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INTRODUCTION -

A thermomolecular pressure theory has been developed in details by Weber⁽¹⁾ and is presently used to calculate the corrections for the measurements of He³ and He⁴ vapour pressure. This theory has been experimentally confirmed by several workers^(2, 3, 4, 5) and more recently by Roberts and Sydoriak⁽⁶⁾, who measured the ratio p_c/p_w for He³ down to 1.9°K (p_c is the pressure of the cold point, p_w that of the warm point). Down to this temperature there is a very good agreement with the theory and it is surprising not to find sensible deviations from a strictly classical theory due to the strong non-ideality of He³ and He⁴ gases. We have extended the measurements of the thermomolecular pressure of He³ to temperatures lower than 1°K in order to control whether this agreement is further confirmed.

Before examining our results we will briefly recall a few elements of the theory. The most general equation obtained by the Weber's theory is the following:

(x) - The measurements here reported have been the argument of a doctoral thesis of one of the authors (A.F.) discussed at the University of Rome in 1965.

2.

$$(1) \quad \frac{dp}{dT} = \frac{K_1 6\eta^2}{\rho T} \frac{1}{R^2 + 4K_2 R \gamma_M}$$

where p is the pressure, T the temperature, R the radius of the pipe, η and ρ are the viscosity and the gas density, K_1 and K_2 are constants and γ_M the slip coefficient of Maxwell's theory, which is of the same order of magnitude of the mean free path. We remember that by integrating equation (1), using expressions for η and ρ from the kinetic theory, one obtains the well known expression which states that the ratio of the extreme pressures p_c/p_w is only a function of T_c , T_w , R/λ_c and R/λ_w (λ is the mean free path).

For He⁴ Weber and Schmidt have obtained the following relation by fitting the constants with experimental data:

$$(2) \quad \log \frac{p_c}{p_w} = \frac{1}{2} \log \frac{T_c}{T_w} + 0,18131 \log \frac{Y_c + 0,1878}{Y_w + 0,1878} + \\ + 0,41284 \log \frac{Y_c + 1,8311}{Y_w + 1,8311} + 0,15823 \log \frac{Y_c + 4,9930}{Y_w + 4,9930}$$

where:

$$Y = \frac{R}{\lambda} = \left(\frac{273,15^\circ\text{K}}{T} \right)^{1,147} \frac{R_p}{13,42}$$

(R in cm and p in μ Hg). This is the equation used by Roberts and Sydoriak in their paper. With the help of an electronic computer, they have determined the ratio p_c/p_w for different T_c and T_w and for different values of R_p . These evaluations, as said before, have been confirmed by their measurements in He³ down to 1,9°K.

The present situation can then be summarized in the following points:

- a) Equation (2) has been deduced from Weber's general equation and the constants have been fitted with experimental data for He⁴.
- b) Experimental data for He⁴ with $T_c < 4^\circ\text{K}$ do not exist.
- c) Experimental data for He³ fit equation (2) down to 1,9°K, although at this temperature He³ is already far from an ideal behaviour.

Furthermore we want, to point out that Roberts and Sydoriak made their measurements with only one pipe ($R = 0,211$ cm) and that, for given T_c and T_w , in order to change p_c and p_w (and then the product R_p) the total quantity of gas in the apparatus had to be changed.

EXPERIMENTAL DATA -

In order to measure the thermomolecular pressure, we have preferred an experimental situation in which p_c is the vapour pressure of a liquid He^3 bath. In this way the measure of p_c becomes a measure of T . In this case it is not necessary to make any correction for adsorption of He^3 on the walls since the bath acts as an infinite gas source. On the other hand in order to have more data for the same T_c (and then the same p_c) it is necessary to change the radius of the pipe. We used stainless steel pipes of 5 different diameters: $2R = 0.48, 0.31, 0.18, 0.07, 0.03$ cm. The measurements with the last pipe are not very reliable for obvious experimental difficulties: equilibrium times to have a meaningful data were of about 2 hours at the lowest temperatures; the data obtained are in a substantial qualitative agreement with the whole set of measurements, but will not be reported in this report.

The diameters chosen, cover, for every temperature, a range of Rp_w large enough to fit our data in the Weber's theory through the calculations made by Roberts and Sydoriak.

The apparatus consists in a copper cell in which the He^3 is condensed^(o) containing, immersed in the liquid, two carbon resistor thermometers, one used for thermoregulation and the other for the measurement of temperature. The cell is soft soldered to a He^3 refrigerator and communicates with the room temperature section through three pipes of different diameters in parallel: the measure of p_w is made alternatively for each of the three pipes with the same manometers, in the same conditions of temperature and quantity of condensed He^3 . The measurements have been made in different runs, combining the 5 diameters chosen, 3 by 3, in different ways, so to check the data obtained in different runs.

The measurement of T_c (from which p_c is deduced) is performed using two Allen and Bradley carbon resistor thermometers (10 ohm, 1/2 W). They have been calibrated against the He^3 vapour pressure measured statistically in the He^3 refrigerator. The He^3 pumping line has a diameter of 12 mm and the correction for thermomolecular pressure is rather small, down to 0.4°K . Since the thermometers were immersed in a He^3 bath different from the bath whose vapour pressure was measured, a correction was made to take into account the thermal boundary resistance, following the data given by Fairbank and Lee⁽⁷⁾. Once corrected the data fitted very well the relation $\log R = a - b \log p$, obtained by Cunsolo et al.⁽⁸⁾. According to these authors and looking at

(o) - We have used He^3 of the following purity: 99.86% total He, 100.00% He^3 in He, supply by Monsanto Res. Co.

4.

the dispersion of our data, it seems reasonable to assume that the maximum error in the determination of the temperature was $\Delta T = \pm 0.005^\circ\text{K}$. From this, the error in the value of p_c can be deduced. During the measurements the temperature T_c was kept constant with the help of an electronic thermoregulator driven by one of the two carbon resistor thermometers in the cell. The ΔT due to thermoregulation was sure less than $5 \cdot 10^{-4} \text{ }^\circ\text{K}$.

In tables I to IV we have collected the experimental data for the 4 tubes used. The errors in the measurement of p_w are given by the calibration of the manometers (we have used a set of 3 manometers: an Hg, an oil manometer and a McLeod with two capillaries of different sensitivities; all calibrated in a conventional way) and by the precision in reading (this often improved by taking the average of repeated measures): these are surely contained within 0.5% for most of the data; when the error is larger than 0.5%, it is written in the tables.

DISCUSSION -

We soon want to note that it is not possible to overlap our data with those obtained by Roberts and Sydoriak. Measuring the vapour pressure of a liquid bath, we dealt with much higher pressures than those met by Roberts and Sydoriak; this means that at temperatures $T_c \approx 1^\circ\text{K}$ we find that the ratio $p_c/p_w \approx 1$, i. e. a thermomolecular effect, is not measurable.

What we have done has been to calculate p_w from the Weber-Schmidt's formula for pipes of diameters like ours and in Figs. 1, 2, 3 we have plotted the curves so calculated together with our experimental data. It is evident that the differences between calculated and experimental values are clearly away from the experimental errors, which we have calculated in a pessimistic way. These differences cannot even be attributed to an error in the measurement of p_c , e. g. to a bad calibration of the thermometers: we found that, in order to fit our data on the calculated curves we should correct T_c in a way which is different for each diameter of the pipes.

We have tried to justify the quoted differences in the scheme of the Weber's theory, taking into account the non-ideality of He^3 at the lower temperatures. We have calculated again p_c/p_w using equation (2) down to a given temperature; below this temperature we have integrated directly equation (1) using for η and ρ the data existing in the literature. Both η and B (the second virial coefficient, used to calculate ρ) have been computed from the work of Keller et al. (9, 10, 11) and are in good agreement with the measurements of Becker et al. (12). The quoted authors, in order to obtain the values of η and B, have used two different kinds of potential: the Lennard-Jones 12-6 (which they call LJ1) and an exp-6 poten

TAB. I

$$R = 3.5 \times 10^{-2} \text{ cm}$$

$$T_w = 294 \pm 2^\circ\text{K}$$

T_c °K $\Delta T_c = \pm 0.005$	P_c (mm Hg)	P_w (mm Hg)	P_c/P_w
0.805	3.03 ± 0.10	3.05	0.99
0.679	1.16 ± 0.05	1.35	0.88
0.645	0.85 ± 0.04	9.80×10^{-1}	0.88
0.590	0.49 ± 0.02	6.80×10^{-1}	0.74
0.511	0.19 ± 0.01	3.50×10^{-1}	0.55
0.467	$(0.95 \pm 0.09) \times 10^{-1}$	2.20×10^{-1}	0.43
0.440	$(0.61 \pm 0.06) \times 10^{-1}$	1.75×10^{-1}	0.34
0.370	$(0.14 \pm 0.02) \times 10^{-1}$	$(8.58 \pm 0.20) 10^{-2}$	0.17
0.347	$(0.80 \pm 0.10) \times 10^{-2}$	$(6.40 \pm 0.20) 10^{-2}$	0.13

TAB. II

$$R = 9.0 \times 10^{-2} \text{ cm}$$

$$T_w = 294 \pm 2^\circ\text{K}$$

T_c °K $\Delta T_c = \pm 0.005$	P_c (mm Hg)	P_w (mm Hg)	P_c/P_w
0.755	2.12 ± 0.08	2.14	0.99
0.695	1.33 ± 0.05	1.39	0.97
0.580	0.44 ± 0.03	5.20×10^{-1}	0.87
0.540	0.27 ± 0.02	3.70×10^{-1}	0.75
0.521	0.21 ± 0.01	3.00×10^{-1}	0.74
0.504	0.17 ± 0.01	2.55×10^{-1}	0.68
0.450	$(0.73 \pm 0.07) \times 10^{-1}$	1.35×10^{-1}	0.55
0.416	$(0.38 \pm 0.04) \times 10^{-1}$	$(9.20 \pm 0.10) 10^{-2}$	0.41
0.397	$(0.27 \pm 0.03) \times 10^{-1}$	$(6.80 \pm 0.10) 10^{-2}$	0.39
0.378	$(0.17 \pm 0.02) \times 10^{-1}$	$(5.30 \pm 0.10) 10^{-2}$	0.32
0.370	$(0.14 \pm 0.02) \times 10^{-1}$	$(4.40 \pm 0.10) 10^{-2}$	0.32
0.360	$(0.11 \pm 0.02) \times 10^{-1}$	$(4.05 \pm 0.15) 10^{-2}$	0.27
0.349	$(0.84 \pm 0.13) \times 10^{-2}$	$(3.30 \pm 0.05) 10^{-2}$	0.26

6.

TAB. III

$$R = 1.55 \times 10^{-1} \text{ cm}$$

$$T_w = 294 \pm 2^\circ\text{K}$$

T_c °K $\Delta T_c = \pm 0.005$	P_c (mm Hg)	P_w (mm Hg)	P_c/P_w
0.755	2.12 ± 0.08	2.14	0.99
0.695	1.33 ± 0.05	1.36	0.98
0.580	0.44 ± 0.03	5.10×10^{-1}	0.89
0.529	0.23 ± 0.02	2.90×10^{-1}	0.85
0.521	0.21 ± 0.01	2.70×10^{-1}	0.83
0.506	0.17 ± 0.01	2.30×10^{-1}	0.77
0.450	$(0.73 \pm 0.07)10^{-1}$	$(1.20 \pm 0.01)10^{-1}$	0.62
0.416	$(0.38 \pm 0.04)10^{-1}$	$(7.00 \pm 0.04)10^{-2}$	0.54
0.397	$(0.27 \pm 0.03)10^{-1}$	$(5.60 \pm 0.20)10^{-2}$	0.48
0.375	$(0.16 \pm 0.02)10^{-1}$	$(4.10 \pm 0.10)10^{-2}$	0.40
0.370	$(0.14 \pm 0.02)10^{-1}$	$(3.60 \pm 0.10)10^{-2}$	0.39
0.359	$(0.11 \pm 0.02)10^{-1}$	$(3.00 \pm 0.10)10^{-2}$	0.36
0.348	$(0.82 \pm 0.13)10^{-2}$	$(2.65 \pm 0.10)10^{-2}$	0.33

TAB. IV

$$R = 2.4 \times 10^{-1} \text{ cm}$$

$$T_w = 294 \pm 2^\circ\text{K}$$

T_c °K $\Delta T_c = \pm 0.005$	P_c (mm Hg)	P_w (mm Hg)	P_c/P_w
0.759	2.18 ± 0.08	2.20	0.99
0.673	1.10 ± 0.05	1.13	0.97
0.645	0.85 ± 0.04	8.70×10^{-1}	0.97
0.511	0.19 ± 0.01	2.25×10^{-1}	0.85
0.482	0.12 ± 0.01	1.65×10^{-1}	0.78
0.451	$(0.74 \pm 0.07)10^{-1}$	1.07×10^{-1}	0.71
0.427	$(0.48 \pm 0.04)10^{-1}$	$(8.00 \pm 0.20)10^{-2}$	0.62
0.383	$(0.20 \pm 0.02)10^{-1}$	$(4.10 \pm 0.03)10^{-2}$	0.49
0.347	$(0.80 \pm 0.10)10^{-2}$	$(2.20 \pm 0.02)10^{-2}$	0.37
0.333	$(0.54 \pm 0.08)10^{-2}$	$(1.73 \pm 0.02)10^{-2}$	0.32
0.321	$(0.38 \pm 0.06)10^{-2}$	$(1.50 \pm 0.05)10^{-2}$	0.26
0.317	$(0.34 \pm 0.06)10^{-2}$	$(1.30 \pm 0.05)10^{-2}$	0.26

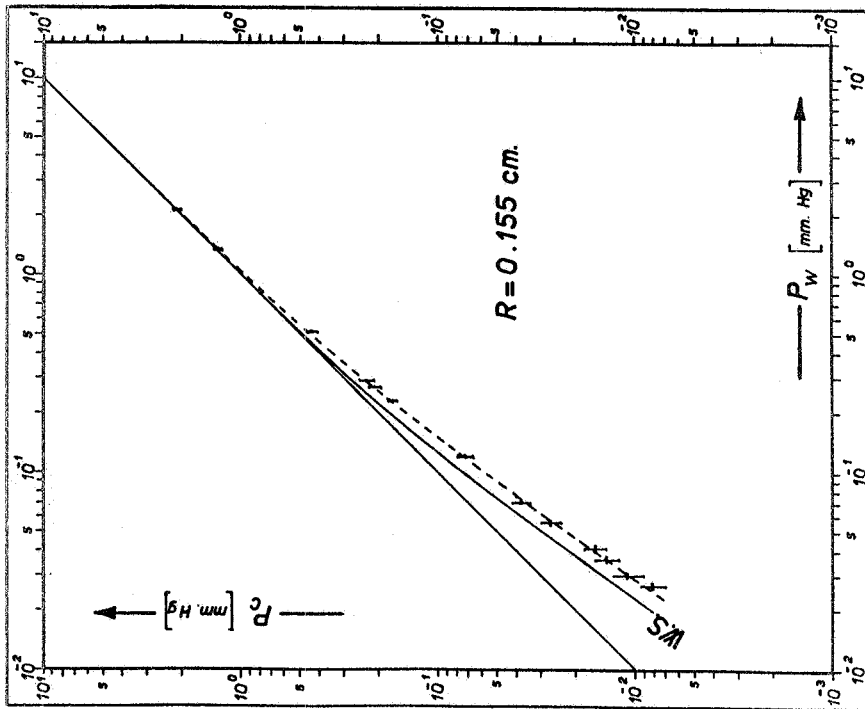


FIG. 1 - He³ thermal molecular pressure for a tube of 0.155 cm Radius. $T_w = 294 \pm 2^\circ\text{K}$; P_c is the vapor pressure of the bath et gives directly T_c (the T_c range is from $\approx 0,35$ to $\approx 0,8^\circ\text{K}$). The straight line is $P_c = P_w$ i.e. absence of thermal pressure; the full line W.S. represents eq. (2); the points with errors and dotted line are the present measurements.

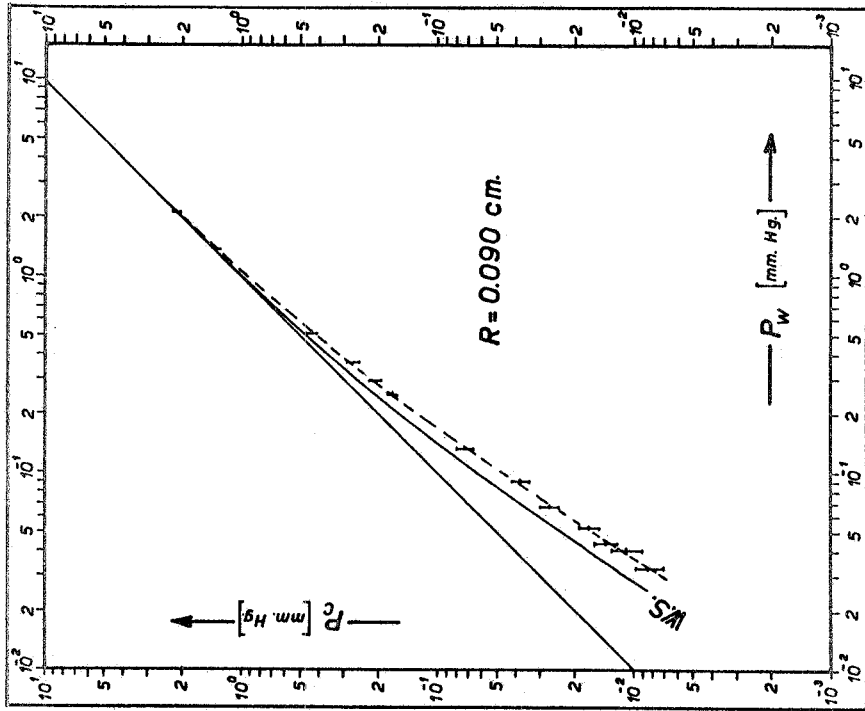


FIG. 2 - He³ thermal molecular pressure for a tube of 0.090 cm Radius. For the specifications see fig. 1.

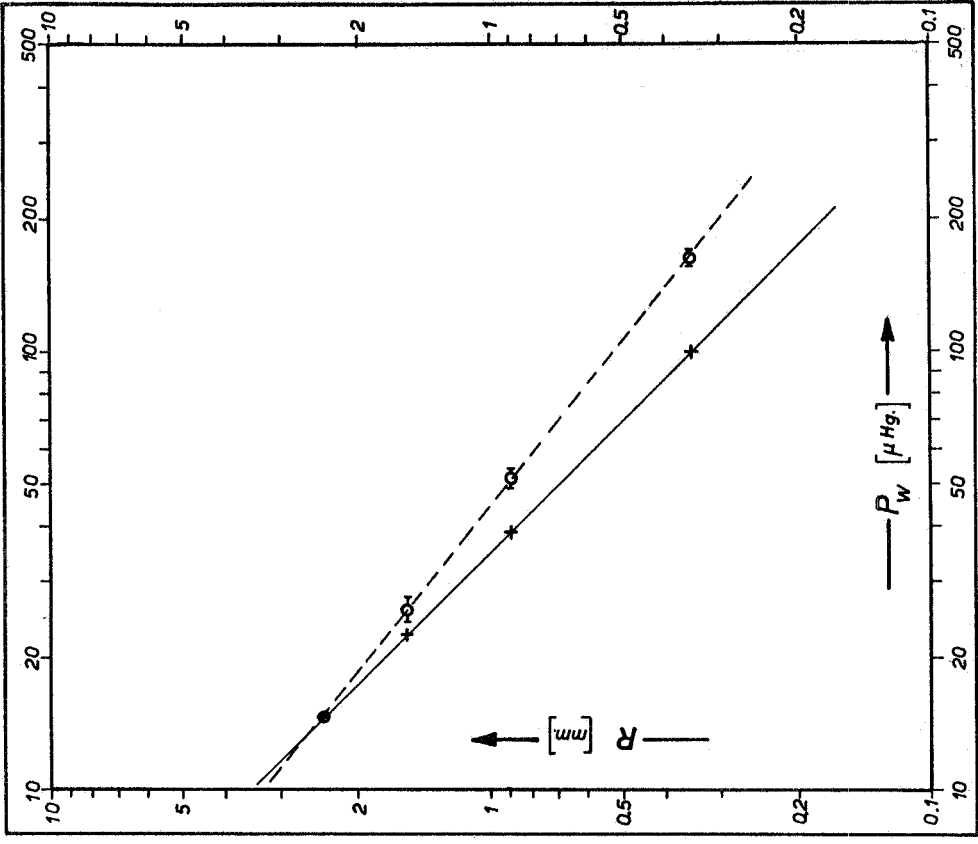


FIG. 4 - $R - P_w$ relations for $P_c/P_c = \text{const.}$ from eq. (2) (full line) and from present experiment (dotted line).

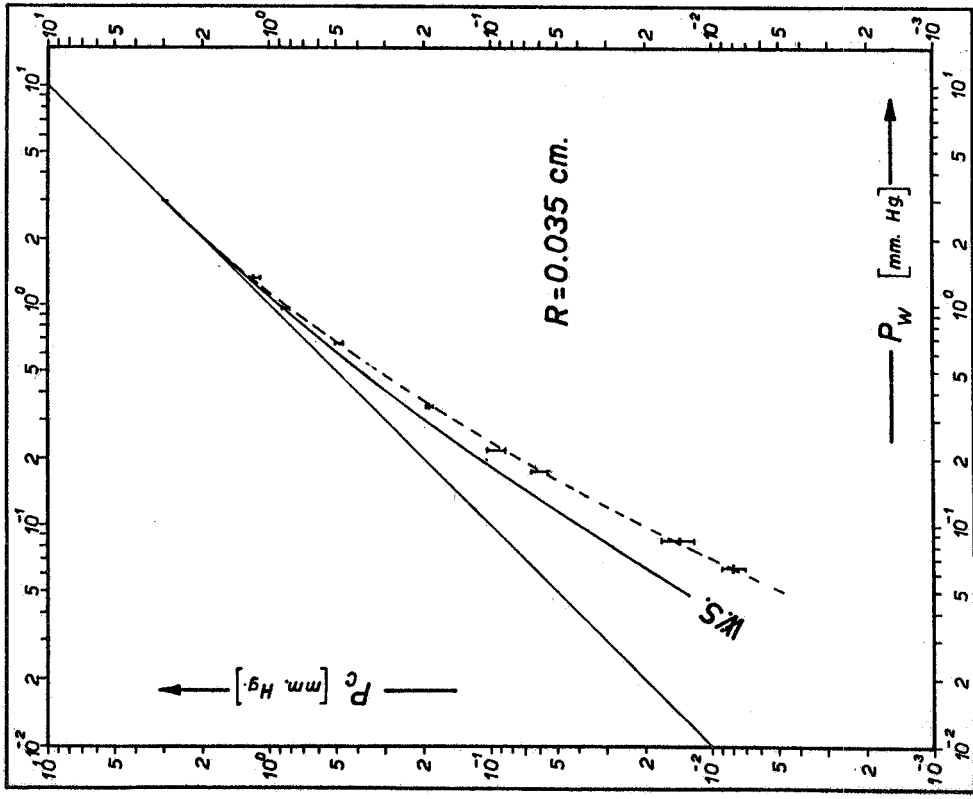


FIG. 3 - He^3 thermal molecular pressure for a tube of 0.035 cm Radius. For the specifications see fig. 1.

tial (indicated by MR5). We have used both sets of values, with a numerical integration of equation (1) which started from T_c going up to 2.2°K ; from this point on we have used equation (2) up to $T_w = 294^{\circ}\text{K}$. The values so obtained are listed in Table V in the columns $p_w(\text{LJ1})$ and $p_w(\text{MR5})$. Furthermore, we have used the same quoted values to integrate equation (1) up to 10°K , still using equation (2) from this point up to (room) temperature: these values are still listed in table V in the column $p_w(\text{LJ1})^x$.

Looking at Table V it is clear that taking into account the real behaviour of He^3 in eq. (1) did not eliminate the sensible differences between the calculated and the experimental values; it must be pointed out however that the correction acted in the proper direction, since the corrected values come closer than the uncorrected ones to the experimental data.

TAB. V

P_w values calculated for T_c and p_c (given in the first and second column) from eq. (2) and eq. (1) in different ways (see in the text) and corresponding $P_w(\text{exp.})$.

$$R = 3.5 \times 10^{-2} \text{ cm}$$

$$T_w = 294 \pm 2^{\circ}\text{K}$$

T_c $^{\circ}\text{K}$	p_c (μHg)	$p_w(\text{R.S.})$	$p_w(\text{MR5})$	$p_w(\text{LJ1})$	$p_w(\text{LJ1})^x$	$p_w(\text{exp.})$
0.35	8.62	37.51	38.45	39.09	47.51	65
0.36	11.17	43.98	-----	-----	52.46	73
0.37	14.30	51.25	-----	-----	58.36	82
0.38	18.11	59.43	-----	-----	64.98	91
0.40	28.12	78.64	78.97	79.24	82.66	113
0.45	72.69	147.53	147.76	147.85	149.10	190
0.55	308.54	421.41	421.90	421.90	421.60	470
0.60	544.49	663.72	664.69	664.69	663.74	700
0.65	893.09	1009.36	1009.76	1009.76	1009.03	1030

At this point one is tempted to make a deeper criticism to the Weber's theory. This is also justified by the following observation about our data. As said before, referring to eq. (2), the ratio p_c/p_w is a function of the variables: Rp_c , Rp_w , T_w and T_c . Looking at our experimental procedure, if we keep constant T_c and T_w , also p_c is fixed. In this case p_c/p_w is a monotonous single-valued function of Rp_w . If we choose any value of the ratio p_c/p_w it follows that $Rp_w = \text{const}$. In fig. 4 we have plotted the hyperbole obtained from eq. (2), as calculated by Roberts and Sydoriak (in fact, the data are taken at slightly different in the range of

$T_c : 0.34 - 0.41^\circ\text{K}$), for the 4 radii which we have used (full line). We have also deduced from our data the values of R and p_w which we obtain keeping fixed $(p_c/p_w)_{\text{exp}}$ in the same temperature interval for T_c and taking one normalization point with the above curve (dotted line). It is easily seen that we do not obtain $(Rp_w)_{\text{exp}} = \text{const}$. The data fit very well a straight line in a $\log R$ vs $\log p_w$ plot and the empirical relation that is obtained is $R^n p_w = \text{const}$, with $n = 1.25$. This result, we hope confirmed by other measurements, demonstrates that it is no longer possible to use Weber's theory by a better fitting of the constants of eq. (2), but that the theory must be modified in its analytical structure.

USE OF THE PRESENT DATA -

With our experimental data we have drawn the graphs of Figs. 5 and 6, which can be used immediately to make thermomolecular pressure corrections as function of T_c and R .

Fig. 5 allows the direct reading of the temperature if one uses a liquid He^3 bath connected to the room temperature manometer through a pipe whose radius equals one of those used by us.

Fig. 6 allows interpolations to make corrections for pipes of different diameters. We would not advise to make extrapolations out of the range of diameters explored by us.

We believe that in the curves drawn in these two figures the errors should be of the order of 8 - 10%.

In Fig. 5 the curve for $R = \infty$ is obviously the vapor pressure curve taken from the Roberts and Sydoriak' T_{62} temperature scale. We want to point out however, that this scale, if the present measurements are taken into account, should be revised, since the thermomolecular pressure corrections made by Roberts and Sydoriak were made on the basis of eq. (2) and then are not the proper ones.

ACKNOWLEDGEMENTS -

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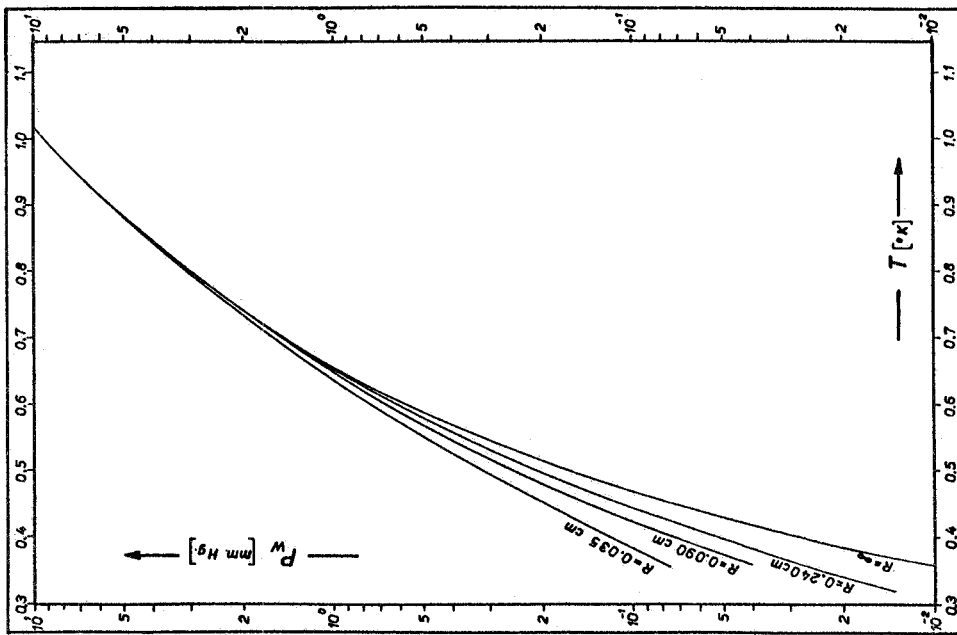


FIG. 5 - P_w - T_c curves from experimental points, for three pipe radius and $T_w = 294 + 20K$. The curve $R = \infty$ is the vapor-pressure curve of Roberts and Sydnor.

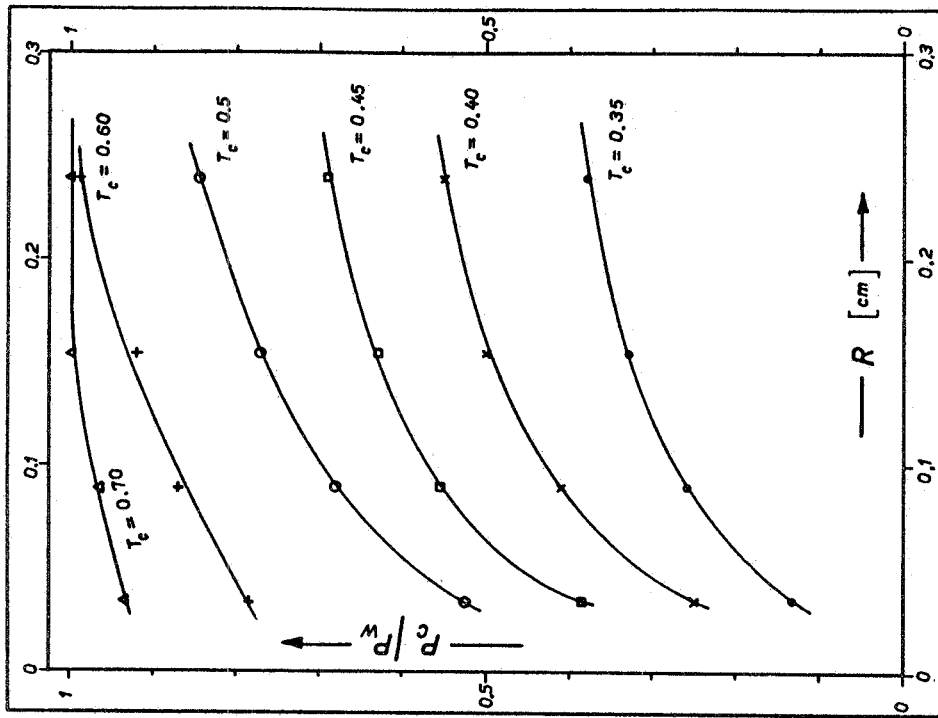


FIG. 6 - P_c/P_w ratios versus pipe radius R for several cold temperatures and fixed $T_w = 294^{\circ}K$.

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