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Remarks on the Saturation of a Superconvergence Sum Rule in π^0 Photoproduction.

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The problem of the approximate saturation of superconvergence relations has been recently investigated by several authors (1-5).

The usual procedure is to consider only the contribution of a few nearby low-lying particles and resonances to the superconvergence sum rule $(1/\pi)\int \mathrm{Im}\, F(\nu, t)\, d\nu = 0$. This method can then give reasonable results provided that the amplitude $F(\nu, t)$ for fixed t goes to zero rapidly enough as ν approaches to infinity.

In this note we want to discuss the superconvergence properties of an invariant amplitude of π^0 photoproduction, and we will show in fact that, owing to the slow convergence of the sum rule, the approximate saturation by means of a few low-lying states, as much as the present phenomenological analysis of the experimental data allows, seems rather doubtful. Following the classical paper by CHEW, GOLDBERGER LOW and NAMBU (6), the matrix element for single-pion photoproduction can be written as

$$H = \sum_i H^{(i)} g^{(i)} ,$$

$$H^{(i)} = M_A A^{(i)}(\nu, t) + M_B B^{(i)}(\nu, t) + M_C C^{(i)}(\nu, t) + M_D D^{(i)}(\nu, t)$$

with

$$M_A = i\gamma_5(\gamma\varepsilon)(\gamma k) ,$$

(1) P. BABU, F. J. GILMAN and M. SUZUKI: *Phys. Lett.*, **24**, B 65 (1967).
(2) B. SAKITA and K. C. WALI: *Phys. Rev. Lett.*, **18**, 31 (1967).
(3) G. ALTARELLI, F. BUCCELLA and R. GATTO: *Phys. Lett.*, **24**, B 57 (1967).
(4) L. K. PANDE: *Nuovo Cimento*, **48** A, 838 (1967).
(5) G. ALTARELLI and M. COLOCCI: *Nuovo Cimento*, **48** A, 573 (1967).
(6) G. F. CHEW, M. L. GOLDBERGER, F. E. LOW and Y. NAMBU: *Phys. Rev.*, **106**, 1345 (1957).

$$M_B = 2i\gamma_5[(P\varepsilon)(qk)-(Pk)(q\varepsilon)]\,,$$
$$M_C = \gamma_5[(\gamma\varepsilon)(qk)-(\gamma k)(q\varepsilon)]\,,$$
$$M_D = 2\gamma_5[(\gamma\varepsilon)(Pk)-(\gamma k)(P\varepsilon)-iM(\gamma\varepsilon)(\gamma k)]\,,$$

where k and q are respectively the photon and pion four-momenta, $P=\frac{1}{2}(p_1+p_2)$, p_1 and p_2 being the initial and final nucleon four-momenta, and ε is the photon polarization. We define, furthermore,

$$\nu_1 = -\frac{(qk)}{2M}\,,\qquad \nu = -\frac{(Pk)}{M}=\frac{W^2-M^2}{2M}-\nu_1\,,\qquad t=\mu^2-4M\nu_1\,;$$

μ and M are the pion and nucleon masses, and W is the c.m. energy.

The superscript (i) refers as usual to the values $(\pm, 0)$, and

$$g^{(+)}=\delta\alpha_3\,,\qquad g^{(-)}=\tfrac{1}{2}[\tau_\alpha,\tau_3]\,,\qquad g^{(0)}\tau_\alpha\,,$$

where α is the isospin index of the outgoing pion.

Superconvergence sum rules have already been obtained by MUKUNDA and RADHA [7] for the $B^{(-)}$ and the $D^{(-)}$ invariant amplitudes, but they found that the contributions of the nucleon and the N^* $J=\frac{3}{2}^+$ (1.238) do not saturate the sum rules.

The result is not surprising if one believes in the Regge behaviour at high energies of the invariant amplitudes, as given for instance by LOCHER and ROLLNIK [8]. In fact the B and D amplitudes asymptotically go like $\nu^{\alpha-1}$, so that, for small fixed t, these amplitudes do not superconverge, because $\alpha(0)$ (the Regge trajectory at $t=0$ of the ω-meson) is of the order of 0.6. It turns out that the only superconvergent amplitude is $C^{(i)}(\nu, t)$, because of the asymptotic behaviour $\nu^{\alpha-2}$.

Taking into account the dispersion relations for the invariant amplitudes [6], and remembering the crossing properties of the C amplitude and of the $g^{(i)}$ [6] one obtains the two superconvergence sum rules

$$\frac{ef}{2M}(\mu_p^a\pm\mu_n)+\frac{2\mu}{\pi}\int_{\nu_0}^{\infty}\operatorname{Im} C^{(^0_+)}(\nu,t)\,d\nu=0 \tag{1}$$

and, because $C^{(0)}+C^{(+)}=C^{(\pi^0)}$, we have

$$\frac{ef}{2M}\mu_p^a+\frac{\mu}{\pi}\int_{\nu_0}^{\infty}\operatorname{Im} C^{(\pi^0)}(\nu,t)\,d\nu=0 \tag{2}$$

or else

$$\frac{ef}{2M}\mu_p^a+\frac{\mu}{\pi M}\int_{\mu+M}^{\infty}W\operatorname{Im} C^{(\pi^0)}(W,\nu_1)\,dW=0\,, \tag{3}$$

(7) N. MUKUNDA and T. K. RADHA: *Nuovo Cimento*, **44** A, 726 (1966).
(8) M. P. LOCHER and H. ROLLNIK: *Phys. Lett.*, **22**, 696 (1966).

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where μ_p^a and μ_n are the anomalous neutron and proton magnetic moments, $f^2/4\pi = 0.08$, $e^2/4\pi = 1/137$. These sum rules have already been obtained by PANDE [4], and ALTARELLI and COLOCCI [5]. PANDE finds that the sum rules (1) are saturated fairly well by the nucleon and the N^* resonance.

The question naturally arises if the contributions of the higher resonances, which are known to be important in pion photoproduction, add up to zero. ALTARELLI and COLOCCI [5] consider also the contributions of the $J = \frac{3}{2}^-$ (1.518) and the $J = \frac{5}{2}^+$ (1.672) resonances; and assuming the superconvergence of the amplitude and of its derivative with respect to t, at $t \simeq 0$, they obtain values for the electric multipoles E_{2-} and E_{3-} [9] at the resonances, of the same order of magnitude but of opposite sign, *i.e.* negative instead of positive, than those given by the analysis of Salin [10] and Beder [11].

In the present paper we try to saturate the sum sule (3) using a phenomenological multipole analysis of π^0 photoproduction up to $W = 1.8$ GeV done by WALKER at Cal-Tech [12]. In this analysis the experimental data are fitted by six resonances (the N^* $J = \frac{3}{2}^+$ at 1.236 GeV, the $J = \frac{3}{2}^-$ at 1.519 GeV, the $J = \frac{5}{2}^+$ at 1.672 GeV, the $J = \frac{5}{2}^-$ at 1.652 GeV, the $J = \frac{1}{2}^+$ at 1.471 GeV and the $J = \frac{1}{2}^-$ at 1.561 GeV, respectively with multipoles M_{1+} and E_{1+}, E_{2-} and M_{2-}, E_{3-} and M_{3-}, E_{2+} and M_{2+}, M_{1-} and E_{0+}), the Born amplitudes and a «background» introduced in order to have a better fit at certain energies. The resonant multipoles are given in Breit-Wigner form with energy-dependent width. Using the relation between the invariant amplitude $C(\nu, t)$ and the multipoles $M_{l\pm}$ and $E_{l\pm}$ given by CGLN [6], we can thus directly try to saturate the superconvergence relation (3).

We keep ν_1 (that is t) fixed small in order to have always physical values for

$$\cos\theta = \frac{\omega_q k - 2M\nu_1}{kq},$$

where θ is the c.m. scattering angle and $(\boldsymbol{q}, \omega_q)(\boldsymbol{k}, k)$ are the c.m. pion and photon four-momenta. This requirement is easily fulfilled [13] taking the «threshold» value of ν_1 for which $\cos\theta = 0$, namely

$$2M\nu_{10} = \frac{\mu^2}{2}\,\frac{\mu + 2M}{\mu + M}, \qquad\qquad t_0 = \mu^2 - 4M\nu_{10}.$$

The first term in eq. (3) gives 0.272 $(\text{GeV})^{-1}$, while the contribution to the integral given only by the first resonance is -0.276 $(\text{GeV})^{-1}$. The agreement is thus very good, and it is worth mentioning that the value of M_{1+} used here is essentially the Dalitz-Sutherland [14] value, and it is practically confirmed by other estimates [15]. This result agrees therefore with Pande's conclusion [16].

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(9) They take $E_{2-}/M_{2-} = 3$ and $E_{3-}/M_{3-} = 2$ as given by BEDER [11].
(10) P. H. SALIN: *Nuovo Cimento*, **28**, 1294 (1963).
(11) D. S. BEDER: *Nuovo Cimento*, **33**, 94 (1964).
(12) R. L. WALKER: private communication to A. B.
(13) G. JONA-LASINIO and H. MUNCZEK: *Phys. Rev.*, **117**, 585 (1960).
(14) R. H. DALITZ and D. G. SUTHERLAND: *Phys. Rev.*, **146**, 1180 (1966).
(15) I. G. AZNAURYAN and L. D. SOLOVIEV: *Dispersion sum rules and SU_6 symmetry*, II (Joint Inst. for Nucl. Research, Preprint E-2544, 14 January 1966).
(16) The N^* contribution given by ALTARELLI and COLOCCI [5] is somewhat different from ours because they use an energy-independent width Breit-Wigner form for the resonances.

Before discussing the contributions to the sum rule coming from the other multipoles, let us try to give an estimate of the high-energy tail to the integral.

Assuming a Regge behaviour for the $C^{(\pi^0)}$ amplitude as given by LOCHER and ROLLNIK (8), taking $\alpha(0) = 0.6$ the integral from $W \simeq 3$ GeV (8,17) up to infinity gives a contribution to the sum rule of the order of magnitude of 0.1 $(\text{GeV})^{-1}$. The positive sign is fixed essentially by the positive sign of the product $f_{\omega\pi\gamma} f_{\omega NN}$, suggested by group-theoretical calculations and favoured by experiments in photoproduction (11). Such a big contribution is not surprising if one thinks that the integrand in eq. (3) behaves at high energies like $\sim W^{2\alpha-3}$ and therefore the integral like $\sim W^{-1}$.

Considering now the contributions given by the experimental multipoles up to 1.8 GeV we find that the $J=\frac{3}{2}^-$ and $J=\frac{5}{2}^+$ account together for $a \sim -0.085\ (\text{GeV})^{-1}$, whereas the other three resonances give small contributions to the integral, and furthermore they are of different signs, and essentially add up together to zero. The « background », on the other hand adds to the integral about 0.05 $(\text{GeV})^{-1}$, so that the sum rule (3) now gives

$$0.272 - 0.21 = 0\,,$$

which is off by 30%.

Apart from the completely pessimistic conclusion that the C amplitude is not at all superconvergent, one may take several points of view, if one believes in relation (3). We may think that the bad result obtained is mainly due to uncertainties in the experimental fit, and particularly the « background ». On the other hand, the high-energy tail in the integral may have been overestimated (obviously apart from the possible uncertainty of the sign), because the C amplitude, considering the rapid convergence properties compared to the other amplitudes, essentially does not play any role in the fits of the high-energy photoproduction cross-sections.

In our opinion, however, the most important point comes from the fact that in the phenomenological saturation of eq. (3) there is a « gap » of experimental information from $W \sim 1.8$ GeV to $W \sim 3$ GeV, where the cross-section begins to have a Regge behaviour (8,17). If we consider that in this energy region there are the $J=\frac{7}{2}^+$ (1920), and at least two other resonances at 2.19 and 2.36 GeV, we see that these additional contributions may well add up to satisfy the sum rule.

We have tried to evaluate very roughly the contribution of the $J=\frac{7}{2}^+$ (1920) to the integral from the differential cross-sections of π^0 photoproduction at different angles given by ALVAREZ *et al.* (18). We have thus obtained a value for the multipoles E_{3+} and M_{3+} which is of the right order of magnitude to saturate eq. (3).

We can think therefore that the superconvergence relation (3) is probably true, even if we do not have yet enough experimental information to check it precisely; but an approximate saturation by means of a few nearby states seems very unlikely.

We can add one more word for the superconvergence properties at fixed $t=t_0$ of the derivative with respect to t of the amplitude, *i.e.* the sum rule

$$\int \left[\frac{\mathrm{d}}{\mathrm{d}t}\,\mathrm{Im}\,C^{(\pi^0)}(\nu, t)\right]_{t=t_0} \mathrm{d}\nu = 0\,. \tag{4}$$

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(17) P. DI VECCHIA and F. DRAGO: preprint LNF-67/8, 27 February 1967, to be published in *Phys. Lett.*

(18) R. ALVAREZ, Z. BAR-YAM, W. KERN, D. LUCKEY, L. S. OSBORNE and S. TAZZARI: *Phys. Rev. Lett.*, **12**, 707 (1964).

Because of a factor $\log \nu$ in the integrand the integral goes to zero even more slowly than the integral in eq. (2), and in fact we obtain very bad results using Walker's multipoles. This maybe explains why ALTARELLI and COLOCCI (5), assuming the validity of both eqs. (2) and (4), find a sign for E_{2-} and E_{3-} in contradiction with the results of several isobar models and phenomenological analyses, Walker's fit amongst others (19).

We do not know at the present moment if this property of slow convergence holds also for amplitudes different from those of pion photoproduction. For instance, in the case of meson-baryon scattering, it may well be that the doubtful conclusions of BABU *et al.* (1) depend from the fact that the Regge trajectory of the 27-plet at $t=0$, although negative, could be rather small in magnitude.

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(19) Walker's fit gives essentially $E_{2-}/M_{2-} \simeq E_{3-}/M_{3-} = 2$.