COMITATO NAZIONALE PER L'ENERGIA NUCLEARE
Laboratori Nazionali di Frascati

LNF - 68/2
5 Gennaio 1968

M. A. Locci and P. Picchi : SINGLE AND DOUBLE CHARGE EXCHANGE OF PIONS BY LIGHT NUCLEI. -

Servizio Documentazione
dei Laboratori Nazionali di Frascati del CNEN
Casella Postale 70 - Frascati (Roma)

Laboratori Nazionali di Frascati del CNEN
Servizio Documentazione

LNF - 68/2

Nota interna : n. 389
5 Gennaio 1968

M. A. Locci and P. Picchi : SINGLE AND DOUBLE CHARGE EXCHANGE OF PIONS BY LIGHT NUCLEI. -

(Submitted for publication to Nuovo Cimento)

## 1. - INTRODUCTION. -

Nuclear reactions by pions can give valuable information on the many-body structure of a nucleus.

The single charge exchange :

$$\pi^{\pm} + \binom{A}{Z} \longrightarrow \pi^{o} + \binom{A}{Z \pm 1} \tag{1.1}$$

and the double charge exchange :

$$\pi^{\pm} + \binom{A}{Z} \longrightarrow \pi^{\mp} + \binom{A}{Z \pm 2} \tag{1.2}$$

are two examples; the (1. 1) can be used for nuclear models check, the (1. 2) for nuclear correlation study (two nucleons, at least, take part in the reaction). Very little is known about the single charge exchange. The only theoretical work on (1. 1) dealt primarily with the Fermi gas model in the impulse approximation[1]. A great deal of theoretical and experimental work on the other hand has been devoted to the double charge exchange[2].

The purpose of the present article is to express $d\sigma/d\Omega$ for the reactions (1. 1) and (1. 2) for pion energies $E_\pi$ 80-200 MeV by (1p) shell nuclei in terms of the elementary interactions $\pi$-nucleon. The method we use is basically that of Glauber[3], the advantage of which lies in the fact that multiple scattering effects are considered . Its great limitation is the small-angle approximation (for the energy $E_\pi$ that we use the approximations are reasonable for minimum angles $\theta_0 \sim 70^o$).

In sect. 2 we give the basis of the Glauber method; in sect. 3 we present the actual calculations and results for $\pi^+ + B^{11} \longrightarrow \pi^o + C^{11}$ and $\pi^+ + Li^7 \longrightarrow \pi^- + B^7$.

## 2. - THE GLAUBER METHOD. -

Glauber's conditions are $KR \gg 1$ and $V_0/E(K) \gg 1$, where E is the kinetic energy of the incident particle upon a potential V of magnitude $V_0$ and range R.

Under these conditions we are justified in assuming that backward scattering will be very weak, that the wave function of the particle within the volume of the potential may to a good approximation be written in the form :

$$\psi(\vec{r}) = e^{i\vec{k}\cdot\vec{r} - (i/\hbar v)\int_{-\infty}^{z} V(\vec{b}+\hat{K}z')\,dz'}$$

where v is the velocity of the particle, b is its impact parameter.

Now, the scattering amplitude can be written as :

$$(2.1) \qquad F(\vec{K}',\vec{K}) = \frac{-2m}{4\pi\hbar^2}\int e^{-i\vec{k}'\cdot\vec{r}}\, V(\vec{r})\, e^{i\vec{k}\cdot\vec{r} - (i/\hbar v)\int_{-\infty}^{z} V(\vec{b}+\hat{K}z')dz'}\, dz\, d^2b$$

For small scattering angles the vector $\vec{K}-\vec{K}'$ is nearly perpendicular to $\vec{K}$. Assuming it perpendicular the z-integration is simply that of an exact differential and leads to :

$$(2.2) \qquad F(\vec{K}',\vec{K}) = \frac{K}{2\pi i}\int e^{i(\vec{K}-\vec{K}')\cdot\vec{b}} \left\{ e^{-(i/\hbar v)\int_{-\infty}^{+\infty} V(\vec{b}+\hat{K}z')dz'} - 1 \right\} d^2b$$

The fucntion :

$$(2.3) \qquad -(i/\hbar v)\int_{-\infty}^{+\infty} V(\vec{b}+\hat{K}z')dz' = X(\vec{b})$$

represents the total phase shift the wave suffers in traversing V.

In this approximation, the scattering of a particle by n centres in the fixed positions $\vec{r}_i$ is obtained by replacing $X(\vec{b})$ by $\sum X_i(\vec{b}-\vec{b}_i)$ and the single particle wave function by the many particle wave function $n(\vec{q}_1, \vec{q}_2, \ldots, \vec{q}_n)$.

The amplitude is :

$$F(\vec{K}',\vec{K}) = \frac{k}{2\pi i}\int e^{i(\vec{K},\vec{K}')\cdot\vec{b}} \left|n(\vec{q}_1,\ldots,\vec{q}_n)\right|^2 \quad \times$$

$$\times \left[e^{i\sum X_i(\vec{b}-\vec{b}_i)}-1\right] d\vec{q}_1, \ldots, d\vec{q}_n \, d^2b \tag{2.3'}$$

We assume, as a first approximation, that the nuclear ground state wave function factorizes so that $\left|u_i(\vec{q}_1,\ldots,\vec{q}_n)\right|^2$ may be written as :

$$\left|u(\vec{q}_1,\ldots,\vec{q}_n)\right|^2 = \prod_{i=1}^{N} q_i(\vec{q}_i) \tag{2.4}$$

where $q_i(\vec{q}_i)$ is the normalized density for the i-th particle.

With the definition $t_i(\vec{b}-\vec{b}_i) = -1 + e^{X_i(\vec{b}-\vec{b}_i)}$ Eq. (2.3) becomes :

$$F(\vec{K}',\vec{K}) = \frac{K}{2\pi i}\int e^{i(\vec{K}-\vec{K}')\cdot\vec{b}} \left[\sum_{i=1}^{N} \langle t_i(\vec{b})\rangle \; + \right.$$

$$\left. + \sum_{j=1}^{N}\sum_{i\neq j}^{N} \langle t_i(\vec{b})\rangle \langle t_j(\vec{b})\rangle \; \ldots\ldots \right] d^2b \tag{2.5}$$

where

$$\langle t_i(\vec{b})\rangle = \int t_i(\vec{b}-\vec{b}_i)\, q_i(\vec{b}_i)\, d^2b_i \tag{2.6}$$

and

$$q_i(\vec{b}) = \int_{-\infty}^{+\infty} q(\vec{b}_i, z_i)\, dz_i \; . \tag{2.7}$$

For a single scattering centre :

$$F(\vec{K}',\vec{K}) = \frac{K}{2\pi i}\int e^{-i(\vec{K}-\vec{K}')\cdot\vec{b}'}\, t(\vec{b}')\, d^2b' = \frac{K}{2\pi i}\, t(\Delta\vec{K}) \; . \tag{2.8}$$

Eq. (2.8) gives the connection between two-body scattering amplitude $F(\vec{K}',\vec{K})$ and the $t(\Delta\vec{K})$.

Now,

$$t_i(\vec{b}-\vec{b}_i) = \frac{1}{2\pi^2}\int e^{i(\vec{b}-\vec{b}_i)(\vec{K}'-\vec{K})}\; t(\Delta\vec{K})\, d^2\Delta K \tag{2.9}$$

and :

$$(2.10)\quad \begin{aligned} \langle t_i(\vec{b})\rangle &= \int t_i(\vec{b}-\vec{b}_i)\, q(\vec{b}_i)\, d^2 b_i = \\ &= \frac{1}{(2\pi)^2}\int e^{i(\vec{b}-\vec{b}_i)(\vec{K}'-\vec{K})}\, t(\Delta \vec{K})\, d^2\Delta\vec{K}\, q(\vec{b}_i)\, d^2 b_i = \\ &= \frac{1}{(2\pi)^2}\int e^{i\vec{b}\cdot\Delta\vec{K}}\, t(\Delta\vec{K})\, d^2\Delta\vec{K}\int e^{-i\vec{b}_i\cdot\Delta\vec{K}}\, q(\vec{b}_i)\, d^2 b_i = \\ &= \frac{1}{(2\pi)^2}\int e^{i\vec{b}\cdot\Delta\vec{K}}\, t(\Delta\vec{K})\, q(\Delta\vec{K})\, d^2\Delta\vec{K}\ . \end{aligned}$$

The single scattering term $\sum_{i=1}^{N} \langle t_i(b)\rangle$ in Eq. (2.5) is identical to that obtained from the impulse approximation, which is valid under less restrictive assumptions. The higher order terms

$$\Sigma_i \,\Sigma_{j\neq i}\ ,\ \Sigma_i \,\Sigma_{j\neq i}\, \Sigma_{K\neq j}\ \text{ecc.}$$

are double and triple scattering ecc. A comparison of the expressions for double scattering given by Glauber and Watson respectively similar to ours gives a difference of 15% or less for all angles(4). This is a good check for the Glauber method.

## 3. - CALCULATION AND RESULTS. -

### 3. 1. - Single charge exchange.

We considere the following reaction :

$$(3.1)\qquad \pi^+ + B^{11} \rightarrow \pi^o + C^{11}\ .$$

The $B^{11}$ is a (1p) shell nucleus and is well described by a density function(5) :

$$(3.2)\qquad q(r) = \frac{2}{\pi^{3/2}}\ \frac{1}{a_o^3(2+3\alpha)}\ \left(1+\frac{\alpha r^2}{a_o^2}\right)\exp\left(-\frac{r^2}{a_o^2}\right)$$

where $\alpha = \dfrac{A-4}{6}$ .

The root mean square radius of this distribution is given by $\langle r^2\rangle^{1/2} = a_o(5/2 - 4/A)^{1/2}$. The nucleon-nucleon correlations we use are due to the Pauli exclusion principe. A check of the effects of hard correlations will require much more copious pion beams than are presently available.

The amplitudes $f(\vec{K}', \vec{K})$ of the elementary processes $\pi^+p \rightarrow \pi^+p$, $\pi^+n \rightarrow \pi^+n$, $\pi^+n \rightarrow \pi^o p$, $\pi^o n \rightarrow \pi^o n$, $\pi^o p \rightarrow \pi^o p$ (the last two are obtained from elementary considerations in isotopic spin space) are well known.

Substituting into $f(K', K)$ the S plus P-wave forms for pion-nucleon scattering :

(3.3) $$f(\vec{K}', \vec{K}) = \alpha + \beta \vec{K}' \cdot \vec{K}/k^2 = \left[ 1 - \frac{\beta}{2(\alpha+\beta)} \frac{\Delta \vec{k}^2}{K^2} \right] (\alpha+\beta)$$

where for $\pi^+n \rightarrow \pi^+n$ scattering (neglecting the spin flip term)

$$k\alpha = (a_3 + 2a_1)/3$$

$$k\beta = (2a_{33} + a_{31} + 4a_{13} + 2a_{11})/3$$

for $\pi^+p \rightarrow \pi^+p$ :

$$k\alpha = a_3$$

$$k\beta = 2a_{33} + a_{31}$$

for $\pi^o n \rightarrow \pi^o n = \pi^o p \rightarrow \pi^o p$ :

$$k\alpha = (2a_3 + a_1)/3$$

$$k\beta = (4a_{33} + 2a_{13} + 2a_{31} + a_{11})/3$$

for $\pi^+n \rightarrow \pi^o p = \pi^o n \rightarrow \pi^- p$ :

$$k\alpha = (a_3 - a_1)\sqrt{\frac{2}{3}}$$

$$k\beta = (2a_{33} - 2a_{13} + a_{31} - a_{11})\sqrt{\frac{2}{3}}$$

Here $a_i = \exp(i\delta_i)\sin\delta_i$, and the $\delta_{2T,\,2J}$ are the usual phase shifts[6].

Extrapolating Eq. (3.3) into an exponential form we have :

(3.4) $$f(\vec{K}', \vec{K}) = (\alpha+\beta)\, e^{-\Delta\vec{K}^2 \frac{\beta}{2(\alpha+\beta)K^2}}$$

In multiple scattering some difficultes arise,as the elementary amplitude is affected by the virtual behaviour of intermediate pions. An expression for the $T = J = 3/2$ off-shell $\pi$-N scattering amplitude, which is the relevant term in the energy range of interest is[7] :

(3.5) $$f_{33}(u, -\mu^{*2}) \simeq K(-\mu^{*2})\, \psi(-\mu^{*2})\, f_{33}(u, -\mu^2)$$

where $K(-\mu^{*2})$ is the pionic form factor of the nucleon, where :

$$(3.6) \qquad \psi(-\mu^{*2}) = \left(1+\frac{\mu^{*2}}{4m^2}\right) \frac{p_1 B_o(u_r, -\mu^{*2})}{q_1 B_o(u_r, -\mu^{*2})}$$

where m and $\mu$ are the nucleon and pion masses; $\mu^*$ is the effective mass of the pion; $u = (\omega - m)/m$, $\omega$ being the total c. m. energy of the $\pi$-N system; $u_r = 0.314$ is the u value of the 3/2, 3/2 resonance; $p_1 = (p_{10}^2-m^2)^{1/2}$ and $q_1 = (q_{10}^2-m^2)^{1/2}$ are the off-shell and on-shell c. m. momenta, where :

$$p_{10} = (\omega^2+m^2+\mu^{*2})/2 \quad ; \qquad q_{10} = (\omega^2+m^2-\mu^2)/2$$

where :

$$B_o(u, -\mu^{*2}) = 4/3 \, \frac{f^2}{\mu^2} \, \frac{u}{(u+\delta)^2} \left[1 + \frac{2\delta}{u+\delta}\right]$$

with $f^2 = 0.08$ and $\delta = (\mu^{*2} + \mu^2)/2m^2$; $f_{33}(u, -\mu^2)$ is the real amplitude.

A way to determine $\mu^*$ is to consider the virtual particle as a wave packet $\psi(x, t)$ within the nucleus and to assume Gaussian distribution in momentum space :

$$(3.6) \quad \text{i. e.} \quad a(K-Q) = (\Delta p\sqrt{2\pi})^{-1} \exp\left[-\frac{(K-Q)^2}{2(\Delta p)^2}\right]$$

with $\Delta p = (\Delta x)^{-1}$ and $\Delta x = \lambda$, the mean free path of the pion within the nucleus.

The effective distribution is then derived from the momentum distribution through energy-momentum conservation $\Sigma p_i = \Sigma p_f$.

The mean free path $\lambda$ is however long enough that the approximation $f_{3,3}(u, -\mu^{*2}) = f_{33}(u, -\mu^2)$ seems to be reasonably well satisfied except near $E_\pi = 200$ MeV.

Taking that into account the amplitude for the reaction $\pi^+ + B^{11} \rightarrow \pi^o + C^{11}$ can be written as :

$$(3.7) \quad F(\vec{K}', \vec{K}) = \frac{K}{2\pi i}\Big[\int e^{i(\vec{K}-\vec{K}')\cdot\vec{b}} \sum \langle t(\vec{b})\rangle d^2b \; + \int e^{i(\vec{K}-\vec{K}')\cdot\vec{b}} \sum\sum \langle t(\vec{b})\rangle \langle t(\vec{b})\rangle \; + \int e^{i(\vec{K}-\vec{K}')\cdot\vec{b}} \sum\sum\sum \langle t(\vec{b})\rangle \langle t(\vec{b})\rangle \langle t(\vec{b})\rangle + \ldots \text{ ecc}\Big]$$

In fig. 1 we illustrate the two types of terms which can occour in double scattering.

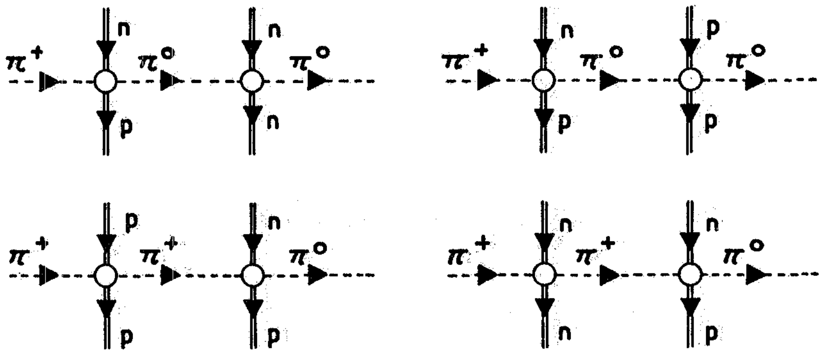


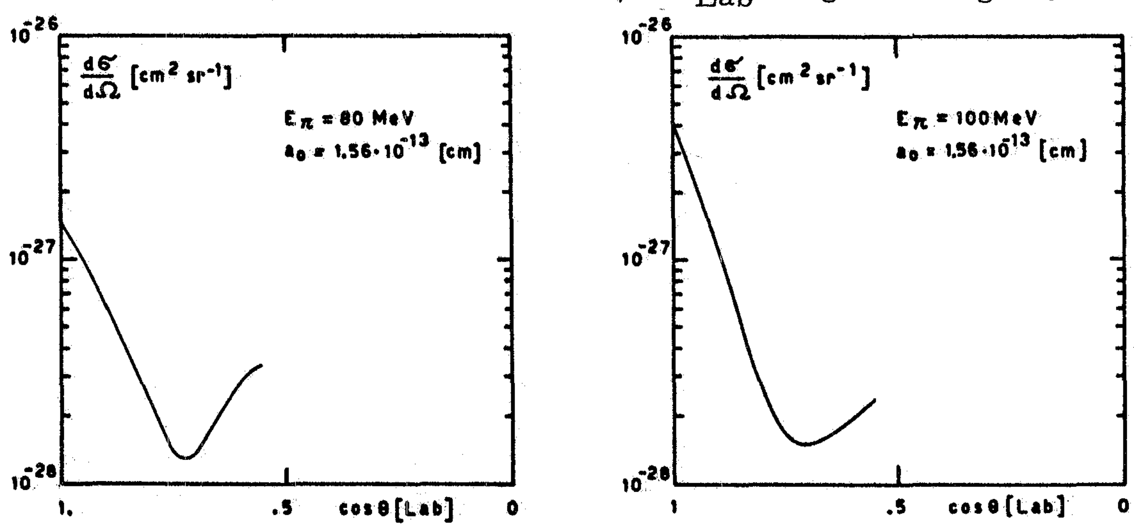

FIG. 1 - Possible contributions to double scattering for single charge exchange.

The single scattering term in Eq. (3.7) is :

$$F_1(\vec{K}',\vec{K}) = \sum f(\vec{K}',\vec{K})\, q(\Delta\vec{K}) \tag{3.8}$$

The interpretation of the term $F_1$ which is identical to that obtained from the impulse approximation, is simple. The interaction matrix element involves an integration over all nucleon coordinates as well as those of the meson. The latter integration leads to the pion-single nucleon scattering amplitude $f(\vec{K}',\vec{K})$, whereas the former becomes a weighting factor $q(\Delta\vec{K})$ (form factor) whose magnitude is determined by the probability that all nucleons remain in their unperturbed states following the collision.

The calculation of the Eq. (3.7) is made for energy $E_\pi$ of 80, 100 MeV. The angular distributions $(d\sigma/d\Omega)_{Lab}$ are given in figs. 2, 3.




FIGS. 2, 3 - The angular distribution $(d\sigma/d\Omega)_{Lab}$ for $\pi^+ + B^{11} \rightarrow \pi^o + C^{11}$ and for $E_\pi$ = 80, 100 MeV.

3.2. - Double charge exchange.

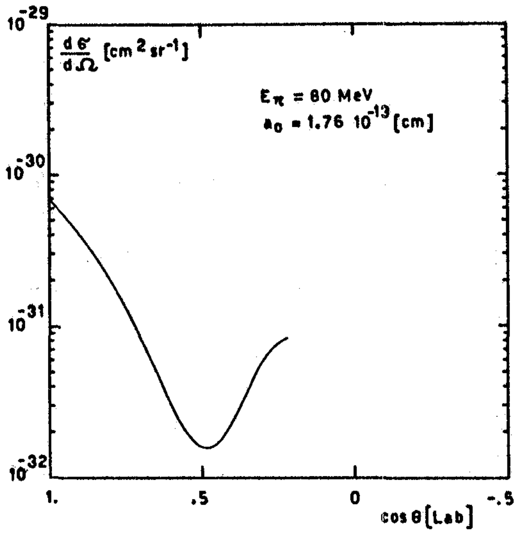
The amplitude for double charge exchange (see 3.1) is :

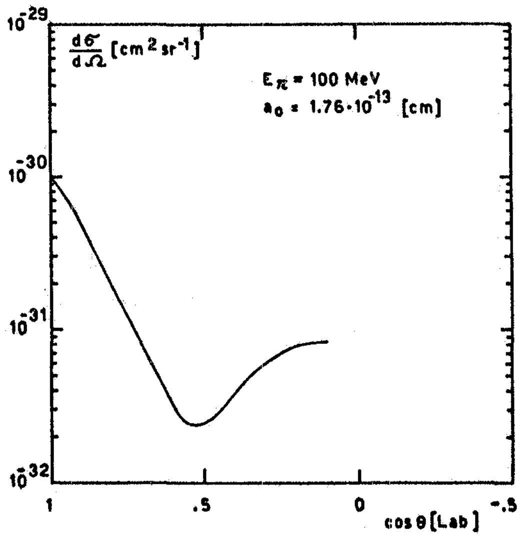
$$F(\vec{K}',\vec{K}) = \frac{K}{2\pi i}\left[\int e^{i(\vec{K}-\vec{K}')\cdot\vec{b}}\left(\Sigma\Sigma\langle t(\vec{b})\rangle\langle t(\vec{b})\rangle\right. + \right.$$

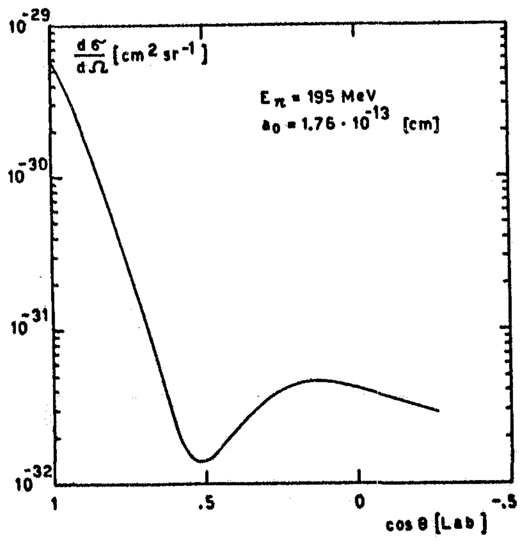
$$+ \int e^{i(\vec{K}-\vec{K}')\cdot\vec{b}}\left(\Sigma\Sigma\Sigma\langle t(\vec{b})\rangle\langle t(\vec{b})\rangle\langle t(\vec{b})\rangle\right. + \tag{3.9}$$

$$+ \Sigma\Sigma\Sigma\Sigma\langle t(\vec{b})\rangle\langle t(\vec{b})\rangle\langle t(\vec{b})\rangle\langle t(\vec{b})\rangle + \ldots \quad \text{ecc.}$$

The shell model says that $Li^7$ is composed of two protons and two neutrons in 1 S state and one proton and two neutrons in 1 P state. The elastic double charge exchange therefore occurs on the pair of 1 p neutrons, so $F_2$ has a single term. The angular distributions $(d\sigma/d\Omega)_{Lab}$ are given in figs. 4, 5, 6 for energy $E_\pi$ of 80, 100, 195 MeV.




FIGS. 4, 5, 6 - The angular distributions $(d\sigma/d\Omega)_{Lab}$ for $\pi^+ + Li^7 \rightarrow \pi^- + B^7$ and for $E_\pi$ = 80, 100, 195 MeV.

It is very difficult to compare our results with experimental data for the absence of bound states in the reaction products. Gilly et al. give for $Li^7$ the forward cross-section $(0.9 \pm 0.1 \times 10^{-28}\ cm^2 sr^{-1})$ for incident meson with kinetic energy of 195 MeV.

By means of the binding energy of the $B^7$ nucleus (16, 6 MeV) and the $\pi^-$ spectrum we can obtain a value for the experimental forward elastic scattering cross-section very near to ours.

Our calculation is also in accordance with data of E. Becher and Z. Maric[2] when they assume that double charge exchange occurs on the 1p neutron pair and that the final nuclear wave function is the same as the initial one.

## 4. - CONCLUSION. -

As one sees, for angles $\theta < \theta_o$, this simple theory can be used as a nuclear model check and as a nuclear correlation study. The lack of experimental data does not allow this control.

## ACKNOWLEDGEMENT. -

It is a pleasure to thank Prof. B. Touschek and Dr. E. Etim who read through the manuscript and suggested many improvements.

## REFERENCES. -

(1) - S. B. Kaufman and C. O. Hower, Phys. Rev. 140, B1272 (1965).
(2) - F. Becker and Z. Maric, Nuovo Cimento 41, B174 (1966); and references therein.
(3) - R. J. Glauber, Lectures in Theoretical Physics (New York, 1959), vol. 1, p. 315.
(4) - M. M. Sternheim, Phys. Rev. 135, B912 (1964).
(5) - R. Hofstadter, Nuclear and Nucleon Structure (New York, 1963), p. 315.
(6) - L. D. Roper, R. Wright and B. T. Feld, Phys. Rev. 138, B190 (1965).
(7) - E. Ferrari and F. Selleri, Phys. Rev. Letters 7, 387 (1961).
(8) - L. Gilly, M. J. Meunier, M. Spighel, J. P. Stroot, P. Duteil and A. Rode, Phys. Letters 11, 264 (1964).