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ENERGY DEPENDENCE OF PHOTONUCLEAR EFFECT AND MUON INTENSITY UNDER ROCK

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Abstract: We attempt to verify the high-energy behaviour of the photo-nuclear cross section σ_γ which is expected to be dominated by the exchange of a Regge pole, by analysing the information on the intensity-depth curve of muons under rock.

We first discuss the nuclear interactions of high-energy muons and compare our calculations with the available data on muons at the accelerators or under rock. Then we compare the muon integral spectrum obtained from under-rock results and from our range-energy relation with the one measured directly at sea level. We conclude that there is no evidence of the supposed energy dependence of the cross section.

1. Introduction

It has recently been suggested [1]) that the data on under-rock cosmic ray muons could serve as a check of the high-energy behaviour of the cross section of such processes, as the pion photoproduction on nucleons, which are assumed to be ruled by the exchange of a Regge pole. In fact the nuclear interactions of muons occur through $\gamma+\mathrm{N} \rightarrow \pi+\mathrm{N}$ processes produced by the associated virtual photons. For high photon energies ($\varepsilon \gtrsim 1$ GeV), these processes are in a Regge pole description controlled by the exchange of the $\rho(J = 1, T = 1)$ and $\omega(J = 1, T = 0)$ trajectories.

If we confine ourselves to consider the ρ-trajectory in the physical region ($|\cos\theta| \leqq 1$), and hence a $-4\varepsilon \leqq t \leqq 0$ range of momentum transfer, it can be shown[2]) that asymptotically

$$\sigma_\gamma \leqq \text{const } \varepsilon^{2[\alpha_\rho(0)-1]}, \tag{1}$$

where the inequality comes from replacing $2[\alpha_\rho(t)-1]$ with $2[\alpha_\rho(0)-1]$. Therefore, if we assume [3]) $\alpha_\rho(0) = \frac{1}{2}$, the cross section σ_γ for photoproduction at energies higher than a proper value ε^* is given by

$$\sigma_\gamma \leqq \text{const } \varepsilon^{-1}, \qquad (\varepsilon > \varepsilon^*). \tag{2}$$

The purpose of this note is to verify this conclusion (2) by computing the range-energy relation for muons and by comparing the muonic spectrum so obtained from the intensity-depth curves with the one measured directly at sea level.

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2. Nuclear interactions of high-energy muons

Let us write the differential cross section for the nuclear inelastic interaction of muons with energy E_μ in the factorized form 4, 5)

$$d^2\sigma_\mu(E_\mu, \varepsilon, \tau)/d\varepsilon d\tau = N(E_\mu, \varepsilon, \tau)\sigma_\gamma(\varepsilon, \tau, E_\mu) = N(E_\mu, \varepsilon, \tau)\sigma_\gamma(\varepsilon)\Gamma(\tau), \tag{3}$$

where τ is the modulus of the squared tetramomentum of the exchanged virtual photon.

The equivalent photon spectrum $N(E_\mu, \varepsilon, \tau)$ is given in an accurate way by the revised Kessler formula 4)

$$N(E_\mu, \varepsilon, \tau) = \frac{1}{\pi}\alpha(\varepsilon^2+\tau)^{-\frac{1}{2}}\tau^{-1}\{1-\varepsilon|E_\mu+\varepsilon^2|2E_\mu^2+\tau|4E_\mu^2-(\varepsilon^2+\tau^2)m_\mu^2|\tau E_\mu^2\}. \tag{4}$$

The choice on $\Gamma(\tau)$ is less reliable; we shall try to check the suggestion of Dayasu *et al.* 5)

$$\Gamma(\tau) = \left[\frac{0.365}{0.365+\tau}\right]^2, \tag{5}$$

which identifies $\Gamma(\tau)$ with an electromagnetic form factor of nucleons of Hofstadter type, featured by $\langle r^2\rangle = 0.8$ fm. $\sigma_\gamma(\varepsilon)$ is the cross section for photoproduction of real photons.

Up to now, the experimental information on $\sigma_\gamma(\varepsilon)$ at high energy is based on the results obtained by Castagnoli *et al.* 6) from the photostars produced in nuclear emulsions by a 1.1 GeV bremsstrahlung beam. We have

$$\sigma_\gamma(\varepsilon) = 0.33\ \mu\text{b/nucleon} \qquad \text{for} \qquad \varepsilon \leqq 1.1\ \text{GeV}. \tag{6}$$

For $\varepsilon \geqq 1.1$ GeV, we can assume a constant cross section

$$\sigma_\gamma(\varepsilon) = 370\ \mu\text{b/nucleon}, \tag{2'}$$

or else admit expression (2′) for $1.1 < \varepsilon < \varepsilon^*$ and then expression (2) for $\varepsilon > \varepsilon^*$. To verify the correctness of eqs. (4)–(6) and (2′), we consider the following physical processes.

2.1. TOTAL INELASTIC CROSS SECTION σ_{in} OF MUONS

The computed value

$$\sigma_{\text{in}} = \int_{\varepsilon_{\min}}^{(E-m_\mu)}\int_{\tau_{\min}}^{\tau_{\max}} \frac{d^2\sigma}{d\varepsilon d\tau}\, d\varepsilon d\tau,$$

where

$$\begin{aligned}\tau_{\min} &= \varepsilon^2 m_\mu^2|E_\mu(E_\mu-\varepsilon),\\ \tau_{\max} &= E_\mu\varepsilon^2|(E_\mu-\varepsilon),\end{aligned} \tag{7}$$

has been compared with results from 2.5 and 5 GeV monochromatic beams of muons

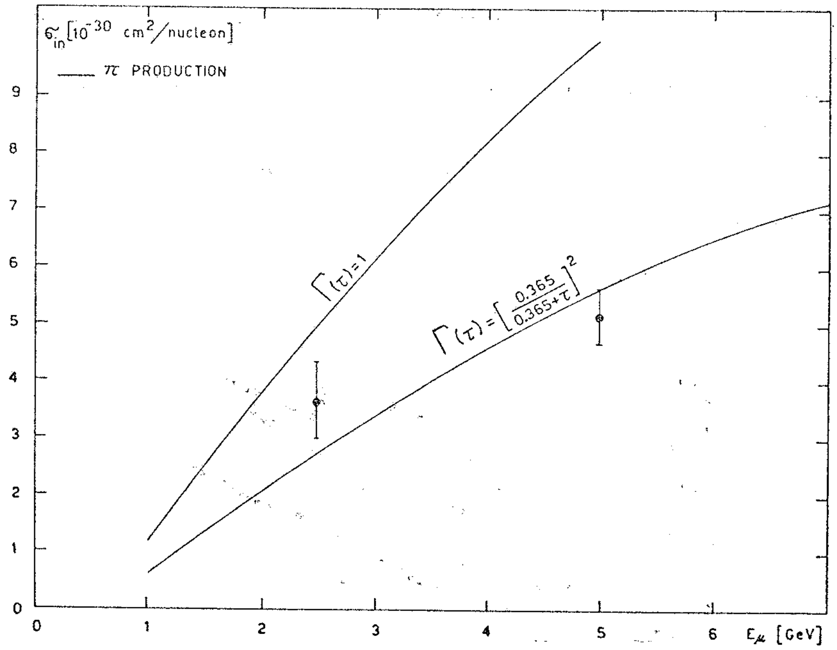

Fig. 1. Total inelastic cross section of muons. The solid lines represent eqs. (7) and (3) computed, respectively, with $\Gamma(r) = 1$ and $\Gamma(r) = [0.365/(0.365+r)]^2$. Experimental points are from Kirk *et al.* 7) for muons of 2.5 and 5 GeV.

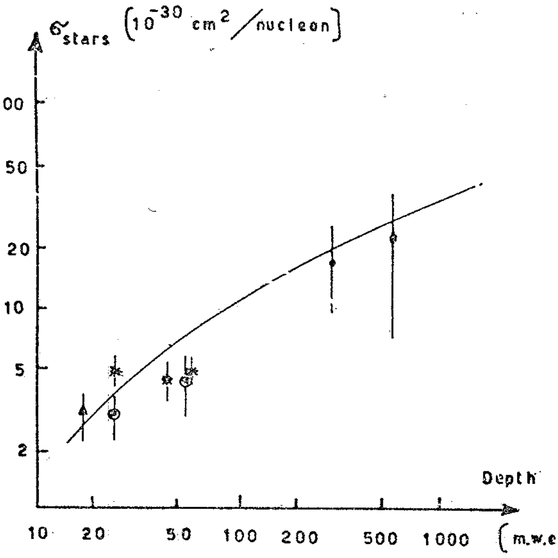

Fig. 2. Available experimental data on the cross section for star production under rock. Avan and Avan 19), ▲ – Kaneko 20), ⊙ – Barron 21), * – George and Evans 9). The solid line is obtained from eq. (8) with the assumption (5) and (2′).

of Kirk *et al.* [7]). With $\varepsilon_{\min} = 140$ MeV we obtain (see fig. 1) good agreement between theory and experiment; on the contrary if we put $\Gamma(\tau) = 1$ there is a clear disagreement. Hypothesis (2) cannot be tested by these experimental data owing to the low value of E_μ.

2.2. CROSS SECTION σ_{st} FOR STAR PRODUCTION UNDER ROCK

We compute

$$\sigma_{\mathrm{sh}}(x) = \int_{E_\mu{}^{\min}}^{\infty} F(E_\mu, x)\mathrm{d}E_\mu \int_{\varepsilon_{\min}}^{E_\mu - m_\mu} \frac{\mathrm{d}\sigma(E_\mu, \varepsilon)}{\mathrm{d}\varepsilon}\mathrm{d}\varepsilon \Bigg/ \int_{E_\mu{}^{\min}}^{\infty} F(E_\mu, x)\mathrm{d}E_\mu, \tag{8}$$

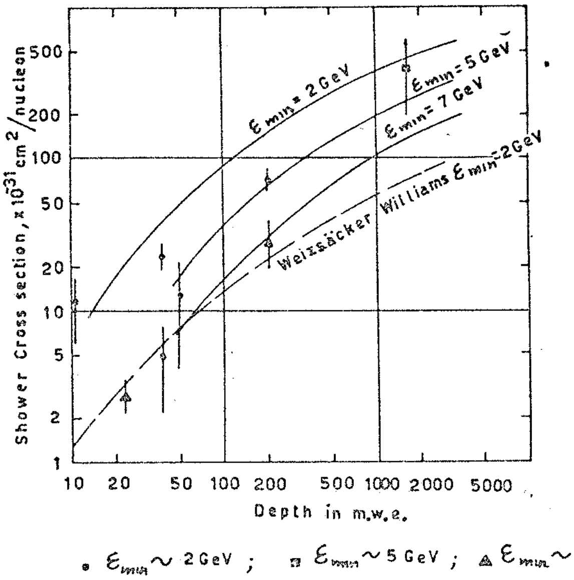

Fig. 3. Cross section for shower production under rock. The available experimental points are given by Kessler and Maze [22]), Amaldi *et al.* [23]), $\varepsilon_{\min} = 2$ GeV, Argau *et al.* [24]), Osaka group [25]), $\varepsilon_{\min} = 7$ GeV and Barret *et al.* [26]), $\varepsilon_{\min} = 5$ GeV. The solid lines are obtained from the eqs. (8), (3) and (4) with the assumptions (5) and (2′) and $\varepsilon_{\min} = 2$, 5 and 7 GeV. The dashed line is obtained with the Weizsäcker-Williams formula instead of eq. (4).

where $F(E_\mu, x)\mathrm{d}E_\mu$ is the μ-spectrum [8]) at a given depth x. With $\varepsilon_{\min} = 140$ MeV we obtain good agreement with the available experimental data (see fig. 2). Taking into account the efficiency of detection discussed by George and Evans [9]), the agreement is still improved.

2.3. CROSS SECTION σ_{sh} FOR SHOWER PRODUCTION UNDER ROCK

We compute eq. (8) for $\varepsilon_{\min} = 2.5$ GeV, thus obtaining the curves of fig. 3. The comparison with the available experimental data shows a satisfying agreement, which still improves if we take into account the sensible uncertainty on the instrumental cut-off values of $\varepsilon_{\min}$. In fig. 3 we report also the curve computed for $\varepsilon_{\min} = 2$ GeV, using the Weizsäcker-Williams formula in place of expression (4).

2.4. DIFFERENTIAL INELASTIC CROSS SECTION $d\sigma/d\tau$ OF MUONS

We compute

$$\frac{d\sigma}{d\tau} = \int_{\varepsilon_{\min}}^{E-m_\mu} \frac{d^2\sigma}{d\varepsilon d\tau} d\varepsilon, \tag{9}$$

where $\varepsilon_{\min}$ has the value $m_\pi + (\tau + m_\pi^2)/2M$ given by kinematic considerations. The comparison carried out in fig. 4 with the experimental data obtained by Kirk *et al.* [7]) from 5 GeV muons shows a very satisfying agreement, taking into account also the fact that for the greatest value of τ the experimental point is probably overestimated.

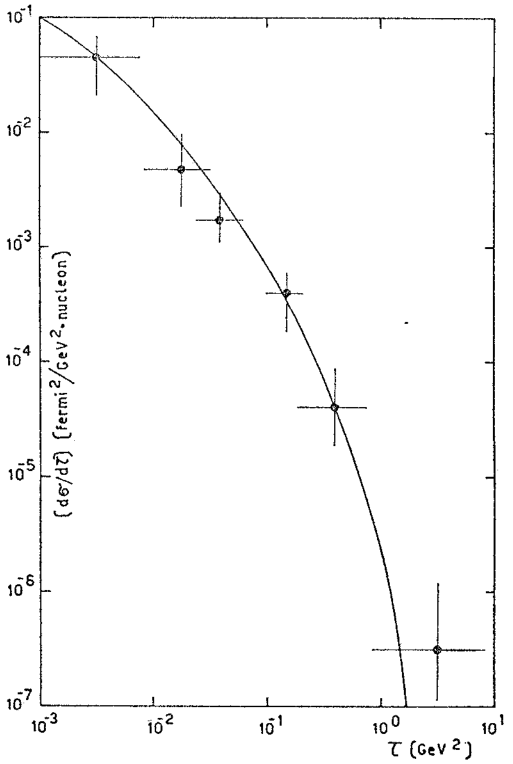

Fig. 4. Experimental points on the inelastic differential cross section $d\sigma/d\tau$ of 5 GeV muons obtained by Kirk *et al.* [17]). The solid line is obtained from eq. (9) with assumptions (4)–(6), $\varepsilon_{\min}$ has the value $m_\pi + (\tau + m_\pi^2)/2M$ from kinematical considerations.

From this discussion we conclude that all our assumptions (4)–(6) are quite well supported by the available experimental data and can thus be used to test eq. (2).

3. Range-energy relation

Let us then choose among expression (2) and (2′) with the following procedure: (i) We compute $(dE_\mu/dx)_{\text{nucl}}$ for underground muons with the aid of the two formulae.

(ii) Thereafter we compute the range-energy relation $R(E_\mu)$. (iii) We transform by means of this $R(E_\mu)$ the experimental intensity-depth curve $I(x)$ into an energy spectrum $I(E_\mu)$ taking into account also the straggling. (iv) We compare this $I(E_\mu)$ with the $I_{\text{s.l.}}$ (E_μ) spectrum obtained at sea level with magnetic spectrographs and other methods.

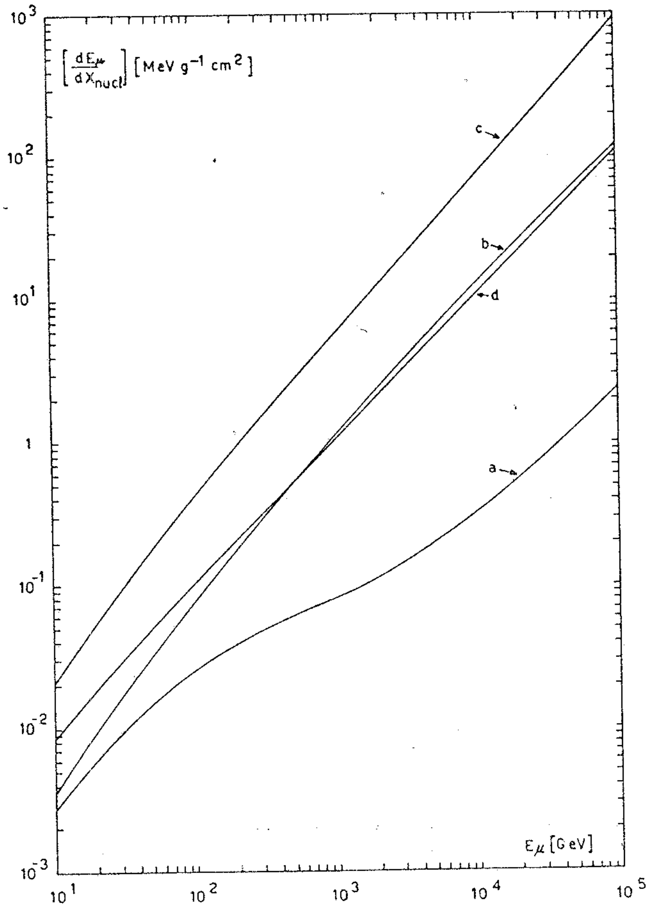

Fig. 5. Energy loss $(\mathrm{d}E_\mu/\mathrm{d}x)_{\text{nucl}}$ for nuclear interactions. The curves relate to different assumptions; a) with eq. (2) and $\Gamma(\tau) = [0.365/(0.365+\tau)]^2$, b) with eq. (2′) and $\Gamma(\tau) = [0.365/(0.365+r)]^2$, c) with eq. (2′) and $\Gamma(\tau) = 1$ and d) with the classical formula of Weizsäcker-Williams for $N_\mu(E, \varepsilon, \tau)$.

(i) In fig. 5 we have:

$$-(\mathrm{d}E_\mu/\mathrm{d}x)_{\text{nucl.}} = K \iint \varepsilon \frac{\mathrm{d}^2\sigma}{\mathrm{d}\tau \mathrm{d}\varepsilon} \mathrm{d}\varepsilon \mathrm{d}\tau$$

computed by eq. (2) (curve a) and eq. (2′) (curve b). Curve c is obtained by introducing in eq. (2) $\Gamma(\tau) = 1$ instead of value (5) and curve d by making use of the classical formula of Weizsäcker-Williams for $N(E_\mu, \varepsilon, \tau)$.

(ii) For the energy losses ($\mathrm{d}E/\mathrm{d}x$) by pair production, bremsstrahlung and collision we used some of our [11, 12]) previous results. Here we only point out that the value of $(\mathrm{d}E/\mathrm{d}x)_\mathrm{p}$ we computed following the quantum theory of Murota *et al.* [13]) taking into account screening effects for large energy transfers ε_t and of asymmetries among members of the pair. We compared our formulae with the recent results on σ_p obtained [14]) in a Wilson chamber at 143 m.w.e. ($E_\mu \approx 32$ GeV).

TABLE 1

Energy of the pair (ε_t, $\varepsilon_t+\Delta\varepsilon_t$) (MeV)	σ_exp (mb/nucleus)	σ_theor	
		Calculation of Chaudhuri *et al.*	our calculation
35– 100	40.5±2.0	41.5	40.1
100– 500	19.6±1.7	19.6	22.8
500–1000	2.7±0.2	1.6	2.5

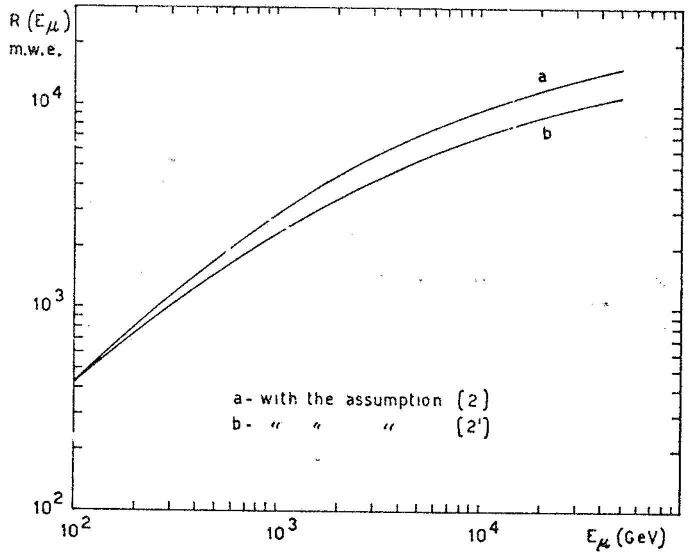

Fig. 6. Range-energy relation curves for muons, a) with assumption (2) and b) with assumption (2').

The computed value is

$$\sigma_\mathrm{p} = \int_{v(\varepsilon_t)}^{v(\varepsilon_t+\Delta\varepsilon_t)} \int_{\varepsilon_t}^{\varepsilon_t+\Delta\varepsilon_t} \frac{\mathrm{d}^2\sigma(\varepsilon_t, v)}{\mathrm{d}\varepsilon_t \mathrm{d}v} \mathrm{d}\varepsilon_t \mathrm{d}v, \tag{10}$$

where we make use of the formulae given in ref. [12]) with the parameter $\alpha = 2$ for the expression of $\mathrm{d}^2\sigma$.

Table 1 shows that these calculations are in good agreement with the experimental results. The previous calculation given in ref. [14]) appears on the contrary out with five times the experimental error for large values of ε_t; this may be due probably to the lack of a lower limit for the energy of each member of the pair.

The calculation of $R(E_\mu)$ with hypotheses (2') and (2) leads to curves a) and b), of fig. 6, respectively.

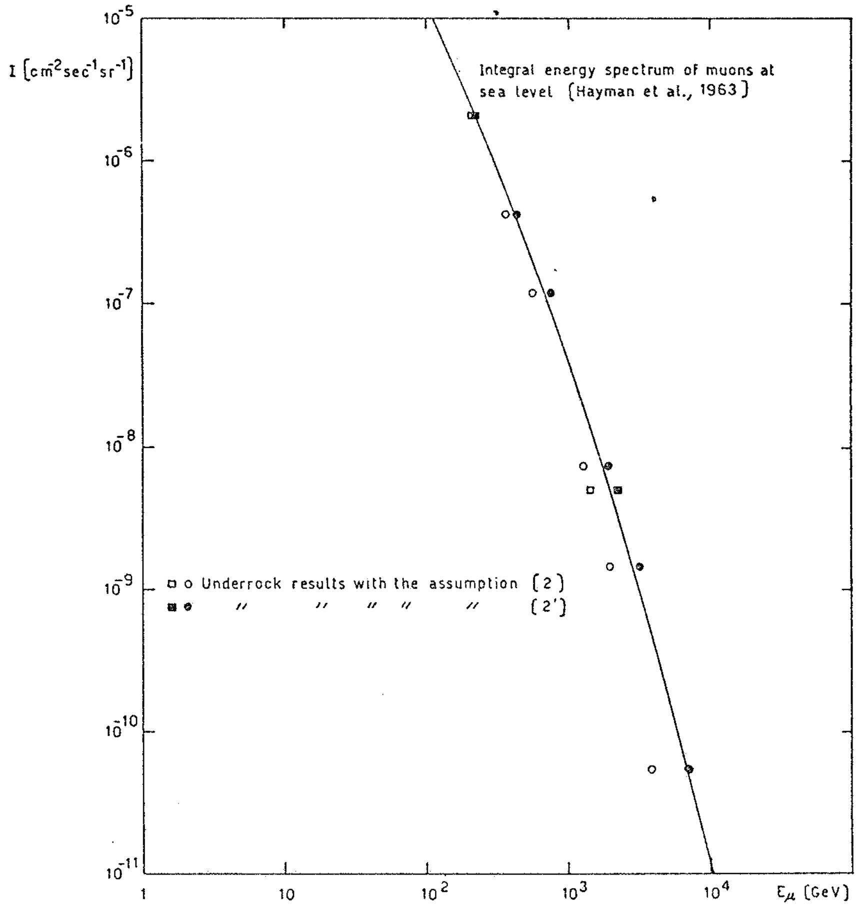

Fig. 7. Integral energy spectrum of muons at sea level. The solid line is obtained from Hayman *et al.* 15). The points represent underground results of Myake *et al.* 16) and Castagnoli *et al.* 17) with the assumptions (2) and (2'), respectively.

4. Comparison between muonic spectra at sea level and intensity-depth results

Fig. 7 shows the results of our calculations. The curve is a plot of the sea-level spectrum $I_{\text{s.l.}}(E_\mu)$ given by Wolfendale *et al.* 15) from the data obtained with magnetic spectrographs and γ-showers.

The experimental points show the results obtained underground by Miyake *et al.* 16) and Castagnoli *et al.* 17). The ordinate of these points gives the intensity measured experimentally and corrected to take account of the straggling effect which we discuss elsewhere [18]). For every depth measured in m.w.e. we have two values of energy, the one obtained from curve (a) and the other from curve (b) of fig. 7.

It turns out that (i) the spectrum obtained from the underground results agrees with the ones obtained at sea level up to the highest energy if we make use of curve $R(E_\mu)$ computed following hypothesis (2′); (ii) the underground experimental points are quite off the ones obtained at sea level following hypothesis (2) and the disagreement grows with increasing energy up to an order of magnitude for $E_\mu \approx 10^4$ GeV.

We can hence conclude that (i) results on under-rock cosmic rays seem not to be in agreement with hypothesis (2); (ii) available results on the nuclear interactions of high-energy muons agree with a sensibly constant value of σ_γ given by eq. (2′).

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