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G. Pancheri: INFRARED RADIATIVE CORRECTIONS FOR RESONANT PROCESSES. -

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1) - INTRODUCTION -

In view of the importance of radiative corrections for the interpretation of the results of colliding beam experiments, E. Etim and B. Touschek(1) proposed a division of labour between theorists and experimentalists. This division of labour was intended to enable two experimentalists to compare the results of their experiments directly and to allow the theorist to concentrate on the essential ultraviolet aspects of the process without entering into the details of the experiments.

This proposal was further elaborated by E. Etim, G. Pancheri and B. Touschek(2) and it was shown that the method should give highly accurate results for a certain class of experiments (in which the statistical error was matched to the momentum resolution) and for processes for which the cross section calculated in lowest order perturbation theory does not rapidly vary with the energy of the colliding particles. M. Greco and G. Rossi(3) have shown that the proposed procedure can be derived from perturbation theory(4 ÷ 7) by means of a canonical transformation applied to the final state of the reaction.

The case of strongly energy dependent processes has been discussed in a very rough approximation in reference (1), but it was not included in (2) since the basic assumption of separability of the infrared process from the high energy part of the interaction cannot be expected to hold in this case. From an experimental point of view the radiative cor rections are of particular importance just in this case of resonant processes. This is due to the fact that the exploration of a resonance calls for a maximum resolution of the energy of the produced particles: this makes the radiative corrections quite big. U. Amaldi, A. de Gasperis and P. Stein[8] have discussed the behaviour of the φ-resonance assuming an energy resolution of 0.5 MeV and applying perturbation theory in a form suggested by P. Kessler[9]. In these conditions the perturbative radiative correction is about -45%; the treatment suggested in refs. (1) and (2) would lead one to estimate that this correction should be -29%. It is seen that the difference is quite considerable and that there is a case for suspecting the applicability of straightforward perturbation theory.

The breakdown of the separability of the low and high energy aspects of the process has the following reason: the condition for separability is that the high energy part of the process takes place in a time interval, which is very short compared to the inverse of the energy lost in the form of radiation. In a resonant process this will generally only be the case as long as the energyloss to the radiation field is contained well within the width of the resonance. If one wants information on the radiative corrections to the form of the resonance curve (and in particular to the form of the "shoulders" of this curve) one has to take explicit account of the finite life time of the intermediate resonant state.

In section 2 we shall give a semiclassical treatment of this phenomenon and it will be shown that one should expect a splitting of the resonance peaks into two: a displaced peak due to the emission of quanta from the electron positron pair of the initial state and a fixed peak corresponding to the emission of quanta from the final state particles. This splitting of the peaks must not be confused with the splitting discussed by Amaldi, de Gasperis and Stein[8], which is characteristic for a measurement of the total apparent centre of mass energy of the resonant state.

Section (3) confirms the results of section 2 in lowest order perturbation theory.

In section 4 a generalization of the results of the previous sections is attempted. The purpose of this generalization is to free the treatment of resonant states from the limitations of perturbation theory.

2) - SEMI CLASSICAL TREATMENT OF THE BREMSSTRAHLUNG EMITTED IN THE FORMATION OF RESONANT STATES -

For non resonant processes the current attributed to the transition from the initial state $|i)$ to the final state $|f)$ can be assumed to be of the form

(1) $$j_c(x) = j_f(x)\,\theta(t) + j_i(x)\,\theta(-t)$$

where j_f and j_i are respectively the currents represented by the charged particles of the final and initial states. It is assumed in (1) that the transition from the initial to the final state takes place in an infinitesimal time interval at $t = 0$.

$j_c(x)$ can be understood as the meanvalue of the quantized current in a scattering state $|s)$ defined by $\lim_{t\to\infty} |s) = f)$.

If the transition $|i) \to |f)$ is resonant it can no longer be assumed that the process takes place in a negligibly short time (negligibly short compared to the reciprocal of the frequency of the emitted radiation). One can take account of the finiteness of the time interval between the creation of the final state at time $t = 0$ and the creation of the resonant state at time $-t_o < 0$ by putting instead of (1)

(2) $$j_c(x) = j_f(x)\,\theta(t) + \overline{j_i(x)}$$

where $\overline{j_i(x)}$ is given by

(3) $$\overline{j_i(x)} = \left(\frac{\Gamma}{2} - i(2E-M)\right) \int_0^\infty dt_o\, j_i(x)\theta(-t-t_o)e^{-\frac{\Gamma}{2}t_o + i(2E-M)t_o}$$

We have chosen the centre of mass system and assumed that the incoming particles (an electron and a positron) have an energy E, that the mass of the resonant state is M and that its width is Γ. Putting

$$j_c(x) = \int j_c(k) e^{i(kx)} d^4k$$

one gets

(4) $$(2\pi)^4 \vec{j}_c(k) = -\sum_f \frac{ie_f \vec{v}_f}{(k\vec{v}_f) - \omega} + S \sum_i \frac{ie_i \vec{v}_i}{(k\vec{v}_i) - \omega}$$

with

(5) $$S = \frac{\frac{\Gamma}{2} - i(2E-M)}{\frac{\Gamma}{2} - i(2E-\omega-M)}$$

Here we have only written the space part of $\vec{j}_c(k)$, e_f, v_f are the charges and velocities of the final state particles, e_i and v_i refer to the initial sta te and equation (4) hold for $\omega = |k_o| = |\vec{k}|$. It is seen that for $(2E-M) \gg \Gamma/2$ and $(2E-M) \gg \omega$ one has S=1 and equation (4) therefore reduces to the Fourier transform of (1).

The Bond-factor β introduced in reference (1) is defined by $dn(\omega) = \beta\, d\omega/\omega$. where $dn(\omega)$ is the average number of photons produced in $|i) \rightarrow |f)$. Using the method explained in the appendix of reference (2) we find

(6) $$\beta = \beta_f + |S|^2 \beta_i$$

Because of charge conjugation invariance there is no interference term.

The knowledge of β allows one to determine the first approximation to the cross section of the resonant process accompanied by the emission of a single photon. Denoting this cross section by $d^3\sigma(\omega)$ one has

(7) $$d^3\sigma(\omega) = \beta \frac{d\omega}{\omega} d^2\sigma_o$$

where $d^2\sigma_o$ is the cross section for the resonant process without the emission of quanta, i.e. calculated in lowest non vanishing order. If the Breit Wigner formula is used for $d^2\sigma_o$ one finds

$$d^2\sigma_o \propto \left(\frac{\Gamma^2}{4} + (2E - M)^2\right)^{-1}$$

Within the range of the validity of the Breit Wigner formula one can therefore write for (7)

(8) $$d^3\sigma(\omega) = \frac{d\omega}{\omega}\left(\beta_f d^2\sigma_o(M) + \beta_i d^2\sigma_o(M+\omega)\right)$$

It is seen from this expression that the cross section for bremsstrahlung will show two peaks (if considered as a function of the machine energy E and for a fixed frequency ω of the emitted radiation): one an undisplaced resonance peak at 2 E = M corresponding to the emission of photons by the particles of the final state and a displaced peak at 2 E = M + ω corresponding to the emission of radiation by the initial state electrons and positrons.

The asymmetry between the initial and final state is due to the fact that radiation is emitted and not absorbed. The emission of radiation from the initial state diminishes the q^2 of the reaction, the emission of the radiation from the final state does not.

For most reactions to be measured with Adone the undisplaced peak should be smaller than the displaced peak since unless the final state particles are also electrons and positrons we shall have $\beta_f <<$ $<< \beta_i$: for the ρ - resonance one has $\beta_e = 0.063$ and $\beta_\pi = 0.011$.

3) - COMPARISON WITH PERTURBATION THEORY -

In this section we compare the expression (8) derived from the semiclassical model with perturbation theory. We calculate the infrared radiative corrections to the lowest order in α for the following process

(9) $$e^+ + e^- \to \rho \to \pi^+ + \pi^-$$

The differential cross section $d^2 \sigma_o$ of this process in the center of mass system of the initial state is given by

(10) $$d^6 \sigma_o = \frac{(2\pi)^{-2}}{16 v \gamma_e^2} \frac{d^3 \vec{p}_3}{2 p_{30}} \frac{d^3 \vec{p}_4}{2 p_{40}} \delta^{(4)}(p_1 + p_2 - p_3 - p_4) \sum_{spin} |B_o|^2$$

where v is the velocity of the electrons, $p_i (i = 1, 2, 3, 4)$ is the four-momentum of the particles, as illustrated in fig. 1, B_o is the matrix element corresponding to the graph in fig. 1 and γ_e is the Lorentz factor of the electrons.

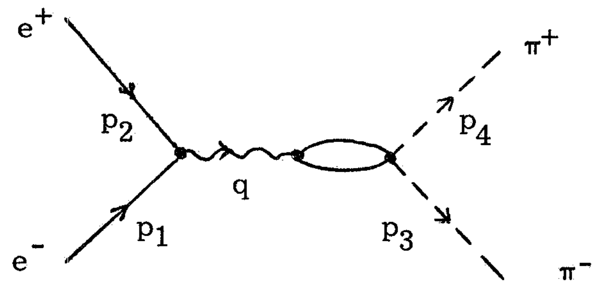

FIG. 1

The expression for $\sum_{spin} M_o^2$ is the following

(11) $$\sum_{spin} |B_o|^2 = \frac{e^4 \left\{ 2\left[(p_3-p_4)\cdot p_1\right]\left[(p_3-p_4)\cdot p_2\right] - \left[m_e^2 + (p_1 \cdot p_2)\right](p_3-p_4)^2 \right\}}{16\, m_e^2 E^4 \left[(4E^2 - M^2)^2 + M^2 \Gamma^2\right]}$$

where m_e is the mass of the electron, M the mass of the ρ, Γ is the width of the ρ-peak and we have indicated with 2E the total energy fournished by the initial state.

One additional photon can be added to the graph of fig. 1 in four different ways as illustrated in fig. 2

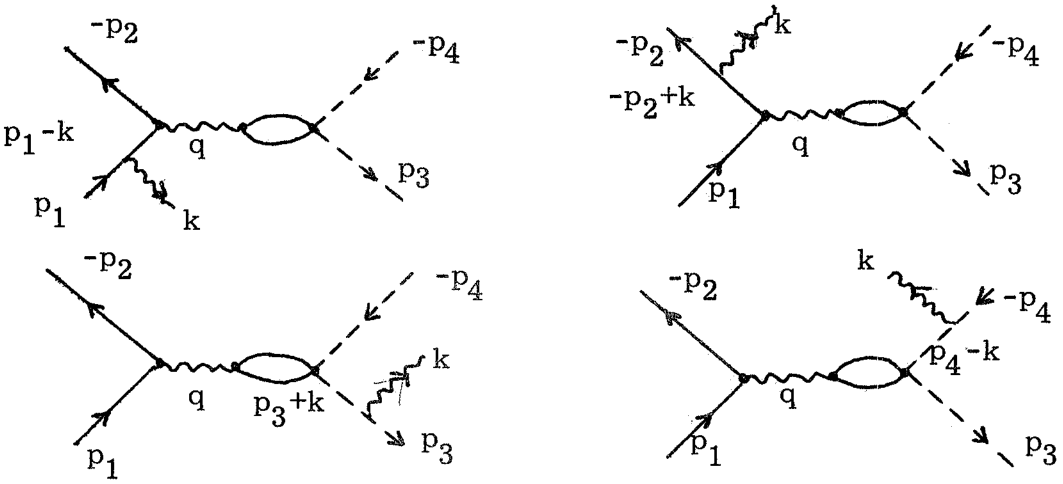

FIG. 2

If one puts

$$|B|^2 = |B_1 + B_2 + B_3 + B_4|^2$$

calling B_i the matrix element wich corresponds to the photon emission by the i-th charged line, one has

$$(12)\quad d^6\sigma = \frac{(2\pi)^{-5}}{16 v \gamma_e^2} \frac{d^3\vec{p}_3}{2p_{30}} \frac{d^3\vec{p}_4}{2p_{40}} \int \frac{d^3\vec{k}}{2\omega} \delta^{(4)}(p_1 + p_2 - p_3 - p_4 - k) \sum_{\substack{spin\\pol}} |B|^2$$

In order to compute $|B|^2$, it must be noted that photon emission by the initial state doesn't interfere with that one by the final state because of the Putzolu theorem(10) so that

$$(13)\qquad |B|^2 = |B_1 + B_2|^2 + |B_3 + B_4|^2$$

The calculation of the matrix elements B_i brings

$$(14)\quad \sum_{\substack{spin\\pol}} |B_1+B_2|^2 \approx \frac{e^6}{16 E^4 m_e^2} \Big\{ 2\left[(p_3-p_4)\cdot p_1\right]\left[(p_3-p_4)\cdot p_2\right] - \left[m_e^2+(p_1\cdot p_2)\right](p_3-p_4)^2 \Big\} \left\{ \frac{(p_1\cdot\epsilon)}{(p_1\cdot k)} - \frac{(p_2\cdot\epsilon)}{(p_2\cdot k)} \right\}^2 |R(E,\omega)|^2$$

(14)
$$\sum_{\substack{\text{spin}\\ \text{pol}}} |B_3+B_4|^2 \approx \frac{e^6}{16E^4 m_e^2} \Big\{ 2\left[(p_3-p_4)\cdot p_1\right]\left[(p_3-p_4)\cdot p_2\right] - \left[m_e^2+(p_1\cdot p_2)\right](p_3-p_4)^2 \Big\} \left\{ \frac{(p_3\cdot\epsilon)}{(p_3\cdot k)} - \frac{(p_4\cdot\epsilon)}{(p_4\cdot k)} \right\}^2 |R(E)|^2$$

where in the numerator we have neglected terms of order ω/E

(15)
$$|R(E,\omega)|^2 = \frac{1}{\left[4E^2(1-\frac{\omega}{E}) - M^2\right]^2 + M^2\Gamma^2}$$

$$|R(E)|^2 = \frac{1}{(4E^2 - M^2)^2 + M^2\Gamma^2}$$

Comparing (15) with (5) it is seen that one has

(16)
$$|S|^2 = |R(E,\omega)|^2/|R(E)|^2$$

near the resonance, where 2E + M in the resonant denominators can be replaced by 2M. Perturbation theory therefore confirms the result (8) to the first order in α.

4) - A POSSIBLE GENERALIZATION TO HIGHER ORDER IN α.

In view of the very high accuracy in the determination of the energy planned for the experiments on resonant annihilation the first order perturbation results cannot be considered sufficient - this was illustrated numerically in the introduction.

A generalization of equation (8) to cover higher powers of the coupling constant must satisfy the requirement that in conditions in which the cross section need not be considered as rapidly variable it should be described by the formula which has been justified in refs. (2) and (3), namely

(17)
$$N d^3\sigma(\omega) = \beta \frac{d\omega}{\omega} \left(\frac{\omega}{E}\right)^{\beta} d^2\sigma_E$$

where of course $\beta = \beta_e + \beta_\pi$. The condition of small variability of the cross section is satisfied for $\omega \ll \Gamma$ as well as for $2E - M \gg \Gamma$ and $\omega \ll 2E - M$. It is not satisfied on the shoulders of the resonance curve.

A further requirement for the desired generalization of (8) is that it should allow for the definition of a $d^2\sigma_E$, which is independent of the experimental resolution. This requirement can only be checked by actually determining the higher contributions with the help of perturbation theory and the problem must be left open till this is done.

A generalization of (8) which satisfies (17) is the following:

(18) $$N d^3\sigma(\omega) = \frac{d\omega}{\omega}\left(\frac{\omega}{E}\right)^{\beta} \left(\beta_f d^2\sigma_E(M) + \beta_i d^2\sigma_E(M+\omega)\right)$$

In the case of $\beta_f = 0$ this expression coincides with the case which has been treated in reference (1). Integrating (18) with the resolution function

(19) $$\rho(\omega) = e^{-\omega^2/2\Delta\omega^2}$$

gives β_i/β times the broadened and slightly shifted resonance curve discussed in reference (1) plus β_f/β times the original zero approximation resonance curve. It can be easily estimated that the error in the application of equation (18) should be at most $\beta_i \beta_f \log(E/\Delta\omega)$, which in the case of the ρ gives about .5%.

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