. We can however make a quantitative comparison: both cost and complexity are approximately proportional to the number G_n of the gate inputs ($G_n = 112$ in fig. 4, $G_n = 88$ in fig. 5).

G_n is given in table 5, for $n = 2$–8 variables, for both P-type and PS-type circuits. The values listed can vary somewhat owing to the different number of inputs of commercially available gates.

Another significant parameter is the propagation delay t'_p. It is obviously given by the propagation delay of each gate, t_p, times the order S of the circuit, i.e. the number of cascaded gates ($S = 2$ in fig. 4, $S = 3$ in fig. 5).

High speed micrologic elements have $t_p = 2$–6 nsec, so that the total delay t'_p lies in the few nanosec range.

The propagation delay should not be identified with

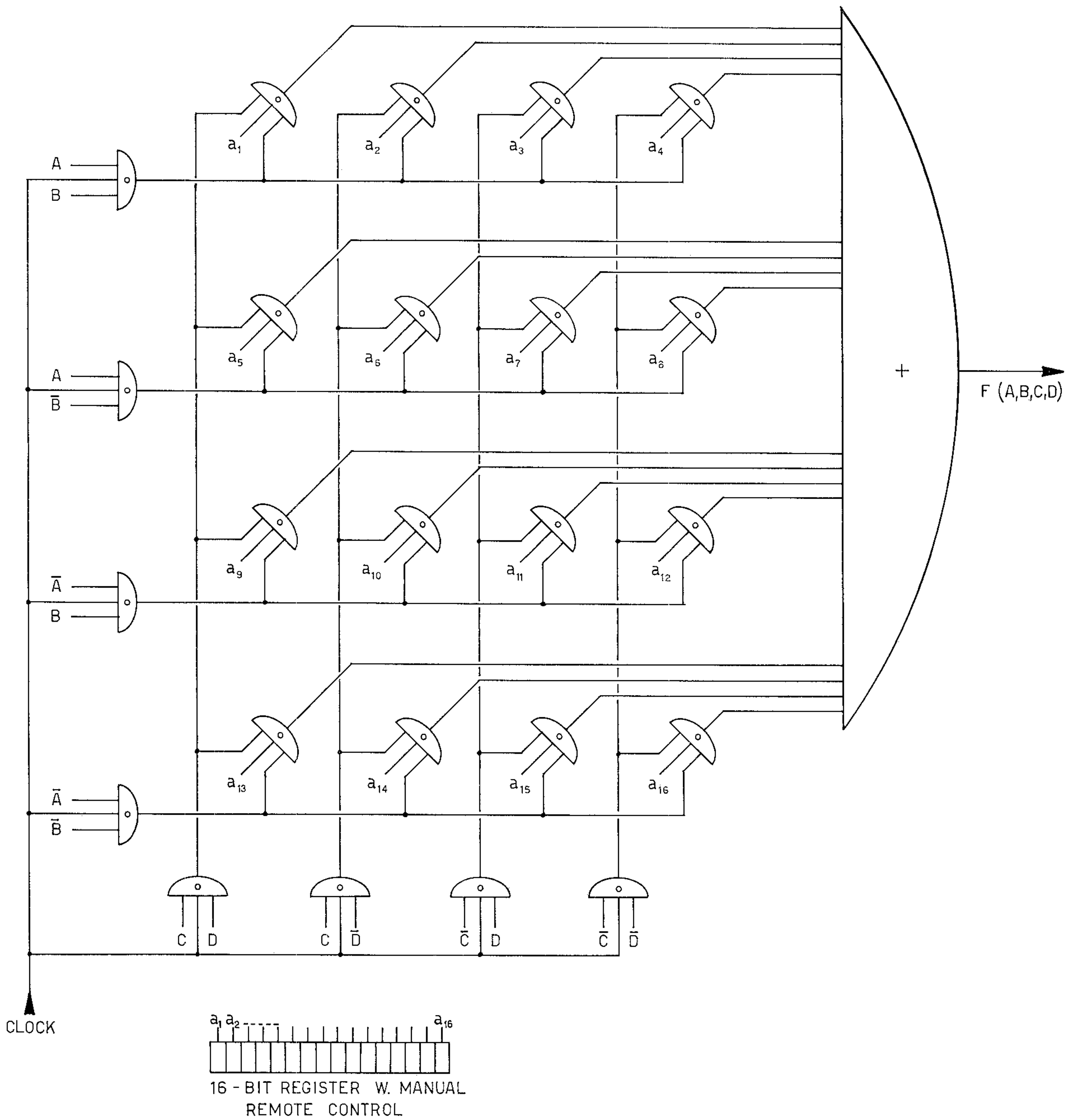

Fig. 5. PS-type diagram to obtain $F(A,B,C,D)$. The $a_i N_i$ terms are obtained by a parallel-series arrangement of AND-gates. Some or all of terms could produce partial outputs.

TABLE 5

Gate inputs number, G_n, for P-type and PS-type diagrams and for different numbers of variables. The higher order S of PS-type diagrams results in a reduced G_n.

No. of variables, n	2	3	4	5	6	7	8	Approx. formula for G_n
P-type diagram $G_n =$	20	48	112	256	576	1280	1516	$G_n = 2^n(n+3)$
PS-type diagram $G_n =$	16	48	88	172	320	624	1184	$G_n = 2^{n/2}(n+2+4\times 2^{n/2})$, ($n$ even)
								$G_n = 4\times 2^n + \frac{1}{2}(3n+7)\cdot 2^{(n-1)/2}$, ($n$ odd)

the resolving time, although both depend on the speed of the logic elements.

A third and very interesting configuration is obtained by taking in eq. (10) $m = n-1$. A typical result is shown in fig. 6, where a function of five variables A,B,C,D,E is obtained from two 4-variables blocks and a simple mixing stage.

One can build therefore correlation modules for n variables only, and then obtain functions of $n+1$, $n+2, \ldots$ variables by small auxiliary units.

However, the logic of fig. 6 has a longer propagation delay than the equivalent P and PS configurations.

4. The complete system

Besides decision circuits, a logic system must include memory elements and input-output devices.

A typical apparatus for multicounter experiments, which includes the decision modules outlined in sect. 3, is shown schematically in fig. 7.

When a master is present, the value ("0" or "1") of

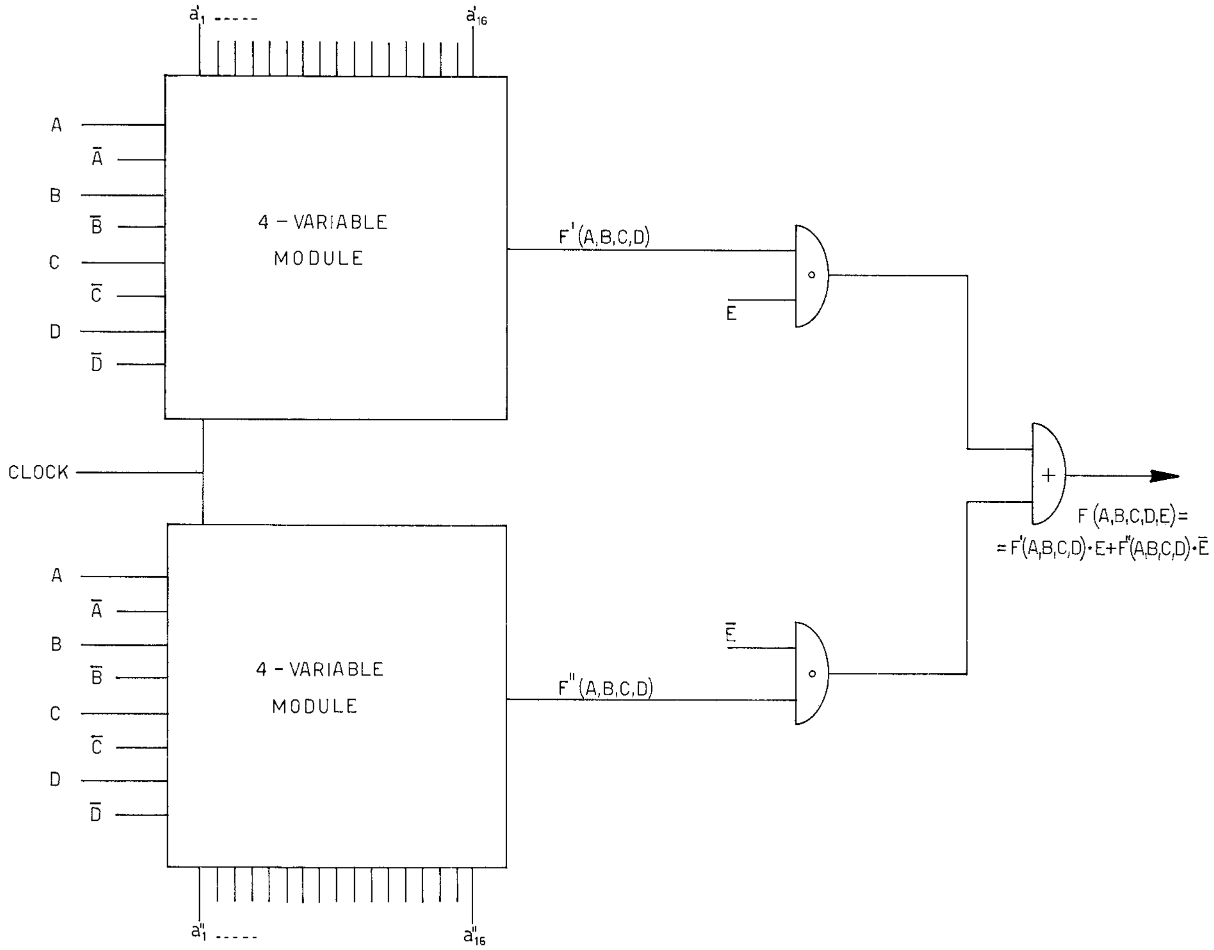

Fig. 6. A function of 5 variables $F(A,B,C,D,E)$ can be obtained by combining two 4-variable modules.

each variable, whether or not a part of the master itself, can be stored in binary memory elements, which are part of the slow correlation unit.

Any needed function of the stored variables can be performed in the latter unit without undergoing the time limitations of the master.

However, the logical modules that perform slow correlations are identical to the fast ones, and it does not pay to build them with slower components.

To evaluate the practical characteristics of this system, one should consider unitary costs (~ 1 $/gate input), the wiring difficulty (~ 5″ × 8″ printed card, for a 4-variables module), the intrinsic gate resolving time (5–20 nsec fwhm), etc.

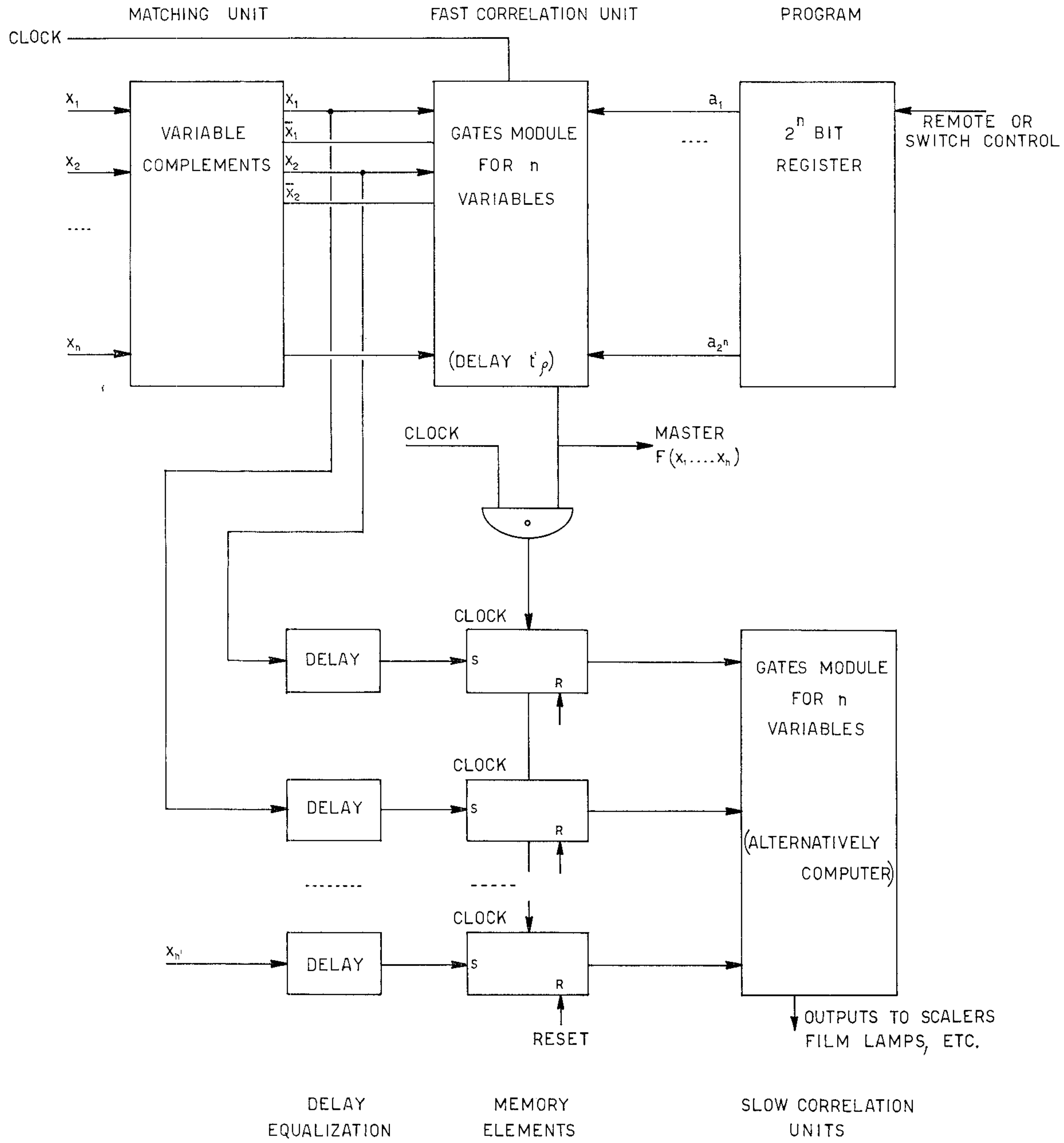

Fig. 7. Block diagram for the fast electronics of a possible experiment. "Correlation units" are used. The outputs $x_1, \ldots, x_n$ of discriminators are complemented, and their voltage levels matched to those of micrologic elements. When a "master" has been obtained from the fast units, a line of memory elements stores some variables for subsequent analysis.

We are now beginning to operate our prototypes, and will soon have the measured characteristics.

5. Conclusion

Integrated circuit elements have attained speed ranges (clock repetition frequency = 50–100 Mc/sec) comparable to those required by many high energy experiments electronics. Furthermore, they are in their young age. Further developments are to be awaited still. On the other side there are little symptoms of great improvements of conventional electronic modules.

These were the starting points in this paper, whose aim it is to show that instead of trying to convert the usual modules to integrated circuitry, one can change the logical organization of experimental block diagrams.

The result is a new "correlation unit" which can give, in a time not much longer than required for a coincidence anti-coincidence operation, the most general logical function of n variables; the unit can be computer programmed.

This performance, unattainable by conventional modules, seems on the contrary simple enough with integrated elements, and offers an exciting improvement of logical flexibility at little cost and complexity increase.

Reference

[1]) M. Phister, Jr., *Logical design of digital computers* (Wiley, New York, 1960).