

Laboratori Nazionali di Frascati

LNF-67/27

C. Mencuccini and A. Reale : REMARKS ON THE  $\eta$  -MESON  
CUSP EFFECT IN  $\pi^0$  PHOTOPRODUCTION.

Estratto da : Nuovo Cimento 48A, 579 (1967)

Remarks on the  $\eta$ -Meson Cusp Effect in  $\pi^0$  Photoproduction.

C. MENCUCINI and A. REALE

*Laboratori Nazionali di Frascati del CNEN - Frascati*

(ricevuto il 10 Febbraio 1967)

In a recent past the knowledge of the  $\pi\mathcal{N}$  system in the energy region of the second resonance ( $\mathcal{N}^{**}$ , 1518 MeV) has been considerably improved, essentially due to some accurate phase-shift analyses of the  $\pi\mathcal{N}$  scattering<sup>(1-5)</sup>.

It turns out that in this energy region strong inelastic effects mainly in  $P_{11}$ ,  $S_{11}$  and  $S_{31}$  states occur, superimposed to the resonant behaviour of the  $D_{13}$  state ( $\mathcal{N}^{**}$ , 1518 MeV). In particular the  $S_{11}$  partial wave seems to be affected by the sharp opening of the rather abundant  $\eta$  production channel according to a cusp mechanism<sup>(6)</sup>, as was pointed out by AUUIL *et al.*<sup>(2)</sup> and further investigated with different degree of success by several authors<sup>(7)</sup>.

A rather convincing evidence has been found that the  $\eta\mathcal{N}$  system mainly passes through a  $J = \frac{1}{2}^-$ ,  $T = \frac{1}{2}$  resonance at threshold. However, some very recent phenomenological analyses<sup>(8)</sup> in which recent experimental results<sup>(9,10)</sup> are taken into account show that a strong interaction in an  $S$ ,  $P$  or  $D$  state can undoubtedly not be ruled out.

---

(<sup>1</sup>) L. D. ROPER: *Phys. Rev. Lett.*, **12**, 340 (1964).

(<sup>2</sup>) P. AUUIL, C. LOVELACE, A. DONNACHIE and A. T. LEA: *Phys. Lett.*, **12**, 76 (1964).

(<sup>3</sup>) P. BAREYRE, C. BRICKMAN, A. V. STIRLING and G. VILLET: *Phys. Lett.*, **18**, 342 (1965).

(<sup>4</sup>) J. CENCE: *Phys. Lett.*, **20**, 306 (1966).

(<sup>5</sup>) L. D. ROPER, R. M. WRIGHT and B. T. FELD: *Phys. Rev.*, **138**, 190 (1965).

(<sup>6</sup>) J. S. BALL and W. R. FRAZER: *Phys. Rev. Lett.*, **7**, 204 (1961).

(<sup>7</sup>) F. UCHIYAMA-CAMPBELL: *Phys. Lett.*, **18**, 189 (1965); A. W. HENDRY and R. G. MOORHOUSE: *Phys. Lett.*, **18**, 171 (1965); J. S. BALL: *The  $\eta$ -nucleon interaction and production in  $\pi$ -nucleon scattering*, UCLA preprint (1966).

(<sup>8</sup>) S. MINAMI: *Phys. Rev.*, **147**, 1123 (1966); F. UCHIYAMA-CAMPBELL and R. K. LOGAN: preprint, University of Illinois.

(<sup>9</sup>) C. BACCI, G. PENSO, G. SALVINI, C. MENCUCINI and V. SILVESTRINI: *Phys. Rev. Lett.*, **16**, 157 (1966).

(<sup>10</sup>) W. B. RICHARDS, C. B. CHIU, R. D. EANDI, A. C. HELMOLZ, R. W. KENNEY, B. J. MOYER, J. POIRER, R. J. CENCE, V. Z. PETERSON, N. R. SEHGAL and V. J. STENGER: *Phys. Rev. Lett.*, **16**, 1221 (1966).

As far as the photoproduction reactions are concerned, due to the rather scanty experimental input available up to a quite recent past, a detailed multipole analysis comparable with the phase-shift analysis of  $\pi\mathcal{N}$  scattering has not yet been done for energies much above the first resonance<sup>(11)</sup>.

The situation around the second resonance is furthermore complicated by the presence of the known puzzle of the position in energy of the experimental maxima for the two reactions  $\gamma p \rightarrow \pi^0 p$  and  $\gamma p \rightarrow \pi^+ n$ . In regard to this particular aspect the possibility of a cusp effect of the  $\eta$  on the single-pion photoproduction was already suggested by REKALO<sup>(12)</sup>. According to his prediction a subtractive anomaly could be expected in the differential cross-section at  $90^\circ$  for the  $\gamma p \rightarrow \pi^0 p$  reaction at the threshold for the  $\gamma p \rightarrow \eta p$  reaction.

On the other hand, a comparative analysis<sup>(9)</sup> has shown that, not too far from the  $\eta$  threshold, the behaviour with the energy of the production of  $\eta$ 's and  $\pi$ 's in the  $T = \frac{1}{2}$  state and also the relative abundance of  $\pi$ 's with respect to  $\eta$ 's for both the initial states  $\pi + \mathcal{N}$  and  $\gamma + \mathcal{N}$  are practically the same. This seems to indicate that a cusp effect due to the  $\eta$  production is reasonably to be expected also in single photoproduction processes.

In this respect among the single-pion photoproduction reactions the  $\gamma + p \rightarrow \pi^0 + p$  seems to be rather suitable to show the effect, due to the absence of the direct-interaction term (present with the charged pions) and because it has an energy behaviour very similar to the energy behaviour of the  $\pi\mathcal{N} \rightarrow \pi\mathcal{N}$  reaction ( $T = \frac{1}{2}$ ).

We maintain here the assumption that the quantum numbers of the state through which the  $\eta$  production occurs, both in production and in photoproduction, are  $l = 0$ ,  $J = \frac{1}{2}$ ,  $T = \frac{1}{2}$ ; the multipole mainly involved in  $\pi^0$  photoproduction is then the electric dipole  $E_{01}^{\pi^0}$ .

It must be noted that looking for an anomaly in the  $\gamma p \rightarrow \pi^0 p$  reaction at the  $\eta$  threshold, because of the steepness of the production cross-section and the relative low values it reaches (factor five on the cross-sections with respect to the  $\gamma p \rightarrow \pi^0 p$ ), an effect is reasonably expected to be strongly energy-dependent and quite small in absolute value. Therefore for an experimental investigation high-energy resolution and good statistics are simultaneously required. Up to now these requirements are reasonably satisfied only for some measurements of differential cross-sections at  $90^\circ$  c.m.s.<sup>(13-15)</sup>.

Keeping this in mind we summarize in Fig. 1 the experimental situation as far as the differential cross-section for  $\gamma p \rightarrow \pi^0 p$  at  $90^\circ$  c.m.s. in the energy region of the second resonance is concerned.

In Fig. 1 the Frascati data (\*) together with the Stanford ones are shown because

<sup>(11)</sup> W. SCHMIDT: *Zeits. f. Phys.*, **182**, 76 (1964); W. SCHMIDT, G. SCHWIDERSKI and H. WUNDER: *Analysis of pion photoproduction in the region of the second pion-nucleon resonance*, in *Proceedings of the International Symposium on Electron and Photon Interactions at High Energies*, Hamburg, 1965. A. DONNACHIE and G. SHAW: *Photopion-production dispersion relations and  $\gamma\pi\pi$  coupling* (to be published in *Ann. of Phys.*).

<sup>(12)</sup> M. P. REKALO: *Sov. Phys. JETP*, **19**, 152 (1964).

<sup>(13)</sup> M. DEUTSCH, C. MENCUCINI, R. QUERZOLI, G. SALVINI, V. SILVESTRINI and R. STIENING: *Proceedings of the Frascati Congress*, 1962, p. 28.

<sup>(14)</sup> C. BACCI, C. MENCUCINI, G. PENSO, A. REALE, G. SALVINI, V. SILVESTRINI, M. SPINETTI and B. STELLA: *Laboratori Nazionali di Frascati Internal Report LNF-66/65* (1965) and to be published in *Physical Review*.

<sup>(15)</sup> H. DE STAEBLER, E. F. ERICKSON, A. C. HEARN and C. SCHAERF: *Phys. Rev.*, **140**, 342 (1965).

(\*) Thanks are due to the authors of ref. (14) who kindly supplied us with the  $90^\circ$  experimental results prior to publication.

these experiments have been performed with comparable resolution,  $\approx \pm 10$  MeV.

At a first examination of the experimental results a sort of structure could be recognized in the region of the rise of the second resonance, that is just at the threshold of the  $\eta$  photoproduction channel.

None of these measurements by itself gives a definite evidence of a clear structure, but all the results together are consistent with this indication.

A possible way through which the opening of the  $\eta$  channel could affect the  $\pi^0$  photoproduction, at threshold, is considered in the following.

In the energy region of the second resonance we consider the state  $l=0$ ,  $J=\frac{1}{2}$ ,  $T=\frac{1}{2}$ , for the following reactions:  $\pi\mathcal{N} \rightarrow \pi\mathcal{N}$ ,  $\pi\mathcal{N} \rightarrow \eta\mathcal{N}$ ,  $\gamma\mathcal{N} \rightarrow \pi\mathcal{N}$ ,  $\gamma\mathcal{N} \rightarrow \eta\mathcal{N}$ ,  $\eta\mathcal{N} \rightarrow \eta\mathcal{N}$  and neglect the  $2\pi$  production as the behaviour of the inelasticity parameter in the  $S_{11}$  partial wave seems to suggest<sup>(2-5)</sup>.

The use of unitarity and time-reversal invariance of the  $S$ -matrix leads to the following relationships:

$$\begin{aligned}\langle \pi\mathcal{N} | SS^+ | \pi\mathcal{N} \rangle &= 1, \\ \langle \eta\mathcal{N} | SS^+ | \pi\mathcal{N} \rangle &= 0, \\ \langle \pi\mathcal{N} | SS^+ | \gamma\mathcal{N} \rangle &= 0, \\ \langle \eta\mathcal{N} | SS^+ | \gamma\mathcal{N} \rangle &= 0, \\ \langle \eta\mathcal{N} | SS^+ | \eta\mathcal{N} \rangle &= 1.\end{aligned}$$

If we put the  $S$ -matrix elements in the form

$$\begin{aligned}\langle \pi | S | \pi \rangle &= Ae^{2i\alpha}, & \langle \pi | S | \eta \rangle &= Be^{i\beta}, & \langle \pi | S | \gamma \rangle &= Ce^{i\gamma}, \\ \langle \eta | S | \gamma \rangle &= De^{i\delta}, & \langle \eta | S | \eta \rangle &= Fe^{2i\varphi},\end{aligned}$$

then we get the following system of equations among different amplitudes and phases:

- (1)  $A^2 + B^2 + C^2 = 1$ ,
- (2)  $AB \exp[i(2\alpha - \beta)] + CD \exp[i(\gamma - \delta)] + BF \exp[i(\beta - 2\varphi)] = 0$ ,
- (3)  $AC \exp[i(2\alpha - \gamma)] + C \exp[i\gamma] + BD \exp[i(\beta - \delta)] = 0$ ,
- (4)  $BC \exp[i(\beta - \gamma)] + D \exp[i\delta] + FD \exp[i(2\varphi - \delta)] = 0$ ,
- (5)  $F^2 + D^2 + B^2 = 1$ .

By taking in to account the relative order of magnitude of the different terms (according to their « strong » or « e.m. » nature) one can easily show that this system

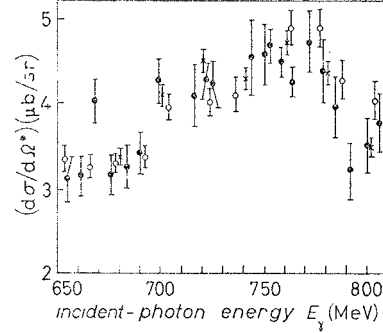


Fig. 1. - The differential cross-section for  $\pi^0$  photoproduction at  $\theta_{\pi^0}^* = 90^\circ$ , in the energy interval (650-810) MeV, as obtained in three experiments performed with good energy resolution:  $\bullet$  Frascati 1966<sup>(14)</sup>;  $\square$  Frascati 1962<sup>(15)</sup>;  $\times$  Stanford<sup>(16)</sup>.

<sup>(14)</sup> R. PREPOST, D. LUNDQUIST and D. QUINN: *Proceedings of the International Symposium on Electron and Photon Interactions at High Energies*, Hamburg, 1965.

is equivalent to the simple equations

$$(6) \quad \begin{cases} C^2 = \frac{(1-A^2)D^2}{1+A^2+2A\cos 2(\alpha-\gamma)}, \\ \frac{\operatorname{tg}(\beta-\delta-\alpha)}{\operatorname{tg}(\gamma-\alpha)} = \frac{1-A}{1+A}. \end{cases}$$

We are interested in the transition amplitude  $Ce^{i\gamma}$ . The inelasticity parameter  $A$  and the phase shift  $\alpha$  of the scattering are taken from ref. (14).

The  $D$  amplitude can be determined from the knowledge of the  $\gamma+p \rightarrow \eta+p$  photoproduction cross-section which has been experimentally measured (16), according to (17)

$$\sigma_{\text{tot}}(\gamma p \rightarrow \eta p) = \frac{\pi \lambda^2 D^2}{2},$$

where  $\lambda$  is the wavelength of the incident photon.

We now assume that both the reactions of photo and pion production of  $\eta$ 's on nucleons are going through a resonant state of a Breit-Wigner type, that is  $\pi N^0 \rightarrow \Delta^* \rightarrow \eta N^0$ ,  $\gamma N^0 \rightarrow \Delta^* \rightarrow \eta N^0$ ; then we can reasonably believe that the energy behaviour is the same for the  $\beta$  and  $\delta$  phases, because it is mainly determined from the energy and the width of the resonant state. We take, according to ref. (7) a value of  $E_\gamma \approx 750$  MeV for the resonance position in photoproduction.

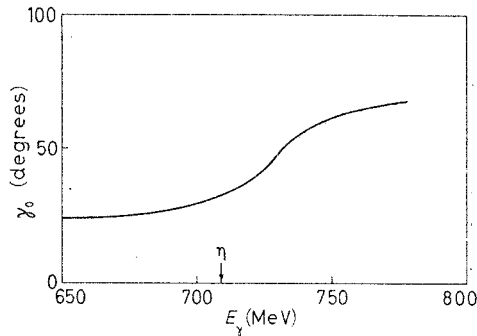


Fig. 2. — The phase of the  $S$ -matrix element for  $\gamma p \rightarrow \pi p$  in the  $T = \frac{1}{2}$ ,  $J = \frac{1}{2}^-$  state vs. the energy of the incident  $\gamma$  (l. s.).

We can then solve eq. (6) for  $C$  and  $\gamma$  by putting  $\beta \approx \delta$  in a small region around 750 MeV.

We can also take profit of the fact that below the  $\eta$  threshold the scattering phase  $\alpha$  and the transition phase  $\gamma_0 = \gamma - \pi/2$  (the  $\pi/2$  term is a consequence of our definition of the

transition elements) are equal (18) so that  $\gamma_0$  is completely determined everywhere.

A graph of  $\gamma_0$  is given in Fig. 2.

Concerning the  $C$  amplitude, let us recall that the total  $E_{0+}^{\pi^0}$  photoproduction amplitude is equal to

$$E_{0+}^{\pi^0} = \frac{2}{3} E_{0+}^{\frac{3}{2}} + \frac{1}{3} (E_{0+}^{\frac{1}{2}} + 3E_{0+}^0);$$

$E_{0+}^{\frac{3}{2}}$ ,  $E_{0+}^{\frac{1}{2}}$ ,  $E_{0+}^0$  are the  $\frac{3}{2}$ ,  $\frac{1}{2}$  isovector parts and the isoscalar part respectively (\*).

(17) C. PELLEGRINI and L. TAU: *Nuovo Cimento*, **16**, 973 (1960).

(18) K. M. WATSON: *Phys. Rev.*, **95**, 228 (1954).

(\*) Because of the Watson theorem, below the  $\eta$  threshold

$$\begin{aligned} \operatorname{Im} E_{0+}^{\frac{3}{2}} &= \operatorname{Re} E_{0+}^{\frac{3}{2}} \cdot \operatorname{tg} \alpha_{31}, \\ \operatorname{Im} (E_{0+}^{\frac{1}{2}} + 3E_{0+}^0) &= \operatorname{Re} (E_{0+}^{\frac{1}{2}} + 3E_{0+}^0) \cdot \operatorname{tg} \alpha_{11}, \end{aligned}$$

where  $\alpha_{31}$  and  $\alpha_{11} (\equiv \alpha)$  are the phase shifts for the  $\frac{3}{2}$  and  $\frac{1}{2}$  isotopic-spin states in  $\pi N^0$  scattering.

For the energies below the  $\eta$  threshold, we take for  $E_{0+}^{\pi^0}$  the values which can be extrapolated from ref. (11), while in the  $\eta\mathcal{N}^0$  resonance region we have

$$E_{0+}^{\pi^0} = \frac{2}{3} E_{0+}^{\frac{3}{2}} + \frac{1}{3} C e^{i\gamma_0},$$

where only the  $E_{0+}^{\frac{3}{2}}$  part (which is not expected to have a fast changing behaviour in our energy region) has been extrapolated from ref. (11).

The real and imaginary parts of  $E_{0+}^{\pi^0}$  are given in Fig. 3A, B respectively.

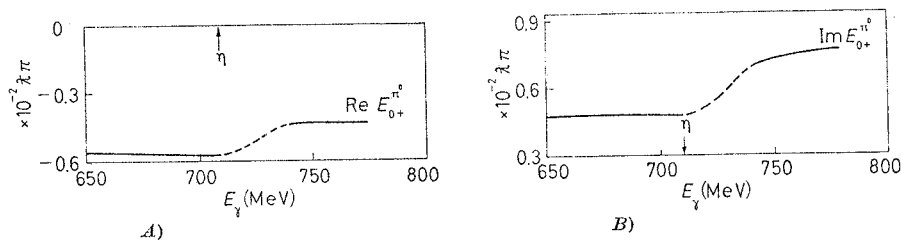


Fig. 3. - The isospin  $\frac{1}{2}$   $E_{0+}$  multipole for  $\gamma p \rightarrow \pi p$  vs. the energy of the incident  $\gamma$  (l. s.).

To get the effect on the cross-section one should know the behaviour of the multipoles which are effective in the region of the second resonance.

In fact, if for instance the assumption is made that only the multipoles  $E_{0+}$ ,  $M_{1+}$ ,  $E_{2-}$ ,  $M_{2-}$  are important, the  $90^\circ$  differential cross-section can be written as

$$(7) \quad \frac{K}{q} \frac{d\sigma}{d\Omega^*} = |E_{0+}|^2 + \frac{5}{2} |M_{1+}|^2 + \frac{5}{2} |E_{2-}|^2 + \frac{9}{2} |M_{2-}|^2 - \text{Re} [E_{0+}^*(E_{2-} - 3M_{2-})] + \\ + 3 \text{Re} M_{2-}^* E_{2-},$$

where  $K$ ,  $q$  are the c.m. momenta of the photon and the meson respectively.

Unfortunately the knowledge of the multipoles is as yet not complete in this region and any attempt to make a quantitative estimate of the  $\eta$  anomaly on the cross-section at threshold, according to (7), is somewhat dependent on the model used for the multipoles involved.

We report elsewhere (14), in a more extensive form, the details of the calculations performed to fit by (7) the experimental results of Fig. 1. Additional information on the other multipoles necessary to build up the cross-section have been taken from ref. (11). We just note here that the calculated anomaly is much smaller than one can argue from the experimental distribution: the perturbation on the  $E_{0+}^{\pi^0}$  multipole due to the sharp opening of the  $\eta$  channel is not great enough to account for the shoulder observed at  $E_{\gamma} = (700 \div 740)$  MeV.

On the other hand the remark has to be made that our model for the  $\eta\mathcal{N}^0$  system could be an oversimplified one. In fact the possibility that the  $\eta\mathcal{N}^0$  resonance could be a  $P_{11}$  state or even a mixture of states cannot be ruled out by the present experimental evidence.

\*\*\*

We wish to thank Prof. G. SALVINI for frequent and valuable discussions on the problem and Profs. M. BASSETTI and C. PELLEGRINI for their helpful remarks. Prof. W. SCHMIDT is also gratefully acknowledged for his criticism. Our thanks are also due to the computer staff and mainly to Dr. G. TRENTA.