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V. Silvestrini, I. Maiani and G. Preparata : A POLE MODEL FOR
$\eta \rightarrow 3\pi$ DECAYS.

A Pole Model for $\eta \to 3\pi$ Decays.

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The experimental situation on the decay rates of the η-particle cannot yet be considered satisfactory, the agreement among the results of different experiments being often only fair.

However, two facts have been cleared up during the last year:

a) The mode $\eta \to \pi^0\gamma\gamma$ is present and important. This decay, first put in evidence by Di Giugno et al. [1], has been later on observed also by Grunhaus [2] and by Strugalsky et al. [3].

b) The decay mode $\eta \to 3\pi^0$ is not very abundant: according to the Berkeley tables [4], it accounts for $(30.5 \pm 2.8)\%$ of the neutral decay modes [5].

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[1] G. Di Giugno, R. Querzoli, G. Troise, F. Vanoli, M. Giorgi, P. Schiavon and V. Silvestrini: *Phys. Rev. Lett.*, **16**, 767 (1966).

[2] J. Grunhaus: *Thesis* (Columbia University, 1966). He gives the following results:

$$\frac{\eta \to \gamma\gamma}{\eta \to \text{neutrals}} = 0.47 \pm 0.06 , \qquad \frac{\eta \to 3\pi^0}{\eta \to \text{neutrals}} = 0.34 \pm 0.04 , \qquad \frac{\eta \to \pi^0\gamma\gamma}{\eta \to \text{neutrals}} = 0.19 \pm 0.08 .$$

[3] Strugalski et al. have presented at the Berkeley Conference (Sept. 1966) the result

$$\frac{\eta \to \pi^0\gamma\gamma}{\eta \to \gamma\gamma} = 0.86 \pm 0.40 .$$

[4] A. H. Rosenfeld, A. Barbaro Galtieri, J. Kirz, W. J. Podolsky, M. Roos, W. J. Willis and C. G. Wohl: *Tables of elementary particle data*, updated August 1966.

[5] It is very hard to make a sensible average of results which are in poor agreement among themselves: not everybody will thus agree with this figure, and the error is in any case to be taken

Both these facts have been considered as difficulties from the point of view of their interpretation in terms of theoretical models. Only very recently, MAIANI and PREPARATA ([8]) have proposed a model which accounts for a large decay rate $\eta \to \pi^0\gamma\gamma$.

As far as point b) is concerned, the difficulty comes about when one considers the ratio $R = (\eta \to 3\pi^0)/(\eta \to \pi^+\pi^-\pi^0)$. Using $(\eta \to 3\pi^0)/(\eta \to \text{neutrals}) = 0.305 \pm 0.028$ and $(\eta \to \text{neutrals})/(\eta \to \pi^+\pi^-\pi^0) = 3.45 \pm 0.30$ ([9]), one gets $R_{\text{exp}} = 1.05 \pm 0.13$.

Several attempts have been made to fit in a model the value of R. We recall that in the absence of final-state interaction R is expected to be ~ 1.7, and that the introduction of a final-state interaction must fit at the same time the ratio R and the distribution on the Dalitz plot for the decay $\eta \to \pi^+\pi^-\pi^0$.

The models used up to now follow two main directions. The first one ([10]) is to assign the π-π interaction to the contribution of a scalar, $T = 0$, di-pion resonance (σ-particle); it is easy to show that R is a function of the width Γ_σ of the σ, R going to 0.5 as Γ_σ goes to zero. The second one, proposed by ONEDA et al. ([11]), is to interpret the final-state interaction as mediated by the ρ.

Both these models were able to predict the value of R and the distribution on the Dalitz plot up to a few years ago, when they were known with large errors. However, they fail to fit a value of R as low as ~ 1, and at the same time the population of the Dalitz plot as it is known to-day.

To explain the situation, the introduction of a $\Delta I = 3$ part in the (electromagnetic) decay $\eta \to 3\pi$ has been considered by VELTMAN et al. ([12]).

We present in the following a very simple model, which accounts for the low

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as an indication. It is worth while in our opinion to recall an experimental fact which is not generally quoted and goes in favour of both points a) and b). Not far above threshold for η production, the cross-section for reaction $\pi^- + p \to 6\gamma + n$ (which is dominated by $\pi^- + p \to \eta + n$) is a factor $1.5 \div 1.8$
$\llcorner\to 3\pi^0$
smaller than the cross-section for reaction $\pi^- + p \to \eta + n$ ([6]): therefore $(\eta \to \gamma\gamma)/(\eta \to 3\pi^0) \geqslant 1.5$.
$\llcorner\to 2\gamma$

In addition $(\eta \to \gamma\gamma)/(\eta \to \text{neutrals}) \simeq 0.40 \div 0.45$ ([1,2,7]).

([6]) L. GUERRIERO: Proc. Roy. Soc., 289, 471 (1966).

([7]) C. BACCI, G. PENSO, G. SALVINI, A. WATTENBERG, C. MENCUCCINI, R. QUERZOLI and V. SILVESTRINI: Phys. Rev. Lett., 11, 37 (1963); W. H. JONES, D. M. BINNIE, A. DUANE, J. P. HORSEY, D. C. MASON, J. A. NEWTH, I. U. RAHMAN, J. WALTERS, N. HORWITH and P. PALIT: Phys. Lett., 23, 597 (1966).

([8]) L. MAIANI and G. PREPARATA: Nuovo Cimento, 48 A, 550 (1967).

([9]) To get this average value, we have used the results of: R. KRAEMER, L. MADANSKY, M. MEER, M. NUSSBAUM, A. PEVSNER, C. RICHARDSON, R. STRAND, R. ZDANIS, T. FIELDS and S. DRENSTEIN: Phys. Rev., 136, B 496 (1964); E. PAULI and A. MULLER: Phys. Lett., 13, 351 (1964) on the ratio $(\eta \to \text{neutrals})/(\eta \to \pi^+\pi^-\pi^0)$; E. PICKUP, D. K. ROBINSON and E. O. SALANT: Phys. Rev. Lett., 8, 329 (1962); P. L. BASTIEN, J. P. BERGE, O. I. DAHL, M. FERRO-LUZZI, D. H. MILLER, J. J. MURRAY, A. H. ROSENFELD and M. B. WATSON: Phys. Rev. Lett., 8, 114 (1962); C. ALFF, D. BERLEY, D. COLLEY, N. GELFAND, U. NAUENBERG, D. MILLER, J. SCHULTZ, J. STEINBERGER, T. H. TAN, H. BRUGGER, P. KRAMER and R. PLANO: Phys. Rev. Lett., 9, 325 (1962); J. BUTTON-SHAFER, M. FERRO-LUZZI, J. MURRAY, M. L. STEVENSON and F. T. SOLMITZ: Proc. of the Intern. Conf. on High-Energy Physics at CERN, 1962, p. 307; B. BUSCHBECK-CZAPP, I. WACEK, W. A. COOPER, A. FRIDMAN, E. MALAMUD, G. OTTER, E. GELSEMA and A. TENNER: Proc. of the Sienna Intern. Conf. on Elementary Particles (Bologna, 1964), vol. 1, p. 166; F. E. JAMES and H. L. KRAYBILL: Phys. Rev., 142, 896 (1966); C. ALFF-STEINBERGER, D. BERLEY, D. COLLEY, N. GELFAND, D. MILLER, U. NAUENBERG, J. SCHULTZ, T. H. TAN, H. BRUGGER, P. KRAMER and R. PLANO: Phys. Rev., 145, 1072 (1966), on the ratio $(\eta \to \text{neutrals})/(\eta \to \text{charged})$; and the accepted value of 0.20 for the ratio $(\eta \to \pi^+\pi^-\gamma)/(\eta \to \pi^+\pi^-\pi^0)$ (see ref. ([4])).

([10]) See, for instance, L. M. BROWN and P. SINGER: Phys. Rev., 133, B 812 (1964).

([11]) S. ONEDA, Y. S. KIM and L. M. KAPLAN: Nuovo Cimento, 34, 655 (1964).

([12]) M. VELTMAN and J. YELLIN: Some comments on the decay of the $\eta(550)$, to be published in Phys. Rev.

value of R and for the observed shape of the distribution in the Dalitz plot, without the need of introducing any sophisticated type of interaction.

We consider the decay $\eta \to 3\pi$ as due to the contribution of the following pole terms:
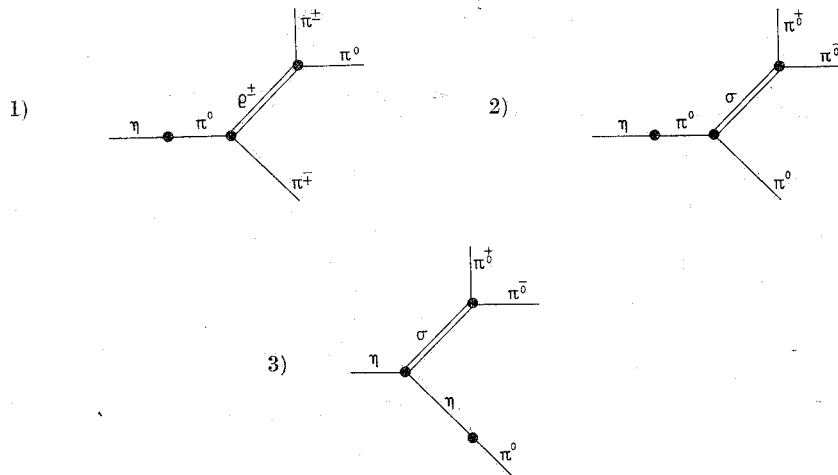
The vertex η-π accounts for the violation of isospin in the decay, and the final-state interaction goes through both the σ^0 and $\rho^{\mp}$ particles. Since we assume C-conservation in the decay, the diagram with the ρ^0 is forbidden.

Diagram 1) gives contribution only to the decay $\eta \to \pi^+\pi^-\pi^0$, while 2) and 3) contribute to both decays $\eta \to 3\pi^0$ and $\eta \to \pi^+\pi^-\pi^0$.

Were the σ an SU_3-singlet, the contributions of diagrams 2) and 3) would cancel each other ($g_{\sigma\pi\pi}$ being equal to $g_{\sigma\eta\eta}$ and the sign of the propagator being opposite). Only the projection of the σ on the SU_3 octet is thus effective in the $\eta \to 3\pi$ decays.

We have no reliable information about the particle which mixes with the σ in the SU_3 scheme: but since its mass is probably much higher (there is some indication of a scalar particle S_0 or ε_0 with a mass of ~ 720 MeV ([4])), we disgregard at this stage its contribution to the $\eta \to 3\pi$ decays.

For a calculation of R we do not need the vertex η-π: the only free parameters are thus the projection $\sin \varphi$ of the σ on the SU_3 octet and the ratio g_1/g_8 between the coupling constants of the $\pi\pi$ system to the SU_3 singlet and octet respectively. If we further assume a nonet model, so that $g_1/g_8 = \sqrt{2}$, we are left with the only free parameter $\sin \varphi$. Small adjustements of the σ mass and width are also possible within the experimental errors ([4]).

According to the diagrams 1), 2) and 3), the amplitude for the decay $\eta \to \pi^+\pi^-\pi^0$ is

$$(1) \quad A_{\mathrm{ch}} \propto \frac{2g_{\sigma\pi\pi}^2/(1 + \sqrt{2}\,\mathrm{ctg}\,\varphi)}{(s_0 - m_\sigma^2) + i\Gamma_\sigma m_\sigma} + g_{\rho\pi\pi}^2 \left\{ \frac{s_+ - s_0}{(s_- - m_\rho^2) + i\Gamma_\rho m_\rho} + \frac{s_- - s_0}{(s_+ - m_\rho^2) + i\Gamma_\nu m_\rho} \right\},$$

where $s_0 = (q_- + q_+)^2$, $s_+ = (q_- + q_0)^2$, $s_- = (q_+ + q_0)^2$ and $q_{\pm}$ is the four-momentum of the $\pi^{\pm}_0$.

For the decay $\eta \to 3\pi^0$, the amplitude is

$$(2) \qquad A_{\text{neutr}} \propto \frac{1}{\sqrt{6}} \sum_j \frac{2 g^2_{\sigma\pi\pi}/(1 + \sqrt{2}\,\text{ctg}\,\varphi)}{(s^j_0 - m^2_\sigma) + i\Gamma_\sigma m_\sigma} \; .$$

The sum over the index j is required to make to amplitude symmetric among the three π^0's.

A rather good fit to the experimental distribution in the Dalitz plot is obtained if we put

$$m_\rho = 758 \text{ MeV}\,, \qquad \Gamma_\rho = 128 \text{ MeV}\,, \qquad g^2_{\rho\pi\pi} = 2.6 \cdot 4\pi\,;$$

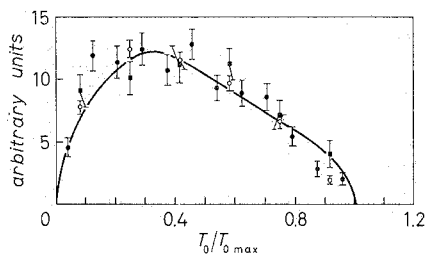$$m_\sigma = 375 \text{ MeV}\,, \qquad \Gamma_\sigma = 70 \text{ MeV}\,, \qquad g^2_{\sigma\pi\pi} = 1.31 \text{ (GeV)}^2$$

Fig. 1. – Decay $\eta \to \pi^+\pi^-\pi^0$. Distribution of the events in the Dalitz plot as a function of the kinetic energy T_0 of the π^0 (in units of $T_{0\text{max}}$). The full line is the distribution expected in our model; ■ ALFF et al. [14]; ○ BALTAY et al. [15]; ● LARRIBE et al. [16].

(as deduced from the used values of m_σ and Γ_σ) and $\varphi \simeq -15°$ $(1 + \sqrt{2}\,\text{ctg}\,\varphi = -4.3)$.

It is worth noticing the agreement between the σ mass and width we need to fit the Dalitz plot, and the ones calculated by FURLAN and ROSSETTI [13].

The result of our calculation is compared with the experimental points in Fig. 1: the full line represents $\int |A_{\text{ch}}|^2 dT$ (where $T = (T_+ - T_-)/\sqrt{3}$) and the experimental points are a summary of the information available to us [14-16]. Our model does not give any appreciable $|T|$ dependence of the population, consistently with experiment.

By integration of $|A_{\text{ch}}|^2$ and $|A_{\text{neutr}}|^2$ over the Dalitz plot, we get the branching ratio

$$R_{\text{th}} = \frac{\eta \to 3\pi^0}{\eta \to \pi^+\pi^-\pi^0} = 1.0$$

in good agreement with the experimental value $R_{\text{exp}} = 1.05 \pm 0.13$. R_{th} includes a correction factor 1.13 to take into account the difference in mass between the charged and neutral pions.

The diagram with the ρ works to depress R in two different ways: it gives an over-all contribution of 20% to the decay rate $\eta \to \pi^+\pi^-\pi^0$, and, more important, it allows to fit the Dalitz plot with a σ as narrow as ~ 70 MeV.

[13] G. FURLAN and C. ROSSETTI: Phys. Lett., 23, 499 (1966).

[14] C. ALFF, D. BERLEY, D. COLLEY, N. GEL'FAND, U. NAUENBERG, D. MILLER, J. SCHULTZ, J. STEINBERGER, T. H. TAN, H. BRUGGER, P. KRAMER and R. PLANO: Phys. Rev. Lett., 9, 325 (1962).

[15] C. BALTAY, P. FRANZINI, J. KIM, L. KIRSCH, D. ZANELLO, J. LEE-FRANZINI, R. LOVELESS, J. McFADYEN and H. YARGER: Phys. Rev. Lett., 16, 1224 (1966). The population of the Dalitz plot was presented by P. FRANZINI at the 1966 Berkeley Conference.

[16] A. LARRIBE, A. LEVEQUE, A. MULLER, E. PAULI, D. REVEL, T. TALLINI, P. J. LITCHFIELD, L. K. RANGAH, A. M. SEGAR, J. R. SMITH, P. J. FISHER and E. PICKUP: Phys. Lett., 23, 600 (1966).

An estimate of the absolute rate for the decay $\eta \to 3\pi$ can also be obtained. To this aim, we need the strength of the η-π e.m. transition element.

By assuming SU_3 symmetry and an octet behaviour of the e.m. current, we get, to second order, the following relation ([17]):

$$\langle \eta | H_{\text{e.m.}} | \pi^0 \rangle = \frac{1}{\sqrt{3}} [(M^2_{K^+} - M^2_{K^0}) - m^2_{\pi^+} - m^2_{\pi^0})] \simeq - (52 \text{ MeV})^2 .$$

Using this value, we obtain

$$\Gamma_{\eta \to \pi^+\pi^-\pi^0} \simeq 140 \text{ eV} .$$

For comparison, we recall that $\Gamma_{\eta \to \gamma\gamma}$, as obtained by SU_3 arguments from the π^0 lifetime, is equal to $\Gamma_{\pi^0 \to \gamma\gamma} \cdot (M_\eta/m_\pi)^3 \cdot \frac{1}{3} \simeq (150 \pm 13) \text{ eV}$, using for the π^0 lifetime the value $\tau_{\pi^0} = (0.91 \pm 0.08) \cdot 10^{-16}$ s ([18]).

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Thanks are due to Prof. N. CABIBBO and Dr. G. DE FRANCESCHI for valuable discussions.

([17]) S. OKUBO and B. SAKITA: *Phys. Rev. Lett.*, **11**, 50 (1963).

([18]) P. STAMER, S. TAYLOR, E. L. KOLLER, T. HEUTTER, J. GRAUMAN and D. PANDOULAS: *Phys. Rev.*, **151**, 1108 (1966); average of Table VI.