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Double Photon Emission in $e^{\pm}e^{-}$ Collisions.

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Summary. — In this work we evaluate, for the process $e^{\pm}+e^{-}\to e^{\pm}+e^{-}+2\gamma$, the angular distribution of the two photons, their energy spectrum and the total cross-section for emitted photons of energy $\geqslant \varepsilon$, in the extreme relativistic limit.

1. – Introduction.

In the attempt to observe two-photon annihilation of electron-positron pairs with ADA (1) a number of events were detected in which the two-photon coincidences between two lead-glass Čerenkov counters could be interpreted as due to the process of double bremsstrahlung

$$e^{+}+e^{-}\to e^{+}+e^{-}+2\gamma\,. \tag{1}$$

If the energy of the incident electrons and positrons is sufficiently high, process (1) would dominate over two-quanta annihilation by a factor of the order $\alpha^2(E/m)^2$ because under the conditions of the experiments the four-momentum transfer between the colliding particles of process (1) is smaller than the corresponding quantity for two-quanta annihilation. This property of the process (1) allowed one to put it on the list of possible monitoring processes, defined as collision processes between electrons and positron sin which the momentum transfer could be considered sufficiently small, so that its description by quantum electrodynamics with unit form factors and unmodified

(1) C. Bernardini, G. F. Corazza, G. Di Giugno, J. Haissinski, P. Marin, R. Querzoli and B. Touschek: *Nuovo Cimento*, **34**, 1473 (1964).
(2) P. Di Vecchia: *Nuovo Cimento*, **45**, 249 (1966).

propagators could be considered accurate. Besides it seemed that process (1) presented some particular advantages with regard to other monitoring processes (²).

The cross-section for process (1) has been calculated previously by BANDER (³) and by BAYER and GALITSKY (⁴), but their results did not agree. We set out to understand the reason for this discrepancy. In view of the lengthiness and complication of the kinematic part of the calculation this investigation was performed along two independent lines by the authors. The results will be compared with those published by other authors, so as to obtain a complete picture of process (1). Our results agree with those obtained by BAYER and GALITSKY.

2. – Results.

The Feynman diagrams contributing to (1) in lowest order are obtained by adding in all possible ways two external photon lines to the two graphs of the elastic scattering and of the annihilation.

In the high-energy and small-angle approximation only the leading graphs in Fig. 1*a* have been considered, the contribution of the graphs of Fig. 1*b* and Fig. 1*c* being considered negligible. This is certainly correct for an experiment which looks mainly at the photons in the forward-backward direction, but is not

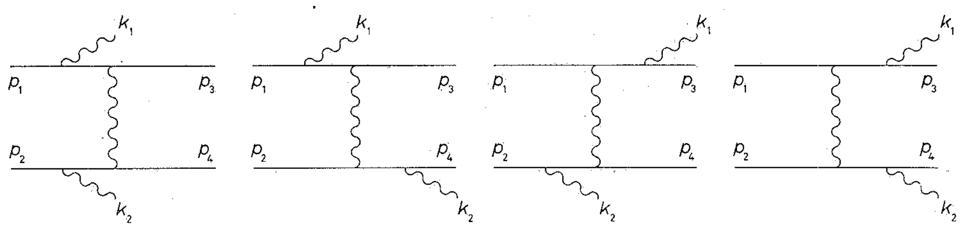

Fig. 1*a*. – Leading graphs.

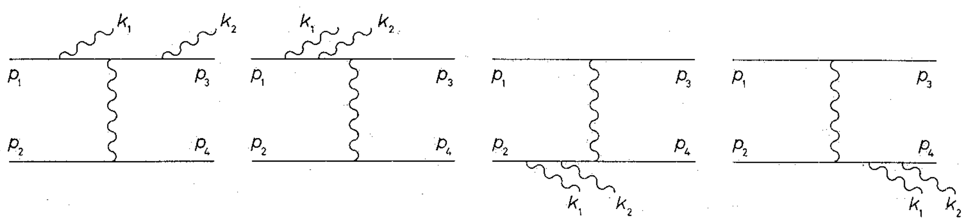

Fig. 1*b*. – Some negligible scattering graphs.

(³) M. BANDER: SLAC-TN-64-93 (1964).
(⁴) V. N. BAYER and V. M. GALITSKY: *JETP Lett.*, **2**, 165 (1965).

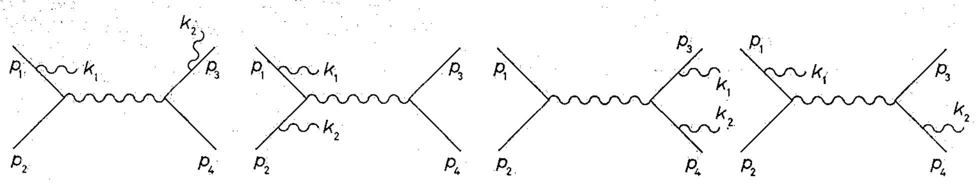

Fig. 1c. – Some negligible annihilation graphs.

so obvious to hold for the total cross-section. The reason that makes the annihilation graphs negligible can be inferred from an inspection of the expression of the photon propagator. Indeed the order of magnitude of the photon propagator in the annihilation graphs is $E(E-\omega_{1,2})$ in c.m.s., while the propagator of the scattering graphs goes to m^2 (m is the electron mass). The resulting cross-section is

$$(2)\qquad \sigma=\frac{(2\pi)^2}{[(p_1p_2)^2-m^4]^{\frac{1}{2}}}\int\frac{\mathrm{d}^3p_3}{E_3}\int\frac{\mathrm{d}^3p_4}{E_4}\int\frac{\mathrm{d}^3k_1}{\omega_1}\cdot\int\frac{\mathrm{d}^3k_2}{\omega_2}\,\frac{\delta^4(p_1+p_2-p_3-p_4-k_1-k_2)}{(p_1-p_3-k_1)^4}\sum_{\text{spin}}\sum_{\text{pol}}\frac{|M|^2}{4}\,,$$

where $p_1(p_2)$ and $p_3(p_4)$ are the four-momenta of the incoming and outgoing electron (positron), k_1, k_2 those of the two photons and the maxtrix element M is

$$(3)\qquad M=\bar{u}(p_3)D_\mu u(p_1)\bar{v}(p_2)C^\mu v(p_4)\,\frac{ie^4m^2}{2(2\pi)^5}$$

with

$$(4a)\qquad D_\mu=\gamma_\mu\frac{\gamma\cdot p_1'-\gamma\cdot k_1+m}{-2(p_1k_1)}\gamma\cdot e_1+\gamma\cdot e_1\frac{\gamma\cdot p_3+\gamma\cdot k_1+m}{2(p_3k_1)}\gamma_\mu\,,$$

$$(4b)\qquad C_\mu=\gamma_\mu\frac{-\gamma\cdot p_4-\gamma\cdot k_2+m}{2(p_4k_2)}\gamma\cdot e_2+\gamma\cdot e_2\frac{-\gamma\cdot p_2+\gamma\cdot k_2+m}{-2(p_2k_2)}\gamma_\mu\,.$$

After elimination of the δ-function the cross-section (2) becomes

$$(5)\qquad \sigma=\frac{(2\pi)^2}{[(p_1p_2)^2-m^4]^{\frac{1}{2}}}\int\frac{\mathrm{d}^3k_1}{\omega_1}\int\frac{\mathrm{d}^3k_2}{\omega_2}\cdot\int\mathrm{d}\Omega\left[\frac{\mathrm{d}|\boldsymbol{p}_3|}{\mathrm{d}E_f}\,\frac{\boldsymbol{p}_3^2}{E_3E_4(p_1-p_3-k_1)^4}\sum_{\text{pol}}\sum_{\text{spin}}\frac{|M|^2}{4}\right]_{\substack{E_i=E_f\\ \boldsymbol{P}_i=\boldsymbol{P}_f}},$$

where $\boldsymbol{P}_i(E_i)$ and $\boldsymbol{P}_f(E_f)$ are the total momentum (energy) of the system in the initial and final states. The calculation of the integral (5) has been performed in the center-of-mass frame, where $E_1=E_2=E$ and $\boldsymbol{p}_1=-\boldsymbol{p}_2$.

4485

In order to obtain information about the high-energy behaviour of the process, the special case in which one photon follows exactly the direction of the incident positron has been treated in detail. The cross-section for this forward-backward emission is [2]

$$(6)\qquad \mathrm{d}\sigma_{\text{F.B.}} = \frac{\alpha^2 r_0^2}{\pi^3}\frac{\mathrm{d}x_1}{x_1}\frac{\mathrm{d}x_2}{x_2}\left\{\frac{4}{3}(1-x_1)(1-x_2) + (1-x_1)x_2^2 + (1-x_2)x_1^2 + x_1^2x_2^2\right\}\gamma^4\,\mathrm{d}\Omega_1\,\mathrm{d}\Omega_2\,,$$

where $x_{1,2} = \omega_{1,2}/E$ and $\mathrm{d}\Omega_{1,2}$ are the elements of solid angle of the two photons. The result (6) is valid only for $\mathrm{d}\Omega_{1,2} \ll \pi/\gamma^2$ and this is too strong a limitation on a detection device.

The total section, which gives the energy distribution of the radiation, has been calculated from (5) independently by the two authors, who found respectively

$$(7a)\qquad \mathrm{d}\sigma = \frac{8\alpha^2 r_0^2}{\pi}\frac{\mathrm{d}x_1}{x_1}\frac{\mathrm{d}x_2}{x_2}\cdot\left\{\frac{9}{4}(1-x_1)(1-x_2) + \frac{3}{2}\left[(1-x_2)x_1 + (1-x_1)x_2^2\right] + x_1^2x_2^2\right\}\qquad \text{(M.G.)}\,,$$

$$(7b)\qquad \mathrm{d}\sigma = \frac{8\alpha^2 r_0^2}{\pi}\frac{\mathrm{d}x_1}{x_1}\frac{\mathrm{d}x_2}{x_2}\left\{(1-x_1)(1-x_2)\left[\frac{5}{4} + \frac{7}{8}\zeta(3)\right] + \left[x_1^2(1-x_2) + x_2^2(1-x_1)\right]\left[\frac{1}{2} + \frac{7}{8}\zeta(3)\right] + \frac{7}{8}\zeta(3)x_1^2x_2^2\right\}\qquad \text{(P.D.V.)}\,,$$

where

$$\zeta(3) = \sum_{n=1}^{\infty}\frac{1}{n^3}\,.$$

The result (7*a*) is given with an error about the order of 5%, and both agree in the limit of low frequencies with the Bloch-Nordsieck cross-section $\mathrm{d}\sigma_{\text{B.N.}}$ [2]:

$$(8)\qquad \mathrm{d}\sigma_{\text{B.N.}} = \frac{8\alpha^2 r_0^2}{\pi}\frac{\mathrm{d}x_1}{x_1}\frac{\mathrm{d}x_2}{x_2}\left[\frac{5}{4} + \frac{7}{8}\zeta(3)\right]\,(*)\,.$$

The cross-sections (6) and (7) have been evaluated in this paper for the case

(*) The numerical coefficient 2, which appears in a previous paper [2], is not correct and must be substituted with 5/4. The same results have been found by BAYER and GALITSKY [5].

[5] V. N. BAYER and V. M. GALITSKY: *Phys. Lett.*, **13**, 355 (1964).

$E_3 \gg m$ where E_3 could be for example the energy of the final electron; the results presented here therefore are wrong only where $E-\omega_{1,2}$ is about the order of m. In each other part of the spectrum the previous results are valid $o(1/\gamma)$ where $\gamma = E/m$. The result (7) agrees with the values of the cross-section given by BAYER and GALITSKY, which is identical to (7*b*).

It has been noted by SIDOROV that the cross-section (7) can be put in the approximation form

$$\mathrm{d}\sigma \simeq \frac{8\alpha^2 r_0^2}{\pi}\frac{\mathrm{d}x_1}{x_1}\frac{\mathrm{d}x_2}{x_2}F(x_1)F(x_2)\,, \tag{9}$$

where $F(x)$ is

$$F(x) = \tfrac{3}{2}(1-x) + x^2\,. \tag{10}$$

This result is very important from the experimental point of view. It says that the energy spectrum of one photon is completely independent from the energy spectrum of the other photon. This separability of the cross-section has been experimentally confirmed with VEP-1 in Novosibirsk ([6]).

The expressions of the forward-backward cross-section (6) and of the total cross-section (7) allow us to calculate the mean angle $\bar{\Theta}$ of the two-photon emission by the process (1)

$$\frac{\gamma^4\bar{\Theta}^4}{8} = \tag{11}$$
$$= \frac{(1-x_1)(1-x_2)\left[\frac{5}{4}+\frac{7}{8}\zeta(3)\right]+\left[x_1^2(1-x_2)+x_2^2(1-x_1)\right]\left[\frac{1}{2}+\frac{7}{8}\zeta(3)\right]+\frac{7}{8}\zeta(3)x_1^2x_2^2}{\frac{4}{3}(1-x_1)(1-x_2)+x_1^2(1-x_2)+x_2^2(1-x_1)+x_1^2x_2^2}\,.$$

$\gamma^4\Theta^4/8$ is a slowly varying function of x_1 and x_2, whose values lie between 1.0 and 1.8.

In order to obtain information about the angular behaviour of the emitted photons, we evaluated the cross-section $\mathrm{d}\sigma(t, x_1, x_2)$ where $t = \gamma\Theta = \gamma R/l$. R is the radius of both the Čerenkov counters and l is the distance between a Čerenkov counter and the crossing point of the electron-positron beams. This resulting cross-section is

$$\mathrm{d}\sigma(t, x_1x_2) = \frac{8\alpha^2 r_0^2}{\pi}\frac{\mathrm{d}x_1}{x_1}\frac{\mathrm{d}x_2}{x_2}\{G_1(t)(1-x_1)(1-x_2) + G_2(t)\left[x_1^2(1-x_2)+x_2^2(1-x_1)\right] + G_3(t)x_1^2x_2^2\}\,. \tag{12}$$

([6]) P. T. GOLUBNICIJ, A. P. ONUCIN, S. G. POPOV and V. A. SIDOROV: presented at the *International Symposium on Electron and Positron Storage Rings, Orsay* September 1966.

The functions G_1, G_2, G_3 are indicated in Table I. This result agrees with an analogous calculation performed by BAYER *et al.* ([7]).

TABLE I.

t	G_1	G_2	G_3	M_1	M_2	M_3
1	0.081	0.065	0.05	0.16	0.065	0.05
2	0.41	0.31	0.24	0.74	0.31	0.24
3	0.74	0.55	0.41	1.27	0.55	0.41
4	1.02	0.74	0.54	1.70	0.74	0.54
5	1.23	0.88	0.64	2.00	0.88	0.64
6	1.39	0.99	0.71	2.25	0.99	0.71
7	1.52	1.07	0.76	2.41	1.07	0.76
8	1.62	1.14	0.80	—	—	—
9	1.70	1.19	0.84	—	—	—
10	1.77	1.23	0.86	—	—	—
11	1.82	1.27	0.89	—	—	—
12	1.87	1.30	0.90	—	—	—
13	1.91	1.32	0.92	—	—	—
14	1.94	1.34	0.93	—	—	—

The cross-section for emitted photons of energy $\omega_{1,2}/E \geqslant \varepsilon$ is obtained by integrating (12) over x_1 and x_2 and turns out to be

$$(13) \qquad \sigma(\varepsilon, t) = \frac{8\alpha^2 r_0^2}{\pi} \left\{ G_1(t) \left[\log\frac{1}{\varepsilon} - 1 + \varepsilon \right]^2 + \right. \\ \left. + G_2(t)(1-\varepsilon^2) \left[\log\frac{1}{\varepsilon} - 1 + \varepsilon \right] + G_3(t) \left(\frac{1-\varepsilon^2}{2} \right)^2 \right\}.$$

A plot of $\sigma(\varepsilon, t)$ is drawn in Fig. 2 for $t \to \infty$. An analogous calculation performed by BANDER ([4]) has given the following result:

$$(14) \qquad \sigma(\varepsilon, t) = \frac{8\alpha^2 r_0^2}{\pi} \left\{ M_1(t) \left[\log\frac{1}{\varepsilon} - 1 + \varepsilon \right]^2 + M_2(t) \left[\log\frac{1}{\varepsilon} - 1 + \varepsilon \right] + \frac{1}{4} M_3(t) \right\}.$$

The functions $M_1(t)$, $M_2(t)$, $M_3(t)$ are indicated in Table I. We believe that (14) is valid only in the limit $\varepsilon^2 \ll 1$, because the cross-section (14) does not go to zero when ε goes to 1. In this approximation the formulas (13) and (14) present the same analytical form with regard to the dependence on t and ε.

([7]) V. N. BAYER, V. S. FADIN and V. A. KHOZE: *Emission of two photons into a given angle in electron collisions*, presented at the *International Symposium on Electron and Positron Storage Rings, Orsay,* September 1966.

4488

Besides, Table I shows an exact coincidence between the functions $G_2(t)$, $G_3(t)$ and $M_2(t)$, $M_3(t)$. The functions $F_1(t)$ and $M_1(t)$, which give the main contribution to the cross-section at the low frequencies, present instead an almost constant discrepancy, whose order is of about 50%.

Therefore our results agree completely with those obtained by BAYER and by BANDER apart from the discrepancy with Bander's calculations, concerning the functions $F_1(t)$ and $M_1(t)$.

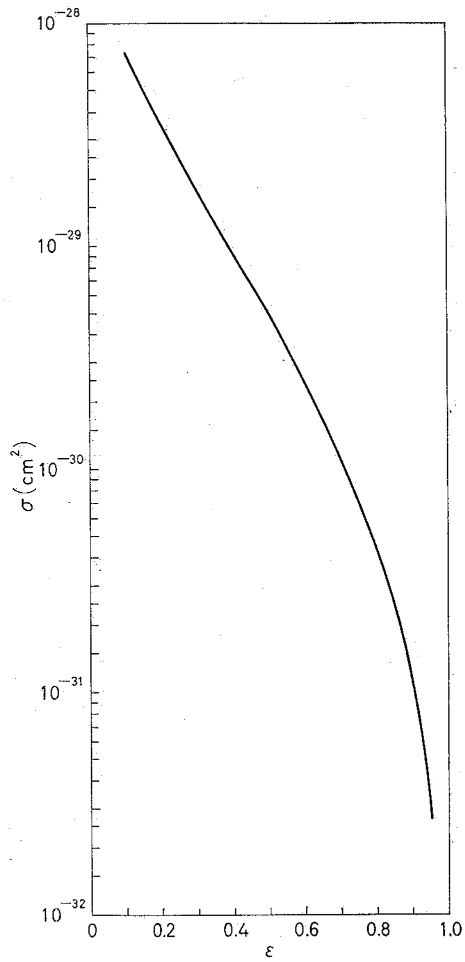

Fig. 2.

A remarkable feature of the cross-section (7) is the absence of a logarithmic term of the form $\log 2\gamma$, which is so characteristic of the single-bremsstrahlung total cross-section ([8]). This logarithmic term does not appear in the cross-section of process (1) because the square modulus of the current matrix element is, because of the transversality of the emitted photons, proportional to the fourth power of the momentum transfer and the probability of a momentum transfer z is proportional to dz/z^3, so that the differential cross-section of the process (1) approaches zero when there is no momentum transfer. The single-bremsstrahlung differential cross-section behaves as dz/z for low momentum transfer and this creates the logarithmic term. The lack of a logarithmic term puts double bremsstrahlung at a disadvantage opposite the background of the single-bremsstrahlung processes created either in collisions with the molecules of the residual gas or in genuine electron-positron collisions ([9]).

([8]) G. ALTARELLI and F. BUCCELLA: *Nuovo Cimento*, **34**, 1337 (1964).

([9]) S. TAZZARI: unpublished paper presented at the *Congressino Adone, Frascati* February 1966.

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APPENDIX

In this Appendix we exhibit some details of the calculation of $d\sigma$ and $d\sigma_{\text{F.B.}}$, of which before we have given only the results. The total cross-section (5) can be written in the following form:

$$\sigma = \frac{2\alpha^2 r_0^2 m^6}{E^2(2\pi)^4}\int \omega_1 \, d\omega_1 \, d\Omega_1 \int \omega_2 \, d\omega_2 \, d\Omega_2 \int d\Omega \left[X \frac{|\boldsymbol{p}_3|^2}{E_3 E_4 (p_1 - p_3 - k_1)^4} \frac{d|\boldsymbol{p}_3|}{dE_f}\right]_{\substack{E_i = E_f \\ \boldsymbol{P}_i = \boldsymbol{P}_f}}, \tag{A.1}$$

where

$$X = \tfrac{1}{4} \sum_{\text{pol}} \text{Tr}\,\{D_\mu \Lambda_-(p_1) \tilde{\bar{D}}_\nu \Lambda_-(p_3)\}\, \text{Tr}\,\{C^\mu \Lambda_+(p_4) \bar{C}^\nu \Lambda_+(p_2)\}\,. \tag{A.2}$$

Since we consider only the terms which in the limit of high energies give the maximum contribution to the cross-section (2), we obtain the following value for the expression (A.2) by making use of well-known trace theorems ([10]):

$$X = \sum_{i=1}^{9} A_i\,, \tag{A.3}$$

where

$$\text{(A.4)} \quad \left\{ \begin{aligned} A_1 &= \frac{1}{2m^4} \sum_{\text{pol}} \left(\frac{p_1 e_1}{p_1 k_1} - \frac{p_3 e_1}{p_3 k_1}\right)^2 \left(\frac{p_4 e_2}{p_4 k_2} - \frac{p_2 e_2}{p_2 k_2}\right)^2 \cdot [(p_1 p_4)(p_2 p_3) + (p_1 p_2)(p_3 p_4)]\,, \\ A_2 &= \frac{1}{2m^4} \sum_{\text{pol}} \left(\frac{p_1 e_1}{p_1 k_1} - \frac{p_3 e_1}{p_3 k_1}\right)^2 \left(\frac{p_4 e_2}{p_4 k_2} - \frac{p_2 e_2}{p_2 k_2}\right) \cdot \\ &\quad \cdot \left[\frac{p_2 e_2}{p_2 k_2}\big((p_1 p_4)(p_3 k_2) + (p_3 p_4)(p_1 k_2)\big) + \right. \\ &\qquad\qquad \left. + \frac{p_4 e_2}{p_4 k_2}\big((p_1 k_2)(p_2 p_3) + (p_1 p_2)(p_3 k_2)\big)\right], \end{aligned} \right.$$

([10]) J. M. JAUCH and F. ROHRLICH: *The Theory of Photons and Electrons* (Reading, Mass., 1959), p. 439.

4490

$$
\text{(A.4)}\quad
\left\{
\begin{aligned}
A_3 = {} & \frac{1}{2m^4}\sum_{\text{pol}}\left(\frac{p_1 e_1}{p_1 k_1}-\frac{p_3 e_1}{p_3 k_1}\right)\left(\frac{p_4 e_2}{p_4 k_2}-\frac{p_2 e_2}{p_2 k_2}\right)\cdot \\
& \cdot\left\{-\frac{p_3 e_1}{p_3 k_1}\left[\frac{p_2 e_2}{p_2 k_2}\big((p_1p_4)(k_1k_2)+(p_1k_2)(p_4k_1)\big)+\right.\right. \\
& \left.+\frac{p_4 e_2}{p_4 k_2}\big((p_1k_2)(p_2k_1)+(p_1p_2)(k_1k_2)\big)\right]- \\
& -\frac{p_1 e_1}{p_1 k_1}\left[\frac{p_2 e_2}{p_2 k_2}\big((p_4k_1)(p_3k_2)+(k_1k_2)(p_3p_4)\big)+\right. \\
& \left.\left.\qquad\qquad+\frac{p_4 e_2}{p_4 k_2}\big((k_1k_2)(p_2p_3)+(p_2k_1)(p_3k_2)\big)\right]\right\}, \\[2ex]
A_4 = {} & \frac{1}{2m^4}\sum_{\text{pol}}\left(\frac{p_1 e_1}{p_1 k_1}-\frac{p_3 e_1}{p_3 k_1}\right)^2\left\{-2m^2\frac{(p_1k_2)(p_3k_2)}{(p_4k_2)(p_2k_2)}-\right. \\
& \left.-\frac{(p_1k_2)(p_2p_3)+(p_1p_2)(p_3k_2)}{p_4k_2}-\frac{(p_1p_4)(p_3k_2)+(p_1k_2)(p_3p_4)}{p_2k_2}\right\}, \\[2ex]
A_5 = {} & \frac{1}{2m^4}\sum_{\text{pol}}\left(\frac{p_1 e_1}{p_1 k_1}-\frac{p_3 e_1}{p_3 k_1}\right)\left\{\frac{p_1 e_1}{p_1 k_1}\left[\frac{(k_1k_2)(p_2p_3)+(p_2k_1)(p_3k_2)}{p_4k_2}+\right.\right. \\
& \left.+\frac{(p_4k_1)(p_3k_2)+(k_1k_2)(p_3p_4)}{p_2k_2}\right]+ \\
& +\frac{p_3 e_1}{p_3 k_1}\left[\frac{(p_1k_2)(p_2k_1)+(p_1p_2)(k_1k_2)}{p_4k_2}+\frac{(p_1p_4)(k_1k_2)+(p_1k_2)(p_4k_1)}{p_2k_2}\right]+ \\
& \left.\qquad+2m^2\left[\frac{(p_1e_1)}{p_1k_1}\frac{(k_1k_2)(p_3k_2)}{(p_4k_2)(p_2k_2)}+\frac{p_3e_1}{p_3k_1}\frac{(p_1k_2)(k_1k_2)}{(p_2k_2)(p_4k_2)}\right]\right\}, \\[2ex]
A_6 = {} & \frac{1}{2m^4}\left\{\frac{(k_1k_2)(p_2p_3)+(p_2k_1)(p_3k_4)}{(p_1k_1)(p_4k_2)}+\frac{(p_4k_1)(p_3k_2)+(k_1k_2)(p_3p_4)}{(p_1k_1)(p_2k_2)}+\right. \\
& +\frac{(p_1k_2)(p_2k_1)+(p_1p_2)(k_1k_2)}{(p_3k_1)(p_4k_2)}+\frac{(p_1p_4)(k_1k_2)+(p_1k_2)(p_4k_1)}{(p_3k_1)(p_2k_2)}+ \\
& +2m^2\left[\frac{(k_1k_2)(p_3k_2)}{(p_1k_1)(p_2k_2)(p_4k_2)}+\frac{(p_1k_2)(k_1k_2)}{(p_3k_1)(p_4k_2)(p_2k_2)}+\right. \\
& \left.+\frac{(k_1k_2)(p_2k_1)}{(p_1k_1)(p_3k_1)(p_4k_2)}+\frac{(p_4k_1)(k_1k_2)}{(p_1k_1)(p_3k_1)(p_2k_2)}\right]+ \\
& \left.\qquad\qquad+\frac{2m^4(k_1k_2)^2}{(p_1k_1)(p_2k_2)(p_3k_1)(p_4k_2)}\right\}.
\end{aligned}
\right.
$$

A_7, A_8, A_9 are obtained from A_2, A_4, A_5 by the following substitutions:

$$
\text{(A.5)}\qquad k_1 \leftrightarrow k_2\,,\qquad p_1 \leftrightarrow -p_4\,,\qquad p_3 \leftrightarrow -p_2\,,\qquad e_1 \leftrightarrow e_2\,.
$$

4491

Before performing the integrations in (A.1) over $d\Omega, d\Omega_1, d\Omega_2$ the following substitutions must be made:

$$\text{(A.6)} \qquad \begin{cases} \boldsymbol{p}_4 = -\boldsymbol{p}_3 - \boldsymbol{k}_1 - \boldsymbol{k}_2 \,, \qquad E_4 = 2E - \omega_1 - \omega_2 - E_3 \,, \\ E_3 = \dfrac{2E^2 - 2E\omega_1 - 2E\omega_2 + \omega_1\omega_2 - \boldsymbol{k}_1\boldsymbol{k}_2}{2E - \omega_1 - \omega_2 - \boldsymbol{\beta}_3\boldsymbol{k}_1 + \boldsymbol{\beta}_3\boldsymbol{k}_2} \,, \end{cases}$$

where according to the previous approximations β and β_3 are

$$\text{(A.7)} \qquad \beta = 1 - \frac{m^2}{2E^2} \,, \qquad \beta_3 = 1 - \frac{m^2}{2(E - \omega_1)^2} \,.$$

Since the energy of the incident particles is very high, the maximum contribution to the integral (A.1) is obtained when one photon follows the incident electron, the other the incident positron, and the scattering angles of the electron and positron are very small.

Therefore it is possible to develop the integrand in a series around the points $\theta = \theta_1 = 0$, $\theta_2 = \pi$ so that in the integral (A.1) are considered only the contributions around the cusp of the differential cross-section. Owing to this

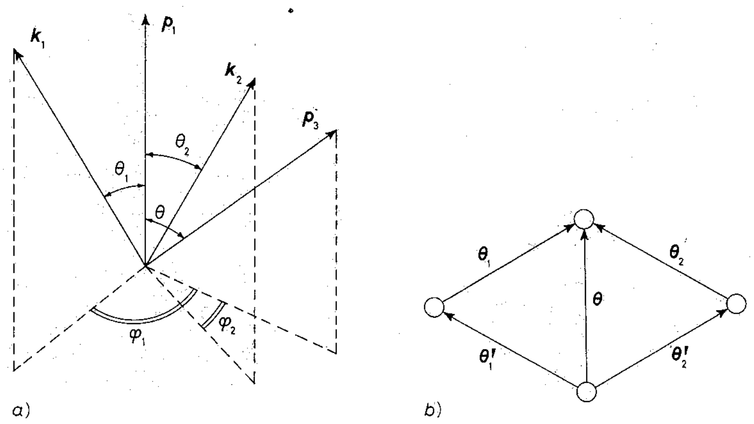

Fig. 3.

development the angles $\theta, \theta_1, \theta_2$ can be considered as vectors (Fig. 3) of the plane orthogonal to the colliding beams and the cross-section (A.1) results

$$\text{(A.8)} \qquad \sigma = \frac{\alpha^2 r_0^2}{\pi^4}\int\frac{dx_1}{x_1}\int\frac{dx_2}{x_2}\int\frac{d\boldsymbol{z}}{z^4}\int d\boldsymbol{x}\,\{2(1 - x_1)I(\boldsymbol{x}, \boldsymbol{z}) + \\ + x_1^2 F(\boldsymbol{x}, \boldsymbol{z})\}\int d\boldsymbol{y}\,\{2(1 - x_2)I(\boldsymbol{y}, \boldsymbol{z}) + x^2 F(\boldsymbol{y}, \boldsymbol{z})\} \,,$$

4492

where

$$\text{(A.9)}\quad \begin{cases} \boldsymbol{x} = \gamma\boldsymbol{\theta}_1\,, \quad \boldsymbol{y} = \gamma\boldsymbol{\theta}_2\,, \quad \boldsymbol{z} = \gamma[(1-x_1)\boldsymbol{\theta} + x_1\boldsymbol{\theta}_1]\,, \\[2mm] I(\boldsymbol{x}, \boldsymbol{z}) = \dfrac{x^2}{(1+x^2)^2} + \dfrac{(\boldsymbol{x}-\boldsymbol{z})^2}{[1+(\boldsymbol{x}-\boldsymbol{z})^2]^2} - \dfrac{2\boldsymbol{x}(\boldsymbol{x}-\boldsymbol{z})}{(1-x^2)[1+(\boldsymbol{x}-\boldsymbol{z})^2]}\,, \\[2mm] F(\boldsymbol{x}, \boldsymbol{z}) = \dfrac{z^2}{(1+x^2)[1+(\boldsymbol{x}-\boldsymbol{z})^2]}\,. \end{cases}$$

If we eliminate the integrations over $\mathrm{d}x_1$, $\mathrm{d}x_2$, $\mathrm{d}\boldsymbol{x}$ and $\mathrm{d}\boldsymbol{y}$, the forward-backward cross-section $\mathrm{d}\sigma_{\text{F.B.}}$ can be obtained provided that we put $\boldsymbol{x} = \boldsymbol{y} = 0$ and integrate over $\boldsymbol{z}$. By making use of polar co-ordinates it is easy to perform the integration over the two photon azimuths $\varphi_{1,2}$ and then on x and y from 0 to $\gamma\Theta$. The resulting cross-section is

$$\text{(A.10)}\quad \mathrm{d}\sigma = \frac{2\alpha^2 r_0^2}{\pi}\frac{\mathrm{d}x_1}{x_1}\frac{\mathrm{d}x_2}{x_2}\int_0^\infty \frac{\mathrm{d}z}{z^3}\{4(1-x_1)J(z,u) + x_1^2 G(z,u\}\cdot \cdot\{4(1-x_2)J(z,u) + x_2^2 G(z,u)\}\,,$$

where

$$\text{(A.11)}\quad \begin{cases} J(z,u) = \dfrac{2+z^2}{2z\sqrt{z^2+4}}\cdot \\ \qquad \cdot \log\dfrac{(z+\sqrt{z^2+4})^4}{4\{z[z+\sqrt{z^2+4}][A(z,u)-1+u(4+z^2)]+4A(z,u)\}} - \\ \qquad -\dfrac{3}{4}+\dfrac{1}{2}u+\dfrac{(2+z^2)u-1}{4A(z,u)}\,, \\[2mm] G(z,u) = \dfrac{z}{\sqrt{z^2+4}}\cdot \\ \qquad \cdot \log\dfrac{(z+\sqrt{z^2+4})^4}{4\{z[z+\sqrt{z^2+4}][A(z,u)-1+u(4+z^2)]+4A(z,u)\}}\,, \\[2mm] A(z,u) = [1-2z^2u+u^2z^2(z^2+4)]^{\frac{1}{2}}\,, \\[2mm] u = \dfrac{1}{1+t^2} = \dfrac{1}{1+\gamma^2\Theta^2}\,. \end{cases}$$

Because of the convergence of the integral (A.10) the upper limit of integration has been set equal to infinity and the calculation has been accouplished analytically only if $u = 0$. Otherwise the integration over z has been performed numerically.

Instead of developing the integrand of (A.1) in a series around the forward-backward direction, we perform the angular integration in another way, shown below. Let us introduce a slightly different definition of the various angles, as in Fig. 4.

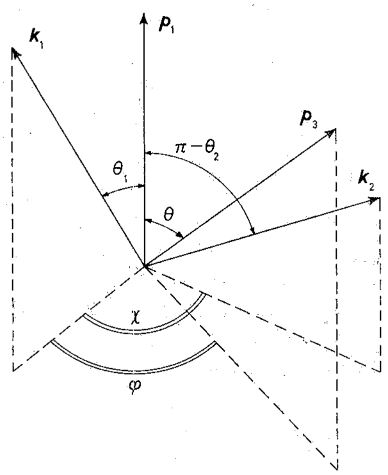

Fig. 4.

By making use of the (A.6) we find following expression for the photon propagator:

(A.12) $(p_1-p_3-k_1)^4=$

$$=\frac{4(\sigma-\delta\cos\theta+\varrho\sin\theta)^2}{\{(2E-\omega_1-\omega_2)+\beta_3\cos\theta(\omega_1\cos\theta_1-\omega_2\cos\theta_2)+\beta_3\sin\theta[\omega_1\sin\theta_1\cos\varphi+\omega_2\sin\theta_2\cos(\varphi-\chi)]\}^2},$$

where

$$\sigma=2(E-\omega_1)^2(E-\omega_2)+[(p_1-k_1)-m^2][2E-\omega_1-\omega_2]\,,$$

$$\delta=\{2(E-\omega_1)(E-\omega_2)(p_1-\omega_1\cos\theta_1)-[(p_1k_1)-m^2][\omega_1\cos\theta_1-\omega_2\cos\theta_2]\}\beta_3\,,$$

$$\varrho=\{2(E-\omega_1)(E-\omega_2)\omega_1\sin\theta_1\cos\varphi+$$
$$+[(p_1k_1)-m^2][\omega_1\sin\theta_1\cos\varphi+\omega_2\sin\theta_2\cos(\varphi-\chi)]\}\beta_3\,.$$

By substituting (A.12) into (A.1), in order to perform the integration over the anomaly θ of the outgoing electron, we must calculate some integrals of this kind

(A.13) $$\int_0^\pi\frac{\sin\theta\,d\theta P(\cos\theta,\sin\theta)}{(\sigma-\delta\cos\theta+\varrho\sin\theta)^2(\alpha-u\cos\theta-v\sin\theta)^h(\alpha'-u'\cos\theta-v'\sin\theta)^k},$$

where $P(\cos\theta,\sin\theta)$ is a polynomial in $\cos\theta$, and h, k can take any value among 0, 1, 2. The terms $(\alpha-u\cos\theta-v\sin\theta)$, $(a'-u'\cos\theta-v'\sin\theta)$ result

4494

from the expressions of (p_4k_2) and (p_3k_1). With the transformation $z=\mathrm{tg}\,(\theta/2)$ we obtain some integrals of the form

$$(\mathrm{A.14})\qquad I_{\alpha,hk}=\int\limits_0^\infty \frac{z^\alpha\,\mathrm{d}z}{(z-z_1)^2(z-z_2)^2(z-z_3)^h(z-z_4)^h(z-z_5)^k(z-z_6)^k},$$

where α is an integer and

$$z_{1,2}=\frac{-\varrho+i\sqrt{\sigma^2-\delta^2-\varrho^2}}{\sigma+\delta},\qquad z_{3,4}=\frac{v\pm i\sqrt{\alpha^2-u^2-v^2}}{\alpha+u},$$

$$z_{5,6}=\frac{v'\pm i\sqrt{\alpha'^2-u'^2-v'^2}}{\alpha'-u'}.$$

The integration can be performed in the complex plane of the variable z; the path is shown in Fig. 5.

We obtain

$$(\mathrm{A.15})\qquad I_{\alpha,hk}=-\frac{1}{\cos\pi\alpha}\frac{\mathrm{d}}{\mathrm{d}\alpha}\left[\sum_{i=1}^{6}\mathrm{Res}\,(z_i)\right].$$

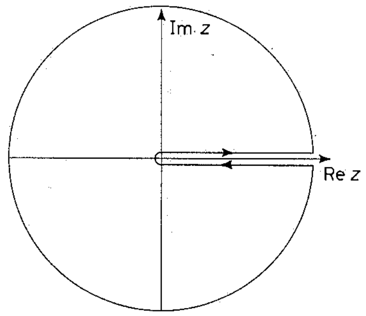

Fig. 5.

For $\theta_1=\theta_2=0$ the function resulting from (A.15), after integration over θ, is independent of φ. This result therefore gives the forward-backward cross-section.

For the general case, we have to perform the integration over the azimuthal angle φ. This can be accomplished in a rather difficult manner, because the large number of terms which come from the integration over θ make the calculation very cumbersome. The result is expressed by the following formula:

$$(\mathrm{A.16})\qquad \mathrm{d}\sigma=\frac{8\alpha^2r_0^2}{\pi}\frac{\mathrm{d}\omega_1}{\omega_1}\frac{\mathrm{d}\omega_2}{\omega_2}\frac{\mathrm{d}(\cos\theta_1)\,\mathrm{d}(\cos\theta_2)}{(1-\beta\cos\theta_1)^2(1-\beta\cos\theta_2)^2}\frac{1}{4\gamma^4}\cdot$$

$$\cdot\left\{\frac{9}{4}\left(1-\frac{\omega_1}{E}\right)\left(1-\frac{\omega_2}{E}\right)+\left(1-\frac{\omega_1}{E}\right)\left(\frac{\omega_2}{E}\right)^2\left[2-\frac{1}{2\gamma^2(1-\beta\cos\theta_2)}\right]+\right.$$

$$+\left(1-\frac{\omega_2}{E}\right)\left(\frac{\omega_1}{E}\right)^2\left[2-\frac{1}{2\gamma^2(1-\beta\cos\theta_1)}\right]+\left(\frac{\omega_1}{E}\right)^2\left(\frac{\omega_2}{E}\right)^2\cdot$$

$$\left.\cdot\left[\frac{3}{4}+\left(1-\frac{1}{2\gamma^2(1-\beta\cos\theta_1)}\right)\left(1-\frac{1}{2\gamma^2(1-\beta\cos\theta_2)}\right)\right]\right\}+B.$$

The term B is not integrated over φ because of its very complicated analytic expression. However it does not contribute to the total cross-section, which is obtained performing the integration over θ_1 and θ_2 in the first part of the right-hand side of (A.16). The dependence of B on θ_1 and θ_2 is of the form $R(\cos\theta_1, \cos\theta_2)\log(\cos\theta_1, \cos\theta_2)$, $R(\cos\theta_1, \cos\theta_2)$ and $\log(\cos\theta_1, \cos\theta_2)$ are a rational and a logarithmical function of $\cos\theta_1$ and $\cos\theta_2$ and do contribute to the angular distributions.

RIASSUNTO

In questo lavoro si calcola, nel limite relativistico estremo, per il processo $e^{\pm}+e^{-}\to e^{\pm}+e^{-}+2\gamma$, la distribuzione angolare dei due fotoni, il loro spettro di energia, e la sezione d'urto totale per l'emissione di fotoni di energia $\geqslant \varepsilon$.

Резюме автором не представлено.

4496