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P. di Vecchia: A NOTE ON DOUBLE BREMSSTRAHLUNG. -

(Nota interna: n. 316)

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The first attempts at monitoring the common volume of the colliding beams in an electron positron storage ring consisted in trying to observe the coincidences caused by two γ-annihilation[(1)]. The experiment, which used lead glass Cerenkov counters to detect γ-rays of more than about 70 MeV energy produced in the collision region by electrons and positrons of about 200 MeV each, revealed the existence of a continuous spectrum for the sum of the energies of the two γ-rays. This continuum was attributed to the process of double bremsstrahlung, viz.:

$$e^+ + e^- \rightarrow e^+ + e^- + 2\gamma \tag{1}$$

It was realized that the γ's of this process should be expected to be emitted mainly into two cones of angular aperture $\Delta\theta = mc^2/E$ (m, E = mass and energy of the electron), both cones centred on the direction of flight of the colliding electrons and positrons. Since process (1) can be conducted in such a way that the four-momentum transfer between the colliding particles could be made arbitrarily small provided that the incident energy is sufficiently large it was concluded that (1) would dominate over two quanta annihilation at very high energies by a factor of the order $\alpha^2(E/mc^2)^2$.

These as yet unproven properties of the process (1) seemed to place it on the list of possible monitoring processes, defined as collision processes between electrons and positrons in which the momentum

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transfer could be considered sufficiently small to justify the assumption, that their description could be trusted to quantum electrodynamics with unit form factors and unmodified propagators. The processes on the list were: backward-forward two quanta annihilation, small angle e^+e^- scattering, single e^+e^- bremsstrahlung.

The following points seemed to be in favour of adopting (1) as a monitoring process:

1) - Its cross section is large compared to the cross section for two quanta annihilation.

2) - The angular dependence of the emitted γ-rays is less critical than in the case of e^+e^- scattering: it is finite in the forward direction, whereas the cross section for scattering is not.

3) - Being observed by a coincidence experiment, its advantage opposite the process of single bremsstrahlung seemed to be that some of the background could be successfully eliminated by using a sufficiently high time resolution of the coincidence.

It must be counted against process (1) that so far neither total nor differential cross section of the process have been determined theoretically.

Bayer and Galitsky[(2)] have given an approximate formula valid for small energies of the emitted γ's: ω_1, $\omega_2 \ll E$. Their result is

(2) $$d\sigma^{(2)} = \frac{8r_o^2\alpha^2}{\pi} \frac{d\omega_1}{\omega_1} \frac{d\omega_2}{\omega_2} \left[\frac{5}{4} + \zeta(3)\right]$$

where $\zeta(3) = \sum_1^\infty n^{-3}$ and $r_o = 2.8 \times 10^{-13}$ cms is the classical electron radius. This result has been checked by the author, who by the use of the Bloch Nordsieck[(3)] method obtained

(3) $$d\sigma^{(2)} = \frac{8r_o^2\alpha^2}{\pi} \frac{d\omega_1}{\omega_1} \frac{d\omega_2}{\omega_2} \left[2 + \zeta(3)\right]$$

The Bayer Galitsky cross section is about 25% less than the one given in (3). It has not been possible to trace this discrepancy between (2) and (3), the latter formula is however borne out by the more detailed calculations presented in the following and by cross checks with the work of Greco on the same subject.

The low energy limit of double bremsstrahlung is not easily accessible to experiment, since we must assume that the background increases rapidly with decreasing energy of the γ's.

In order to obtain information about the high energy behaviour of the process, the special case in which one quantum follows exactly the direction of the electron, the other the direction of the positron has been treated in detail. Only the leading graphs have been considered, so that the following result is valid $0(1/\gamma)$, where $\gamma = E/mc^2$ (γ could be the energy of the final electrons, so that the results presented here will be wrong very near to the "head" of the spectrum). From what was said earlier about the angular distribution, the result should be significant for a detection device of about 1 cm^2 effective area placed at a distance of about 10 metres from the collision region of Adone. (Such a device could be a spark chamber, viewing a larger angular range. The peak cross section could be used as a calibration of this device).

The cross section for the backward forward process is

(4)

$$d\sigma^{(2)*} = \frac{\alpha^2 r_0^2}{\pi^3} \frac{dx_1}{x_1} \frac{dx_2}{x_2} \; \frac{4}{3} (1-x_1)(1-x_2) + (1-x_1)x_2^2 + (1-x_1)x_1^2 + x_1^2 x_2^2 \; d\Omega_1 d\Omega_2 \gamma^4$$

where $x_{1,2} = \omega_{1,2}/E$ and $d\Omega_{1,2}$ are then elements of solid angle of the two γ's. (4) is only valid for

$$d\Omega_{1,2} \ll \frac{\pi}{\gamma^2} \tag{5}$$

The cross section (4) has been checked by using the Bloch Nordsieck method, which gives

$$d\sigma^{(2)*}_{B.N} = \frac{4\alpha^2 r_o^2}{3\pi^3} \frac{dx_1}{x_1} \frac{dx_2}{x_2} \gamma^4 d\Omega_1 d\Omega_2 \tag{6}$$

and it is seen that (6) and (4) agree in the limit $x_{1,2} \rightarrow 0$. A relief map of the energy dependence is given in the figure 1.

A remarkable feature of both the results (3) and (4) is the absence of a logarithmical term of the form $\log 2\gamma$. This puts double bremsstrahlung at a disadvantage opposite the background of chance coincidences due to single bremsstrahlung created either in collision with the molecules of the residual gas or in genuine electron positron collisions(4).

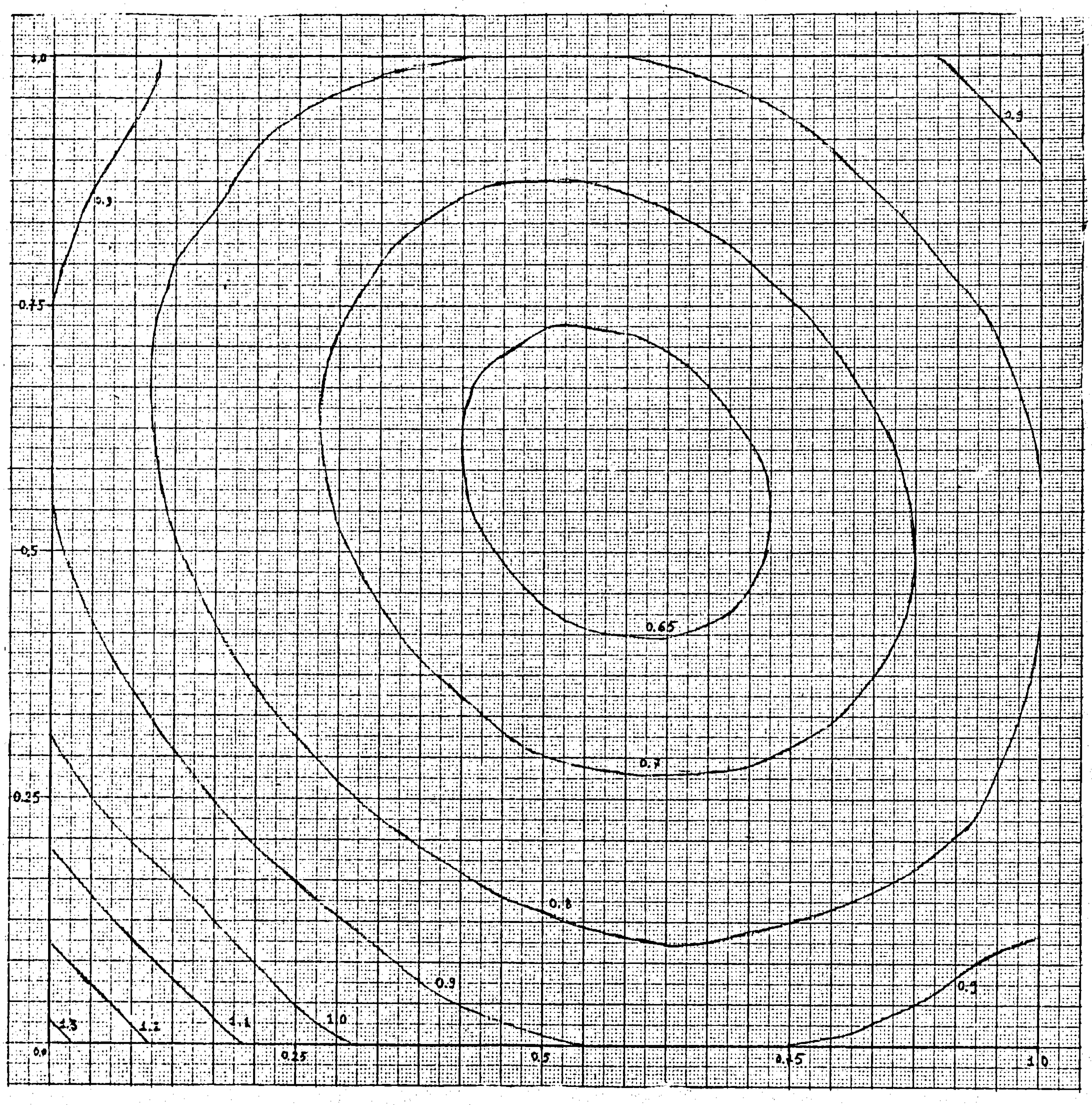

FIG. 1

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(4) - Compare S. Tazzari.