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Photoproduction of π^+ and $\mathcal{N}^* \to \mathcal{N}\gamma$ Magnetic Transition Amplitude.

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An interesting parameter which can be evaluated by means of the pion photoproduction around the P_{33} resonance is the $\mathcal{N}^* \to \mathcal{N}\gamma$ transition magnetic moment, defined as (*)

$$\mathcal{M} = \langle p, m=\tfrac{1}{2}|M_z|\mathcal{N}^*_{\frac{3}{2}}, m=\tfrac{1}{2}\rangle \,. \tag{1}$$

The prediction for (1) given by the SU_6 symmetry [2] is

$$\mathcal{M} = \frac{2\sqrt{2}}{5}\,\mu_v \,; \tag{2}$$

where $\mu_v = \mu_p - \mu_n = 4.706$ (nuclear magnetons).

In doing our evaluation (**) of (1) we follow the treatment of Dalitz and Sutherland [1], who describe the M^3_{1+} amplitude (***) near the resonance with a Breit-Wigner formula

$$M^3_{1+} = \frac{\sqrt{3}}{4}\,\frac{1}{\sqrt{kq}}\,\frac{\Gamma_\gamma^{\frac{1}{2}}\Gamma^{\frac{1}{2}}}{(W-W_R)-i\Gamma/2} \tag{3}$$

(*) In the following we use the notations of [1].

(1) R. H. DALITZ and D. G. SUTHERLAND: *Phys. Rev.*, **146**, 1180 (1966).

(2) M. A. BEG, B. W. LEE and A. PAIS: *Phys. Rev. Lett.*, **13**, 514 (1964).

(**) A short exposition of the method followed in the determination of $\mathcal{M}$ and the results obtained have been previously reported by us at the *International Conference of Dubna (USSR)*, February 7-15 (1967).

See [3], and M. BENEVENTANO: *Pion photoproduction at energy below* 1 GeV (*Review Report*) in the *Proceedings of Dubna Conference* (in press).

(3) M. GRILLI, M. NIGRO, E. SCHIAVUTA, F. SOSO, P. SPILLANTINI and V. VALENTE: *Recent measurements of π^+ photoproduction with coherent bremsstrahlung*, Internal Report LNF (in press).

(***) M^3_{1+} is the magnetic dipole amplitude $T=\frac{3}{2}$, $J=\frac{3}{2}$. For the multipoles we follow the CGLN [4] notations.

(4) G. F. CHEW, M. L. GOLDBERGER, F. E. LOW and Y. NAMBU: *Phys. Rev.*, **106**, 1345 (1957).

with

$$(4)\qquad \Gamma_\gamma = \mu^{*2}\frac{\alpha k^{*3}}{2MW_R}\,, \qquad \mathscr{M} = \mu^*\frac{e}{2M}\,.$$

Since M^3_{1+} is proportional to μ^*, this constant can be evaluated from any experimental result which shows a sufficiently sensitive dependence from the resonating amplitude. In ref. ([1,5]) one makes use of the total cross-section of the π^0 at resonance, since this quantity is estimated to be pratically free from any nonresonating background. The values found by these authors are

$$(5)\qquad \mu^* = (1.26 \pm 0.02)\frac{2\sqrt{2}}{5}\mu_v \text{ (DALITZ, SUTHERLAND) (*)}\,,$$

$$(6)\qquad \mu^* = (1.24 \pm 0.02)\frac{2\sqrt{2}}{5}\mu_v \text{ (AZNAURYAN, SOLOVIEV)}\,.$$

Moreover, from an analysis of the 90° cross-section (integrated over W) ASH *et al.* ([7]) have found

$$(7)\qquad \mu^* = (1.13 \pm 0.004)\frac{2\sqrt{2}}{5}\mu_v\,.$$

1. – It is interesting, in our opinion, to try to estimate the same parameter also from π^+ photoproduction, in order to have an independent test for the values so far obtained. A first approximate estimate of μ^*, starting from the π^+ data (by unpolarized γ-rays), has been given by ([2])

$$(8)\qquad \mu^* \sim 1.6\frac{2\sqrt{2}}{5}\mu_v\,.$$

However, from charged-pion photoproduction by unpolarized γ-rays it is difficult to extract information on the resonating amplitudes because of the presence of a large nonresonating background. An improved value of μ^* can now be obtained by using recent measurements of π^+ photoproduction by linearly polarized γ-rays ([3]). In fact if we describe the photon with two states of polarization, parallel or ortogonal to the « production plane », defined by $\boldsymbol{k}$ (photon momentum) and $\boldsymbol{q}$ (pion momentum), the differential cross-section $\sigma(\theta)$ results from the incoherent sum of two terms corresponding to the two states of polarization

$$(9)\qquad \sigma(\theta) = \tfrac{1}{2}\big(\sigma_\perp(\theta) + \sigma_\parallel(\theta)\big)\,.$$

([5]) I. G. AZNAURYAN and L. D. SOLOVIEV: *Dispersion sum rules and SU_6 symmetry*, II (Joint Inst. for Nucl. Research, preprint E-2544, 14 January 1966).

(*) According to DONNACHIE and SHAW ([6]), if one uses for the nonresonating background the value given by these authors, the result of ([1]) is modified to $\mu^* = (1.24 \pm 0.02)(2\sqrt{2}/5)\mu_v$.

([6]) A. DONNACHIE and G. SHAW: *Low-energy photopion production, SU_6 and the Panofsky ratio* (Report CERN 66/749/5-TH. 673, 6 June 1966).

([7]) W. W. ASH, K. BERKELMAN, C. A. LICHTENSTEIN, A. RAMANANSKAS and R. H. SIEMANN: *Phys. Lett.*, **24** B, 165 (1967).

As the contribution of the resonating multipole M_{1+} is larger (by a factor 4) in $\sigma_\perp$ than in $\sigma_\parallel$, while the most important « background » multipoles contribute approximately in the same way to $\sigma_\perp$ and $\sigma_\parallel$, the $\sigma_\perp$ cross-section is more suitable than σ for our purpose. In particular a background term which does not contribute to $\sigma_\perp$ is the « photoelectric » part of the Born amplitude, whose interference with M_{1+} can be very important (8).

Using the formalism of CGLN (4), we have

$$\sigma_\perp^{(\theta)} = \frac{q}{k}\left[|\mathcal{F}_1|^2 + |\mathcal{F}_2|^2 - 2\,\mathrm{Re}\,\mathcal{F}_1^*\mathcal{F}_2\cos\theta\right] \tag{10}$$

with

$$\begin{cases} \mathcal{F}_1 = E_{0+} + 3(M_{1+} + E_{1+})\cos\theta + \text{higher multipoles}\,, \\ \mathcal{F}_2 = (2M_{1+} + M_{1-}) + \text{higher multipoles}\,, \end{cases} \tag{11}$$

so that at $\theta = 90°$ the relation (10) becomes very simple. In particular, $\sigma_\perp(90°)$ does not contains the electric, resonating, multipole E_{1+} and $\mathcal{F}_1(90°)$ contains only the electric dipole E_{0+}, apart from the small higher multipoles. In the following relations we put

$$\mathcal{F}_1(90°) = f_1\,, \qquad \mathcal{F}_2(90°) = f_2 + \mathcal{F}_{2R}\,, \qquad \mathcal{F}_{2R} = -\frac{2\sqrt{2}}{3}\,M_{1+}^3\,, \tag{12}$$

where f_1, f_2 are « background amplitudes » which, according to the Watson theorem and the $\pi\mathcal{N}$ scattering phase-shifts at our energies, can be considered real. In the following calculations we have evaluated f_1 and f_2 from the Born approximation.

We then get

$$\begin{cases} \sigma_\perp(90°) = \dfrac{q}{k}\left[\Sigma + |\mathcal{F}_{2R}|^2 + 2\,\mathrm{Re}\,(f_2\mathcal{F}_{2R})\right], \\[2ex] \sigma_\perp(90°) = \dfrac{q}{k}\left[\Sigma + |\mathcal{F}_{2R}|^2 + 2f_2|\mathcal{F}_{2R}|\cos\alpha_{33}\right], \end{cases} \tag{13}$$

where

$$\Sigma = |f_1|^2 + |f_2|^2\,. \tag{14}$$

It is very useful to study $\sigma_\perp(90°)$ for the following reasons:

a) There is a simple relation (13) between $\sigma_\perp(90°)$ and $\mathcal{F}_{2R}$.

b) This cross-sections is dominated by $\mathcal{F}_{2R}$.

c) As we shall see, the only amplitude (f_2) which interferes with the resonating term $\mathcal{F}_{2R}$ is small.

2. – Evaluating f_1 and f_2 from the Born approximation we find that $(q/k)\Sigma$ contributes to $\sigma_\perp$ for about $\frac{1}{3}$ of the measured cross-section at resonance.

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(8) A. M. WETHERELL: *Phys. Rev.*, **115**, 1722 (1959).

Moreover the main contribution to Σ ([*]), about 85%, comes from E_{0+} for which the estimate given by the Born term, according to many authors, can be considered quite good in the case of π^+ production ([**]).

Therefore, Σ is evaluated, to a sufficient accuracy ($\approx \pm 10\%$), by taking its Born-term extimate.

Since f_2 is real, the interference term in (13) vanishes at the resonance ($E_\gamma = 345$ MeV) and $\mathcal{F}_{2R}$ can be calculated from

$$\sigma_\perp(90°, 345\ \text{MeV}) - \frac{q}{k}\Sigma = \frac{q}{k}|\mathcal{F}_{2R}|^2 . \tag{15}$$

Taking $\Gamma = 119$ MeV ([1]), $(q/k)\Sigma = 8.7\ \mu\text{b/sr}$ and (see Fig. 1)

$$\sigma_\perp(90°, 345\ \text{MeV}) = (29.1 \pm 1.0)\ \mu\text{b/sr} ,$$

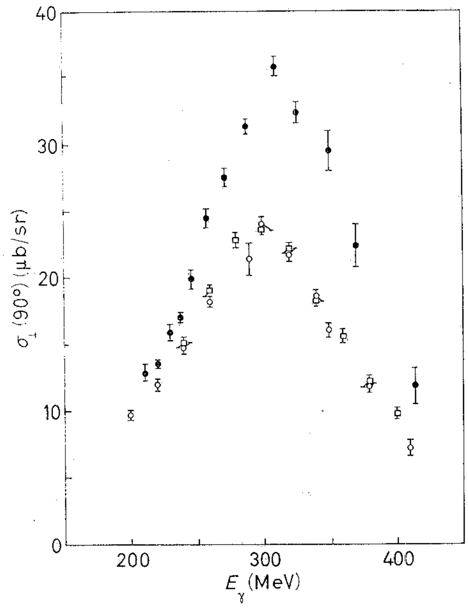

Fig. 1. – $\sigma_\perp(90°, E_\gamma)$ *vs.* E_γ. $\sigma_\perp(90°, E_\gamma)$ is obtained from the relation $\sigma_\perp(90°, E_\gamma) = \sigma(90°, E_\gamma)[1 + A(90°, E_\gamma)]$, combining the experimental data for the asymmetry ratio A (ref. ([3,12])) with the cross-section σ for single π^+ production by unpolarized γ-rays. For σ we have used the values resulting from a Moravcsik fit of the Born data ([13]) (ɸ). For comparison we report also the values (□) obtained for σ by ([14]) with a fit of all available data. The value $\sigma_\perp(90°, 345) = (29.1 \pm 1.0)\ \mu$b/sr (see text) is obtained by means of an interpolation of experimental points.

(*) The amplitude E_{0+} which enters in Σ is not the total multipole amplitude but only that coming from the amplitudes which contribute to $\sigma_\perp$. Precisely we have (see ref. ([9])) $f_1 = (E_{0+} - E^r_{0+})$, where E^r_{0+} (contribution to E_{0+} of the «retardation term») $= -f_r$ (sum in $\mathcal{F}_1$ of «higher multipoles», coming from the retardation terms). Since the corrections to these «higher multipoles» are estimated to be small, the incertitudes to f_1 come from E_{0+}.

(9) M. Nigro and E. Schiavuta: *Analisi delle sezioni d'urto di fotoproduzione di π^+ da γ polarizzati*, Internal Report of Padova University AE-67/1.

(**) See, for example, Fig. 2 in Schmidt ([10]). Precisely we have (in the case of π^+) $|E_{0+}(340\ \text{MeV})|^2 = 2.25 \cdot 10^{-4}(\hbar/\mu c)^2$ in the calculations of ([10]) (whose result for $E^{\pi^+}_{0+}$ coincides pratically with the Born approximation) against a value of $|E_{0+}(340\ \text{MeV})|^2 = 2.0 \cdot 10^{-4}(\hbar/\mu c)^2$ calculated by ([11]) (including Born term $+M^3_{1+}$ contribution$+s$-wave contribution). Moreover, as has been shown in ([6]), this last estimate of E_{0+} agrees with the experiments.

As is known, the fact that $(E^{\pi^+}_{0+}) \approx (E_{0+})_{\text{Born}}$ is a consequence of the fact that, in the static limit, the contribution of the resonant multipole M_{1+}, via the dispersion integral, is rigorously zero in the π^+ case ([4]).

(10) W. Schmidt: *Zeits. f. Phys.*, **182**, 76 (1964).

(11) A. Donnachie and G. Shaw: *Ann. of Phys.*, **37**, 333 (1966).

(12) P. Gorenstein, M. Grilli, F. Soso, P. Spillantini, M. Nigro, E. Schiavuta and V. Valente: *Phys. Lett.*, **23**, 294 (1966).

(13) D. Freytag, W. J. Schwille and R. J. Wedemeyer: *Zeits f. Phys.*, **186**, 1 (1965).

(14) J. T. Beale, S. D. Eckhund and R. L. Walker: *Pion photoproduction data below* 1.5 GeV. Report CTSL-42, CALT-68-108.

we obtain $\Gamma_\gamma = (0.62 \pm 0.03)$ MeV, and from the relation (4)

$$\mu^* = (1.23 \pm 0.03) \frac{2\sqrt{2}}{5} \mu_v . \tag{16}$$

The error due to the subtraction of $(q/k)\Sigma$, which is not included in (16), is contained within 5% even attributing to $(q/k)\Sigma$ an error of 30%.

It is interesting to note that our value for μ^* is compatible with that coming from π^0 photoproduction data, as given by [1,5].

3. – As we have said, our evaluation of μ^* suffers from the presence of a non-negligible background. However, at least on the f_2 part of the background useful information can be obtained looking at the energy dependence of $\sigma_\perp(90°)$. To discuss this point, we can proceed in the following way: with the calculated value of μ^* and taking for f_1 and f_2 the value f_{1B}, f_{2B} being given by the Born term, we calculate, from (13), the quantity $\sigma_\perp - (q/k)\Sigma$ at different energies E_γ. We compare this quantity with the experimental « subtracted » cross-section, obtaining the results reported in Fig. 2. Both the calculated and the experimental $(\sigma_\perp - (q/k)\Sigma)$ have a nice peak and the correct energy dependence of a p-wave near threshold. However, a big disagreement between the two quantities is present, mainly in the region of the experimental peak. In particular, the displacement of the peak certainly means that the interference term $2f_2|\mathcal{F}_{2R}| \cos \alpha_{33}$, which has the general trend shown in Fig. 3, must be much larger than its calculated value. Therefore, f_2 must be strongly increased, and the value of μ^* given by (16) correspondingly changed. In order to get an idea of the order of magnitude of the needed corrections on f_2 and μ^* we write

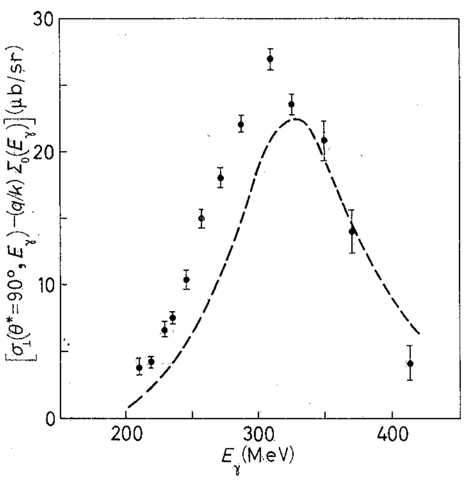

Fig. 2. – « Subtracted » cross-section (see text) $[\sigma_\perp(90°) - (q/k)\,\Sigma_0]$ *vs.* E_γ. ⯁ Experimental points obtained starting from the results of Fig. 1 for $\sigma_\perp(90°)$. — — — Prevision for this « subtracted » cross-section calculated according relation (13) (see text). In both cases for Σ_0 we have used the value obtained by the Born approximation.

$$f_2 = c_2(W) f_{2B} , \qquad \Sigma = |f_{1B}|^2 + |f_{2B}|^2 + (c_2^2(W) - 1) f_{2B}^2 = \Sigma_0 + (c_2^2(W) - 1) f_{2B}^2 \tag{17}$$

and, since $\mathcal{F}_{2R}$ is proportional to μ^*,

$$\mathcal{F}_{2R} = \Delta\mu \mathcal{F}_{2R}^0 , \tag{18}$$

where $\mathcal{F}_{2R}^0$ is calculated from (2) and (3) with the value of $\mu^* = 1.23(2\cdot\sqrt{2}/5)\mu_v$ precedently obtained. The quantities $c_2(W_R)$ and $\Delta\mu$ are the constants which we try to determine.

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From the relations (13), (17) and (18) we have

$$(19)\qquad \sigma_\perp - \frac{q}{k}\Sigma_0 = \frac{q}{k}\left[(c_2^2(W)-1)f_{2B}^2 + \Delta\mu^2|\mathcal{F}^0_{2R}|^2 + c_2(W)\,\Delta\mu\,2f_{2B}\,\mathrm{Re}\,\mathcal{F}^0_{2R}\right].$$

Equating to zero the derivative $(\mathrm{d}/\mathrm{d}W)\cdot(\sigma_\perp - (q/k)\Sigma_0)$ at the position of the experimental peak ($W = W_p$), and neglecting the derivative $(\mathrm{d}/\mathrm{d}W)c_2(W)$ in comparison with the very strong derivatives $(\mathrm{d}/\mathrm{d}W)|\mathcal{F}^0_{2R}|^2$ and $(\mathrm{d}/\mathrm{d}W)\cdot(2\,\mathrm{Re}\,f_{2B}\mathcal{F}^0_{2R})$, we get the approximate relation (*)

$$(20)\qquad \frac{c_2(W_p)}{\Delta\mu} = -\frac{[(\mathrm{d}/\mathrm{d}W)|\mathcal{F}^0_{2R}|^2]_{W=W_p}}{(\mathrm{d}/\mathrm{d}W)[2f_{2B}\,\mathrm{Re}\,\mathcal{F}^0_{2R}]_{W=W_p}} = 2.3\,.$$

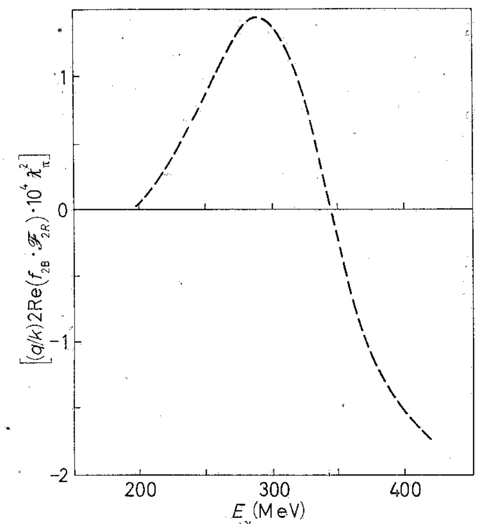

Fig. 3. – Interference term $(q/k)[2\,\mathrm{Re}\,(f_{2B}\mathcal{F}_{2R})]$ *vs.* E_γ. The amplitude $\mathcal{F}_{2R}$ is calculated according to the relations (12) and (3), λ is the Compton wavelength of the pion.

From this calculated value of $c_2/\Delta\mu$ and from the relation (19) we have $\Delta\mu = 0.95$, that means a change in μ^* of 5%.

Since in deriving the relation (20) it was only required to get the peak in the experimental position, it is interesting to ask what happens to the calculated values of $(\sigma_\perp - (q/k)\Sigma_0)$ if we use a modified value for f_2. Figure 4 shows the results of a calculation in which we have taken for f_2 the Born-term contribution multiplied by a constant factor $c_2(W) = \mathrm{const} = 2$ and $\mu^* = 1.19\,(2\sqrt{2}/5)\,\mu_v$. One sees that the agreement of the calculation with the experimental results is quite good, in spite of the approximations introduced. We therefore conclude from this analysis that the disagreement observed in Fig. 2 can be explained

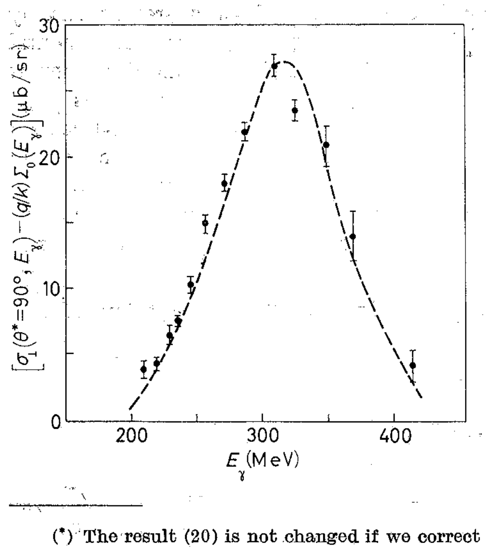

Fig. 4. – Comparison of the experimental « subtracted » cross-section (♦) (see caption of Fig. 2) with that calculated with the reported values for f_2/f_{2B} and for μ^* (dashed curve).

(*) The result (20) is not changed if we correct also f_1 for a slowly varying factor, since f_{1B} has a very weak energy dependence.

by introducing a correction of f_2 which changes the value of μ^* obtained from (16) of only 3%.

It is important to observe that the same effect could never be obtained by changing, in a resonable way, the amplitude f_1, as the disagreement shown in Fig. 2 is strongly energy-dependent.

To have a more definite conclusion about the value of μ^* we must wait for the completion of the analysis of all the data obtained, at different angles and energies, by means of polarized γ-rays ([3]). In fact, it seems possible to obtain from this analysis a more accurate evaluation of f_1 and f_2.

4. – Summarizing, the main conclusions of the present paper are:

1) From an analysis of the π^+ photoproduction data by linearly polarized γ-rays, in the region of P_{33} resonance, we have found

$$\mu^* = (1.23 \pm 0.03)\,\frac{2\sqrt{2}}{5}\,\mu_v\,.$$

This evaluation of μ^* agrees with the one obtained by some analysis of π^0 photoproduction data by unpolarized γ-rays (see relations (5) and (6)).

All these determination have to be compared with the SU_6 prediction $\mu^* = (2\sqrt{2}/5)\mu_v$.

2) An analysis of $\sigma_\perp(90°, E_\gamma)$ as a function of E_γ (Fig. 2) seems to require a big correction on the interference term $(2f_2\,\mathrm{Re}\,\mathscr{F}_{2R})$, if we evaluate f_2 by means of the Born approximation.

For this reason we have evaluated μ^* at resonance ($E_\gamma = 345$ MeV), where the term $2f_2\,\mathrm{Re}\,\mathscr{F}_{2R} = 0$.

However, we have tested that the required correction on f_2 does not change significantly our calculated value for μ^*.