Laboratori Nazionali di Frascati

LNF-67/19

D. Fabiani and M. Puglisi : DETUNING EFFECT COMPENSATION IN THE CAVITY RESONATORS FOR PARTICLES ACCELERATING MACHINES.

Estratto da : Alta Frequenza 36, 137 (1967)

DETUNING EFFECT COMPENSATION IN THE CAVITY RESONATORS FOR PARTICLES ACCELERATING MACHINES

D. FABIANI - M. PUGLISI (*)

Thermal variations and mechanical vibrations can change slightly the geometrical dimensions, and consequently the tuning of a cavity resonator.

In this paper it is shown that, with the aid of a suitable negative feedback, it is possible to stabilize, against the effect of small tuning variations, the amplitude and the phase of the voltage developed across the gap of the resonator.

1. – Introduction.

The normal method that is used to give energy to the particles circulating in an accelerator, with the exception of the betatron, is that of arranging along the orbit of the particles one or more resonant cavities that, excited in a proper way, can deliver the wanted energy to the beam.

The maximum energy $\Delta\varepsilon$ that a particle owning a charge q can gain crossing the lenght l of an accelerating gap is given by:

$$\Delta\varepsilon = K\,q\,l\,E \tag{1}$$

where K is a suitable factor that takes into account the non uniformity of the field in the gap, while E is the maximum value of the field developed by the resonator.

In the orbital accelerating machines the value of $\Delta\varepsilon$ can be very large while, both for reasons depending on the dimensioning of the resonator, and to minimize the transit time effect, the value of « l » is maintained at all times very small with respect to the wave length of the radio frequency field.

The values of « l » being small and the values of $\Delta\varepsilon$ being very large, very high values of accelerating field can be requested and this fact give rise to very difficult design problem for the resonator.

Nevertheless, placing several resonators along the particles orbit, we can reduce, at will, the voltage in each gap and the design of the resonators became very simple.

Multyplying the number of resonators takes on the problem of phasing.

The difficulty presented here, common to all accelerators where more than one resonant cavity is employed, becomes particularly severe in the case of the storage rings (1), where the exact crossing point of the intersecting beams depends strongly on the phase of the field in the various accelerating gaps.

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(1) The storage rings are constant magnetic field machines into which are injected, and then maintained in orbit for a period of time up to ten hours, two beam of equal particles or two beam of particles having equal mass but opposite charge, to observe the reactions that arise from the colliding beams.

The plant of the electron and positron storage ring « Adone » [2], that is now in the completion stage at the National Laboratories of Frascati, is given in fig. 1.

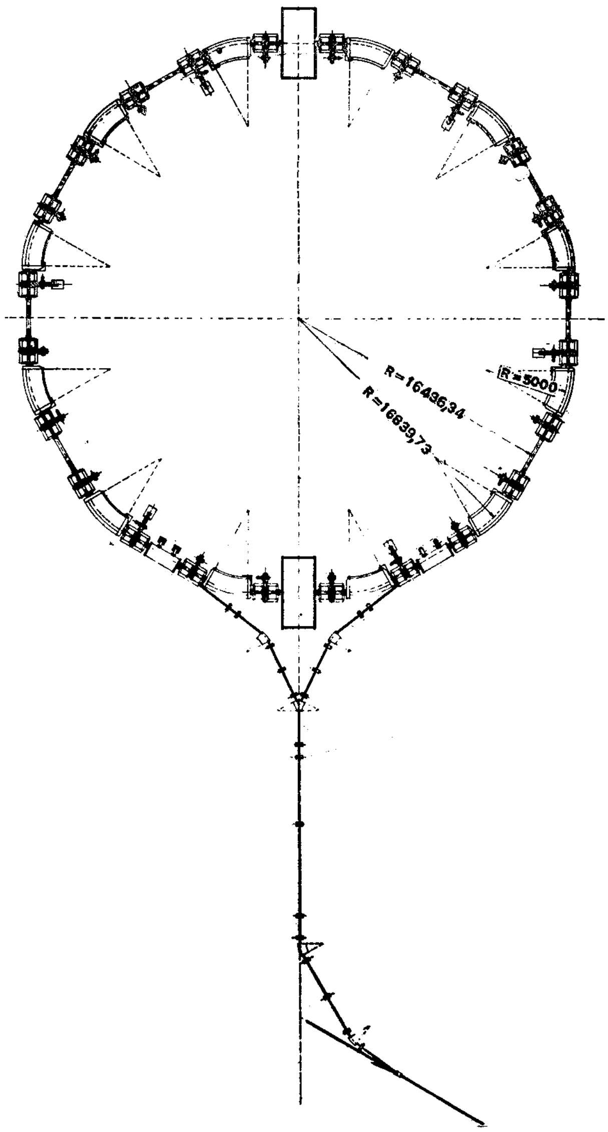

Fig. 1. — The plant diagram of the storage ring «Adone».

Fig. 2 indicates the geometrical sizes of one of the 4 resonators designed to mantain the particles in orbit.

In the following paragraphs we will look upon the difficulty connected with the necessity of maintaining constant both amplitude and the phase of the electrical field in each resonator and we will indicate what seems to us a simple solution of the problem.

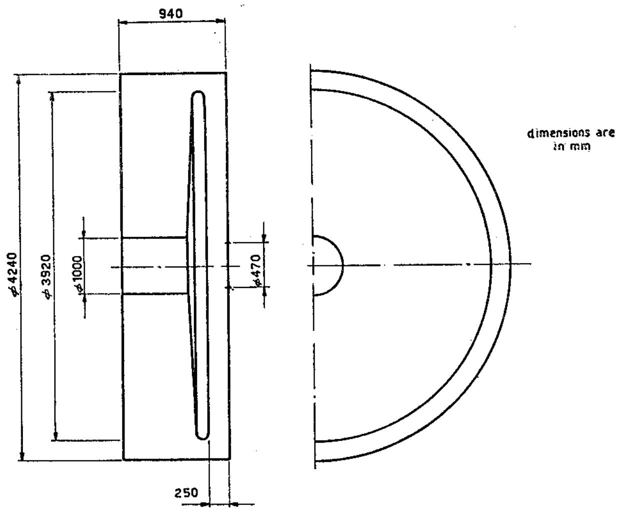

Fig. 2. — Geometrical sizes of one of the resonators of « Adone »

2. – Statement of the problem.

As previously mentioned, the already narrow tollerances on the amplitude and phase of the electric field in each accelerating gap, become more pressing, when precise and fixed crossing points for circulating beams are requested.

It is well known that a tuning variation of a resonator (fed at a fixed frequency) is associated with a variation in the amplitude and phase of the voltage developed by resonator itself.

There are many reasons for wich a detuning can occur namely:

1°) Load variations due to the fluctuations in the accelerated beam.

2°) Variations in the mechanical sizes of the resonator due to thermal variations and mechanical vibrations.

It is evident therefore that it is not sufficient to drive the « *n* » cavities (4 in the case of « Adone ») with prescribed phase signals from a single generator, but it is necessary to compensate automatically, in some way, the effects due to the above mentioned causes. A measure of the criticity degree for tuning an ideal resonator (having an infinite mechanical rigidity) is given approximately from the value of the quality factor of the resonator. (The resonators more commonly used has a quality factors ranging from 5000 to 10 000).

As a real resonator can not be rigid, the problem became more complicate and the tuning criticity for a resonator increases increasing the elasticity of the structure.

The tuning perturbations can be slow or fast.

Geometric deformations due to thermal variations are always very slow (from several seconds to several hours) and therefore can be controlled by servos that, modifying the position of a metallic element placed in the resonator, allow continuous tuning.

The main problem is the compensation for geometric perturbation due to mechanical vibrations and for detuning that are due to loading variations introduced by fluctuation in the accelerated beam. In fact for the fast perturbations it is not possible to follow solutions based on the use of mechanical equipments.

The more conceptually simple solution is based upon the employement of materials with electrical or magnetical variable permittivity, as barium titanate condensers or ferrite.

This solution is always expensive because the large value of the currents to be controlled that require a large amount of reactive material; moreover the heavy losses in the reactive material request more powerful plants.

Another solution, that has the same drawback, is to load the resonator with a reactance tube that, suitably driven, could compensate a change in tuning.

The fact that in the accelerators and especially in those of great power, the power that the amplifier can deliver [2] is always greater, (of a factor two or more) of that strictly demanded gave us the idea to study whether it was possible to employ the extra power of the amplifier to compensate for the fast tuning variation of the cavity.

Namely, as we have just a factor two on the real power that the power amplifier can deliver to the cavity in normal tuning condition, it is possible to compensate the effect of a detuning so large that, in absence of a control, would introduce a phase shift of $\pm 45°$ on the voltage developed by the resonator.

3. – Amplifier-cavity performance analysis.

We will indicate with ν_{0r} and with Q_0 respectively the resonant frequency and the quality factor of the resonator for the considered excitation mode [3], and let ν_0 the frequency value of the impressed field.

In the frequency range defined from the relation (2):

$$\Delta\nu = \pm \frac{\nu_{0r}}{Q_0} \cong \frac{\nu_0}{Q_0} . \tag{2}$$

The behaviour of the resonator can be approached with the behaviour of a lumped resonating circuit.

Between the two equivalent approximations (resonant circuit with series losses and resonant circuit with parallel losses) it seemed more convenient to take the parallel losses approximation because the main losses in the resonators employed in the large machines are due to the beam. While the series losses (due to the power wasted for the Joule effect on the wall of the resonator) are less important.

Fig. 3 shows the adopted scheme, with the stated hypothesis and simplification for the power amplifier and cavity.

With V_0 and ϱ, the e.m.f. and the output impedence of the generator (power amplifier), are indicated.

L_1 indicates the inductance that is used for coupling

(2) This is for consider the transient conditions that might require very large instantaneous power from the radio frequency plants. Abrupt switching-off of one resonator gives a more common example of the situation; in this situation in fact, the machine can remain working only exciting with a larger voltage the remaining resonators.

(3) The frequency of the field impressed to the resonator does not correspond, as we will see, to the own resonant frequency of the resonator (excited in the same mode). The difference between ν_0 and ν_{0r} ($\nu_{0r} = 1/\sqrt{L_0 C_0}$) is due to the coupling between cavity and amplifier. The difference of these two frequencies is very small and for the numerical evaluation of the quality factor ν_0 and ν_{0r} can be considered equals.

via the mutual inductance M, the power amplifier to the resonant cavity that is approximated with the parallel resonant circuit L_0; C_0; R.

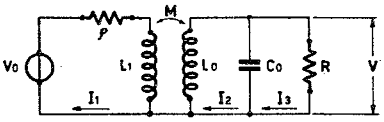

Fig. 3. — Equivalent scheme for the circuit « power amplifier-cavity ».

The resistance R takes into account both the resistive load, due to the beam circulating in the machine, and the self losses of resonator.

We will now make the hypothesis that the variations in the tuning frequency of the resonator are due only to variation on the capacity C_0, this while no generality is lost in the results, simplifies the mathematical analysis and the results themselves nicely follow the real development of the phenomena.

Writing the circuit equations in the steady state con-

we obtain for L_1 and M^2:

$$(3)\quad \begin{cases} L_1 = \dfrac{R_{eq}}{R}\left[L_0 - C_0 R^2 (1 - \omega_0^2 L_0 C_0)\right] \\[2ex] M^2 = \dfrac{R_{eq}}{R}\left[L_0^2 + \left(\dfrac{R}{\omega_0}\right)^2 (1 - \omega_0^2 L_0 C_0)^2\right]. \end{cases}$$

If these condictions are verified then the voltage accross the condenser is:

$$(4)\quad \begin{cases} V = V_0 \dfrac{\sqrt{R R_{eq}}}{\varrho + R_{eq}} e^{j\varphi} \\[2ex] \operatorname{tg} \varphi = \dfrac{R (1 - \omega_0^2 L_0 C_0)}{\omega_0 L}. \end{cases}$$

It is important for us to know wich variations undergoes modulus and the phase of the voltage V when the capacity value of the resonator change from the value C_0 to the value $C_0 + \Delta C$, where ΔC indicated the amplitude of the perturbation.

Calculating V in the more general case, we find:

$$(5)\quad \begin{cases} V = V_0 \dfrac{-\omega_0 M R}{\omega_0^3 (C_0 + \Delta C) R (L_1 L_0 - M^2) - \omega_0 (L_0 \varrho + L_1 R) + j \left\{R \varrho [1 - \omega_0^2 L_0 (C_0 + \Delta C)] - \omega_0^2 (L_1 L_0 - M^2)\right\}} e^{j\varphi} \\[2ex] \operatorname{tg} \psi = \dfrac{R \varrho [1 - \omega_0^2 L_0 (C_0 + \Delta C)] - \omega_0^3 (L_1 L_0 - M^2)}{\omega_0 (L_0 \varrho + L_1 R) - \omega_0^3 (C_0 + \Delta C) R (L_1 L_0 - M^2)}. \end{cases}$$

dition we find the characteristic matrix:

$$\Delta = \frac{1}{j \omega_0 C_0} \begin{vmatrix} \varrho + j \omega_0 L_1 & -j \omega_0 M & 0 \\ \omega_0^2 M C_0 & 1 - \omega_0^2 L_0 C_0 & -1 \\ 0 & -1 & 1 + j \omega_0 R C_0 \end{vmatrix}.$$

It is necessary remember that in the relations (5) the values of L_1, L_0 and M must be regarded as constants and that, in particular, the values of L_1 and M are defined by relations (3).

Following the various substitutions one finds that in the more general case the (5) can be rewritten as follows:

$$(6)\quad \begin{cases} V = V_0 \dfrac{\sqrt{R \cdot R_{eq}}}{\varrho + R_{eq}} \sqrt{\dfrac{1 + \operatorname{tg}^2 \varphi}{\left[1 + \dfrac{R_{eq}}{R_{eq} + \varrho} \omega_0 R C_0 \dfrac{\Delta C}{C_0} \operatorname{tg} \varphi\right]^2 + \left[2 \dfrac{R_{eq}}{R_{eq} + \varrho} \operatorname{tg} \varphi - \dfrac{\varrho}{R_{eq} + \varrho} \omega_0 R C_0 \dfrac{\Delta C}{C_0}\right]^2}}\, e^{j\varphi} \\[3ex] \operatorname{tg} \psi = \dfrac{2 \dfrac{R_{eq}}{R_{eq} + \varrho} \operatorname{tg} \varphi - \dfrac{\varrho}{\varrho + R_{eq}} \omega_0 R C_0 \dfrac{\Delta C}{C_0}}{1 + \dfrac{\varrho}{\varrho + R_{eq}} \cdot \dfrac{R_{eq}}{\varrho} \omega_0 R C_0 \dfrac{\Delta C}{C_0} \operatorname{tg} \varphi}. \end{cases}$$

First of all it is necessary to compute the parameters L_1 and M so that the amplifier can work on a purely resistive load, the values v_0, C_0, L_0 and R are known.

Let $\varrho + R_{eq}$ the total resistance that the generator must see when the resonator is in normal operating condition.

R_{eq} is then the resistive load value that the coupling circuit transfers to the generator and this load must be selected in order to optimize the behaviour of the amplifier tube.

Putting therefore the condition for the load:

$$I_1 = \frac{V_0}{\varrho + R_{eq}}$$

The matched condition occurs when L_1 and M are so selected that for $\Delta C = 0$ we must have $R_{eq} = \varrho$.

Putting the matched condition (that occur only when $\Delta C = 0$ the (6) are reduced to the (7) and we have:

$$(7)\quad \begin{cases} V = \dfrac{V_0}{2} \sqrt{\dfrac{R}{\varrho}} \cos \xi \, e^{j(\varphi - \xi)} \\[2ex] \operatorname{tg} \xi = \dfrac{\omega_0 R C_0}{2} \cdot \dfrac{\Delta C}{C_0}. \end{cases}$$

From equation (7) the following conclusions arise:

1°) The output voltage has a phase that is defined from the difference of two angles φ and ξ.

The angle φ depends only from the selected conditions for the operation of the unperturbed resonator and is therefore regarded as a constant, once defined the circuital parameters; vice versa the angle that depends both on the percentage detuning value, and on the quality factor value of the resonator (neglecting the difference between ω_0 and ω_{0r} it is possible to have $Q_0 = \omega_0 R C_0$), can change only if ΔC change and it vanishes for $\Delta C = 0$.

2°) The amplitude of the output voltage of the resonator is given, except for a multiplication constant, by the cosine of the angle ξ.

4. – THE CORRECTING CIRCUIT.

The very simple relation that connects the phase and the amplitude of the output voltage of the resonator suggests how can be calculated the electrical network that, inserted in a feed-back loop containing amplifier and cavity, can give the wanted correction.

Let's consider the block diagram shown in fig. 4.

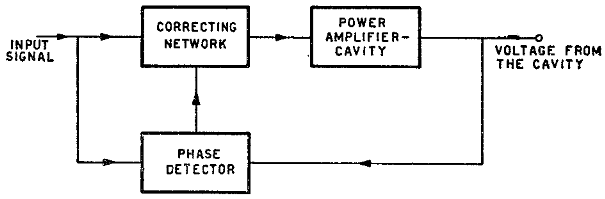

Fig. 4. — Block diagram of the R.F. plant with the correcting network.

The radio frequency signal that had to drive the amplifier linked to the cavity, enters into the correcting network and therefore suitably changed in amplitude and phase drives the power amplifier.

A phase shift detector reads the phase shift between the driving voltage and the voltage developed across the resonator and produces a proportional voltage.

This voltage, namely the error signal, controls phase and amplitude of the R.F. output voltage from the correcting network.

As the given voltage of the resonator varies as $e^{-j\xi} \cos \xi$, to obtain the wanted compensation it is necessary and sufficient that the signal from the correcting network varies as $e^{j\vartheta}/\cos \vartheta$, where the angle ϑ must be proportional

The transfer function for the network shown is:

$$\frac{V_u}{V_i} = \frac{(1 - \omega^2 L C) + j \omega r C}{(1 - \omega^2 L C) + j \omega (R + r) C} .$$

Multiplying numerator and denominator for the same factor

$$- j \frac{1}{\omega r C} \cdot \frac{r}{r + R}$$

and putting:

$$\operatorname{tg} \beta = \frac{1 - \omega^2 L C}{\omega r C} \qquad \operatorname{tg} \alpha = \frac{r}{r + R} \operatorname{tg} \beta$$

we obtain:

$$\frac{V_u}{V_i} = \frac{r}{r + R} \cdot \frac{\cos \alpha}{\cos \beta} e^{j(\alpha - \beta)} . \tag{8}$$

It results that the given transfer function the more exactly will follows the wanted function the smaller will be the value of α as regards to the value of β.

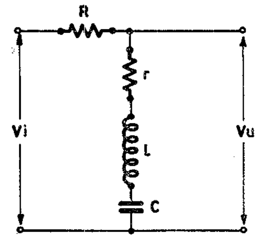

Fig. 5. — The correcting network.

It is important to remind that α became smaller if the ratio R/r increase and we will see below that a satisfactory compromise can be obtained making $R/r \gg 100$.

The correcting network will work in the prescribed manner if we can control the value of β by the signal coming from the phase detector and this can be achieved, very simply, substituting the condenser « C » with a suitable varactor.

To evaluate the effect due to the proposed circuit, we assume that the angle α has negligible value as regards to angle β.

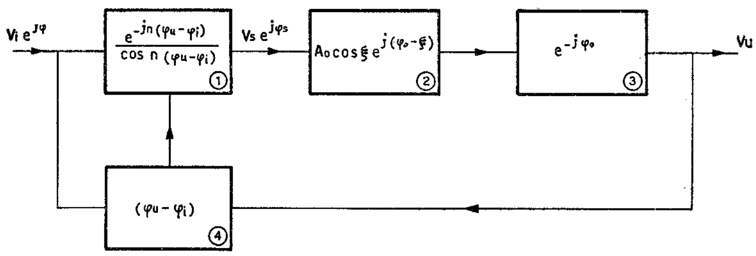

Fig. 6. — Functional block diagram of the feedback loop.

to the phase difference between the driving voltage and the voltage developed across the resonator.

In fig. 5 is drawn a very simple network capable to approach in a satisfactory way the wanted transfer function.

As we have already stated a suitable circuit design allows this approximation.

Consequently we take into consideration the block diagram in fig. 6.

Block 1 contains the correcting network and « n »

indicates the value of the angular amplification. Block 2 contains the power amplifier coupled with the cavity. A_0 indicates the value of the imperturbed voltage gain between the voltage at the gap of the resonator and the voltage that drives the amplifier, while angle ξ already defined, takes into account the effects of the perturbations that originates in the resonator.

Block 3 contains a delay-line that introduces a phase shift of 360 - φ_0 to compensate the rotation of phase (fixed) introduced from the coupling between the power amplifier and the cavity.

It is evident that, provided a suitable angular amplification, we can minimize at will the deviation from the prescribed values of the amplitude and phase of the output voltage.

5. – Experimental results.

Concluding what has been said in the previous paragraphs, we list here the following details of the prototype of the control device built for the radio frequency plants of « Adone ».

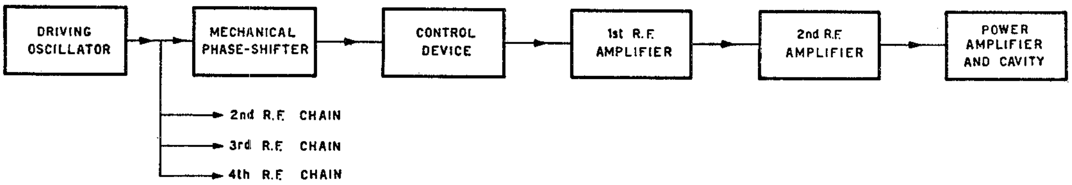

Fig. 7. — Block diagram of the R.F. plant of Adone.

Block 4 contains the phase detector.

We will now indicate with $V_s e^{j\varphi_s}$ the input voltage of the amplifier.

Considering the mathematical expressions of the transfer functions for the delay-line and for the group « amplifier and cavity », we obtain:

$$V_u e^{j\varphi_u} = V_s e^{j\varphi_s} A_0 \cos\xi \, e^{-j\xi} .$$

The complete description of the radio frequency plant will be given in another article.

In this work it is sufficient to remember that such a plant is set up in 4 accelerating cavities each one is fed by an amplifying chain; these are excited, with a suitable phase, by a common oscillator.

The block diagram of the R.F. plant, that feeds a final amplifier, is given in fig. 7, while the main

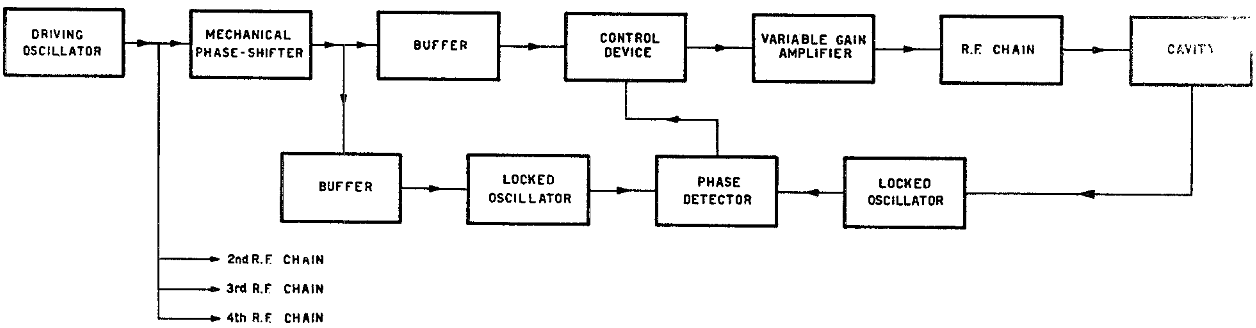

Fig. 8. — Detailed diagram of the control device included in the radio frequency plant.

On the other hand, the output voltage is:

$$V_s e^{j\varphi_s} = V_i e^{j\varphi_i} \frac{e^{-jn(\varphi_u - \varphi_i)}}{\cos n(\varphi_u - \varphi_i)}$$

combining the two relations we find:

$$V_u e^{j\varphi_u} = V_i A_0 \frac{\cos\xi}{\cos n(\varphi_i - \varphi_u)} e^{j[(n+1)\varphi_i - n\varphi_\omega - \xi]}$$

solving:

$$\text{(9)} \quad \begin{cases} \varphi_u = \varphi_i - \dfrac{\xi}{n+1} \\ V_u = V_i A_0 \dfrac{\cos\xi}{\cos \dfrac{n}{n+1}\xi} . \end{cases}$$

characteristics are indicated in the following table:

— Input voltage: 1.2 V_{rms} (input impedence = 60 Ω);
— Output voltage: 200 V_{rms} (output impedence = 60 Ω);
— Mid band working frequency: 8,57 MHz;
— Band width: 800 kHz;
— Rise time of the amplifying chain: 3 μs;
— Rise time of the group « power amplifier cavity »: 0,24 ms.

The block diagram of the control device included in each amplifying R.F. chain, is given in fig. 8. The driving oscillator gives a radio frequency voltage of about 1,2 V_{rms} that feeds the mechanical phase shifters and this, in turn, excite via a buffer stage the locked oscillators giving the reference voltage (of constant amplitude) to the phase detector.

The voltage « to be controlled » is taken with a loop

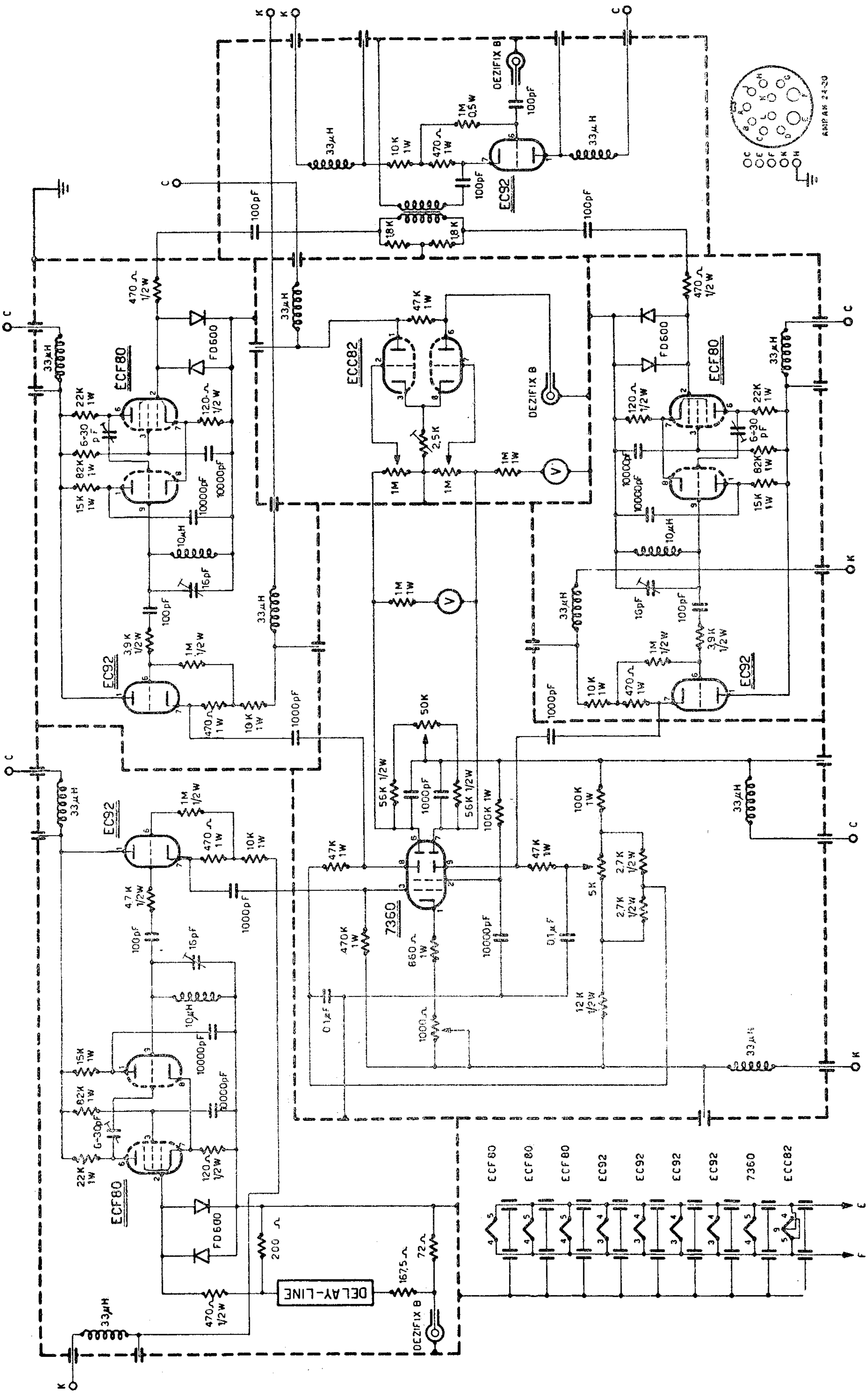

Fig. 9. — Electrical diagram of the phase detector.

from the cavity and then feed to a third locked oscillator whose output (always of constant voltage) gives the other signal to the phase detector.

radio frequency voltage, coming from the control device is feed to the main amplifying R.F. chain via a low power amplifier whose gain can be manually adjusted.

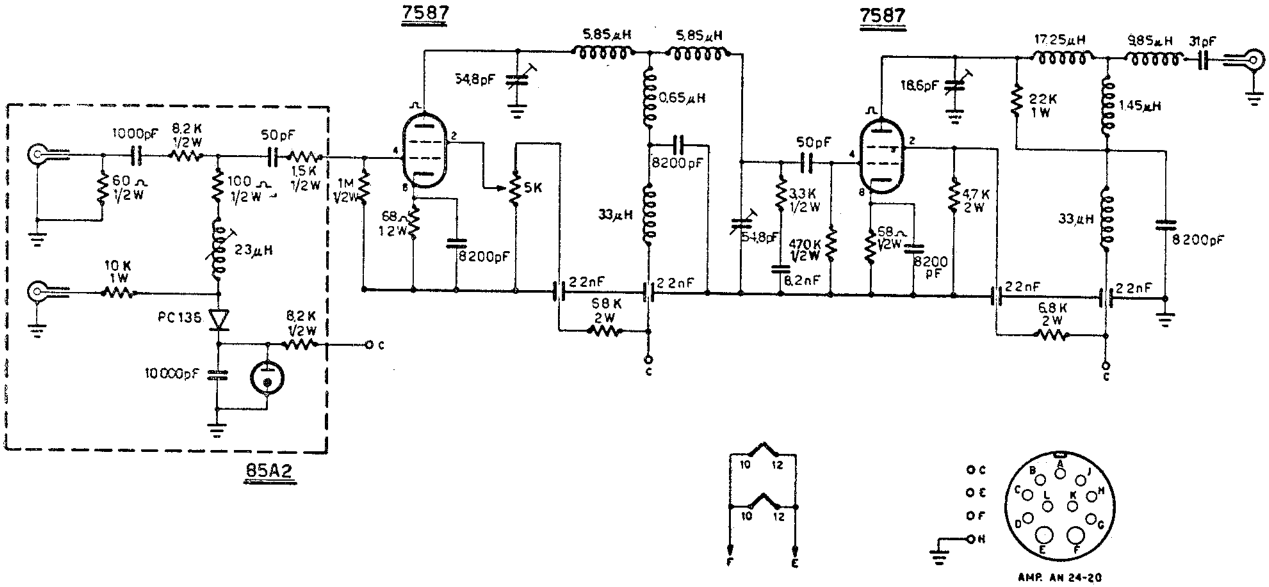

Fig. 10. — Electric diagram of the phase and amplitude controlled R.F. amplifier.

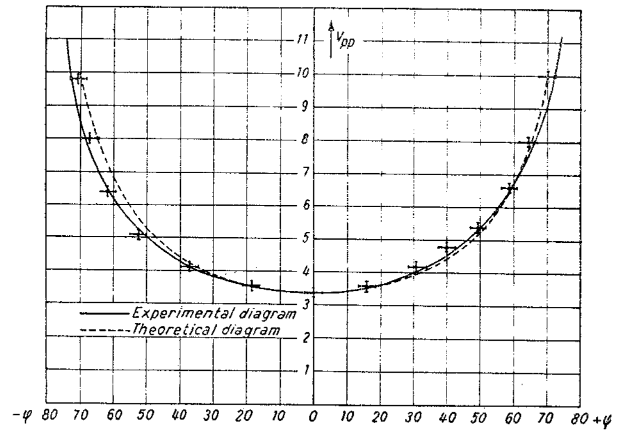

Fig. 11. — Amplitude of radio frequency voltage coming from the correcting network as a function of phase shift between the input and output signals.

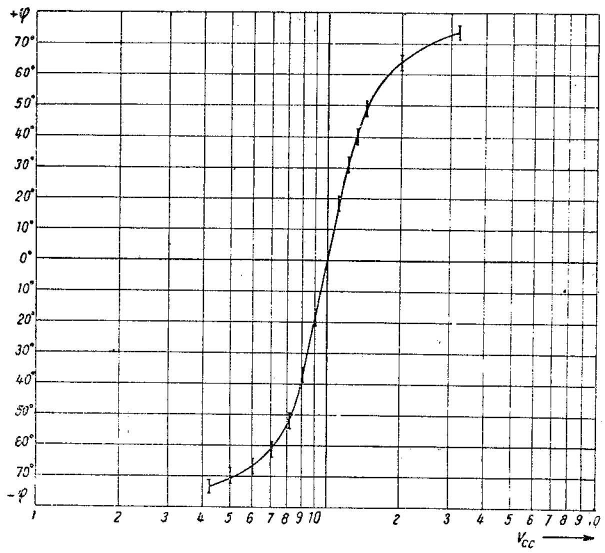

Fig. 12. — Phase of the voltage from the correcting network versus the bias voltage V_{cc}.

The output voltage of the phase detector, suitably amplified, gives a variable bias to the varactor that

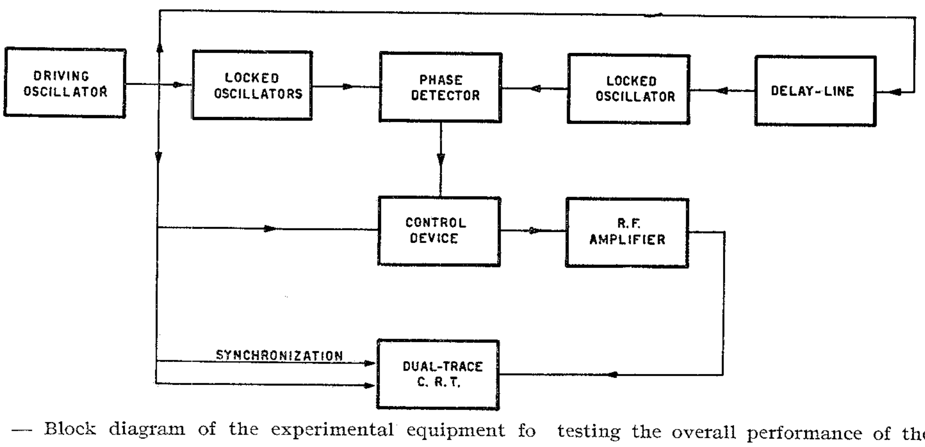

Fig. 13. — Block diagram of the experimental equipment fo testing the overall performance of the system.

controls, as we have already seen, the amplitude and the phase of the R.F. voltage coming from the pilot. The

The electric diagrams of the tested controling devices are shown in figs. 9 and 10.

In figs 11 and 12 are shown the results of the measurements made on the control device.

The photos 14, 15, 16 show the measurements made on the control device simulating a detuning effect of the cavity with a delay-line; the block diagram of the measuring equipment is shown in fig. 13.

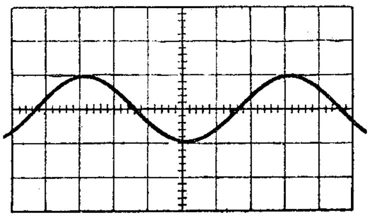

Fig. 14. — Oscillogram of the input and output signals in the measuring equipment showed in fig. 13 when the two signals are equal in amplitude and phase (t = 20 ns/cm).

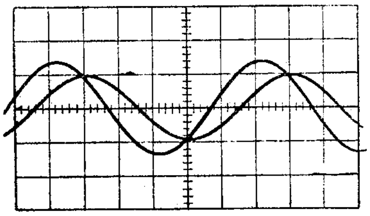

Fig. 15. — Experimental situation as in fig. 14, but now the signal from the control device is (artificially) led, with respect to the signal coming from the pilot, of about 49°±6°. The ratio between the amplitudes of the signals is 0,68 which corresponds (following the cosin law) to about 47° in agreement with the measurement (t = 20 ns/cm).

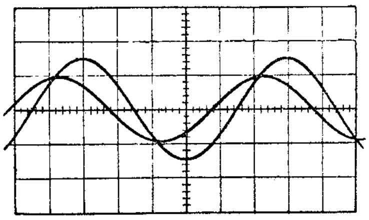

Fig. 16. — Experimental device as in fig. 14, the output signal of the control device is (artificially) lagged respect to the signal coming from the pilot of about 47° ± 6°. The ratio between the amplitudes of the signals is 0,64; this is also in agreement with the theoretical previsions (t = 20 ns/cm).

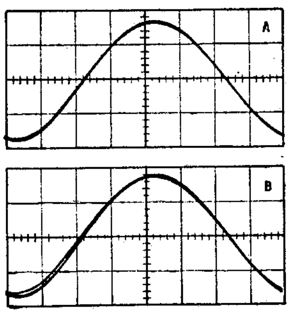

Fig. 17. — *A* - Resonant cavity tuned; Control device ON; *B* - The same conditions as in *A*; Control device OFF.

The results of the measurements made on the whole radio frequency and cavity system are given in the oscillographic photos reported in the pictures 17, 18 and 19. (When the control device included in the chain, is ON or when is OFF).

The various parameters are selected to give coinciding traces (for the voltages coming from the cavity and from the pilot) on the C.R.T. when the cavity is perfectly tuned.

It is necessary to bear in mind that the error assigned to the phase measurements is due to the trace width in

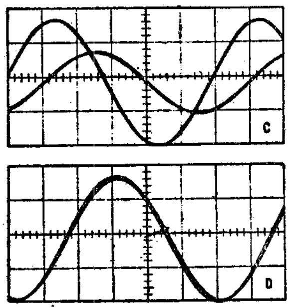

Fig. 18. — *C* - Resonant cavity detuned; Control device OFF. The phase shift between the pilot and cavity signals is nearly 71° ± 6°. - *D* - Same conditions as in *C* but now the control device is ON.

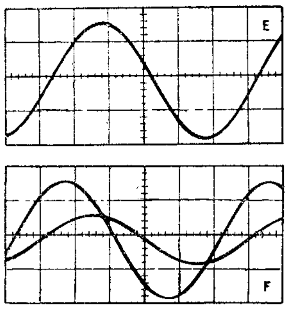

Fig. 19. — *E* - Resonant cavity detuned; The control device ON. *F* - The same conditions as in *E*. The phase shift between the pilot signal and the cavity signals is 53° ± 6°. Control device OFF.

the photos. (We note that, in the photos 17, 18 and 19, given previously, the amplitude variations of the output signal does not follow exactly the law $V = V_{rif}/\cos \varphi$. This is due to the fact that the R.F. amplifier chain, that we had at our disposal during the tests, could not follow exactly the very large amplitude variations required by the system.

Conclusions.

Among the many control system, that have been so far employed, this one, that as long as we known was never proposed for the R.F. plants of accelerating machines, seems us the better because with only one feedback loop allows a practically perfect compensation of the amplitude and phase of the voltage of the cavity.

Such a system, based only on a phase measurement, is inoperative against the cavity voltage variations that do not depend upon a detuning of the cavity and so allow the regulation of the cavity voltage without any change on the feed-back system.

ACNOWLEDGEMENTS.

The authors wish to express their heartfelt thanks to prof. E. Gatti for the many constructive and illuminating discussions on the whole argument.

We also wish to thank P. I. Giordano for the realization of the prototype of the control device and for the long series of measurements made.

The paper was first received 15th *september* 1966.

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