COMITATO NAZIONALE PER L'ENERGIA NUCLEARE
Laboratori Nazionali di Frascati

LNF - 64/25
29 Maggio 1964.

C. Pellegrini and C. Schaerf: SOME CONSIDERATIONS ON THE MAXIMUM AMOUNT OF INFORMATION OBTAINABLE FROM ELECTRON COLLIDING BEAMS EXPERIMENTS.

(Nota interna: n. 245)

Servizio Documentazione
dei Laboratori Nazionali di Frascati del CNEN
Casella Postale 70 - Frascati (Roma)

Laboratori Nazionali di Frascati del CNEN
Ufficio Documentazione

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C. Pellegrini and C. Schaerf: SOME CONSIDERATIONS ON THE MAXIMUM AMOUNT OF INFORMATION OBTAINABLE FROM ELECTRON COLLIDING BEAMS EXPERIMENTS.

The possibility of obtaining informations on the electromagnetic form factors of particles and on the limit of validity of Quantum Electrodynamics by means of electron-positrons colliding beams experiment has been widely discussed(1). We want to add here some remarks showing that, in order to obtain a maximum of information, it is necessary to perform experiments at certain angles and energies with both electron-positron and electron-electron colliding beams. To explain what we mean by maximum information let us consider the three reactions:

(1) $e^+ + e^- \rightarrow a + \bar{a}$

(2) $e^+ + e^- \rightarrow e^+ + e^-$

(3) $e^- + e^- \rightarrow e^- + e^-$

where, for simplicity, we assume a to be a spin zero particle.

After subtraction of the radiative corrections and neglecting the contributions from the two photon channel, the cross section for the three processes can still differ from the one calculated in the first Born approximation because of the form factor of the $\gamma a \bar{a}$ vertex or because of the vacuum polarization effects of the strong interacting particles which modify the photon propagator or because of the breakdown of Quantum Electrodynamics (QED). Following the conventional procedure we introduce a form factor $F_a(q^2)$ for the $\gamma a \bar{a}$ vertex, a form factor $F_e(q^2)$ to describe the effect of a breakdown of QED at the $\gamma e^+ e^-$ vertex(2), a function $P_b(q^2)/q^2$

for a similar breakdown of the photon propagator, and a function $P_V(q^2)/$ $/q^2$ for the change in the photon propagator due to vacuum polarization effects.

The cross sections for the three processes can now be written respectively as:

$$\frac{d\sigma_a}{d\Omega} = \frac{d\sigma_a}{d\Omega}(BORN)\left|F_e(q^2)\right|^2\left|P(q^2)\right|^2\left|F_a(q^2)\right|^2 \tag{4}$$

for process(1), where:

$$P(q^2) = P_V(q^2)\,P_h(q^2)$$

and:

$$\left.\frac{d\sigma}{d\Omega}\right|_{e^+e^-} = \frac{\pi r_e^2 m_e^2}{E^2}\left\{\frac{1}{8}(1+\cos^2\vartheta)\left|G(4E^2)\right|^2 + \right.$$

$$\left. + \frac{1}{4}\,\frac{1+\cos^4\frac{\vartheta}{2}}{\sin^4\frac{\vartheta}{2}}\left|G(-4E^2\sin^2\tfrac{\vartheta}{2})\right|^2 - \frac{1}{2}\,\frac{\cos^4\frac{\vartheta}{2}}{\sin^2\frac{\vartheta}{2}}\,\mathrm{Re}\left[G(4E^2)\,G(-4E^2\sin^2\tfrac{\vartheta}{2})\right]\right\} \tag{5}$$

for processes (2), where:

$$G(q^2) = F_e(q^2)\,P(q^2)\,F_e(q^2)$$

and:

$$\left.\frac{d\sigma}{d\Omega}\right|_{e^-e^-} = \frac{\pi}{4}\,\frac{r_e^2 m_e^2}{E'^2}\left\{\frac{1+\cos^4\frac{\vartheta}{2}}{\sin^4\frac{\vartheta}{2}}\left|G(-4E'^2\sin^2\tfrac{\vartheta}{2})\right|^2 + \right.$$

$$\left. + \frac{1+\sin^4\frac{\vartheta}{2}}{\cos^4\frac{\vartheta}{2}}\left|G(-4E'^2\cos^2\tfrac{\vartheta}{2})\right|^2 + \frac{2}{\sin^2\frac{\vartheta}{2}\cos^2\frac{\vartheta}{2}}\,G(-4E'^2\sin^2\tfrac{\vartheta}{2})\,G(-4E'^2\cos^2\tfrac{\vartheta}{2})\right\} \tag{6}$$

for process (3). $G(q^2)$ is real for $q^2 < 0$; it can be imaginary otherwise.

It is interesting to note that in formula (5) $G(q^2)$ appears for space-like and time-like values of the four-momentum transfer and therefore it is not possible to obtain separate values of:

$$\left|G(4E^2)\right|^2 \text{ and } \left|G(-4E^2\sin^2(\vartheta/2))\right|^2$$

from an analysis of the angular distribution in a way similar to what is done in electron-proton scattering to separate the charge and magnetic form factors of the proton.

With a particular choice of the scattering angles we can obtain from formulas (5) and (6)

$$\left.\frac{d\sigma}{d\Omega}\right|_{e^+e^-}^{\vartheta=\pi} = \frac{1}{4}\pi\,\frac{r_e^2 m_e^2}{E^2}\left\{\left|G(4E^2)\right|^2 + \left|G(-4E^2)\right|^2\right\} \tag{7}$$

for reaction (2) at $\vartheta = \pi$ and

(8) $$\left.\frac{d\sigma}{d\Omega}\right|_{e^-e^-}^{\vartheta=\frac{\pi}{2}} = \frac{9}{2}\pi \frac{r_e^2 m_e^2}{E'^2} G^2(-2E'^2)$$

for reaction (3) at $\vartheta = \pi/2$.

It is now obvious that choosing $E' = \sqrt{2}\,E$ we can obtain from (7) and (8) $G^2(-4E^2)$ and $|G(4E^2)|^2$ and from this the ratio

$$|F_a(q^2)|^2 / |F_e(q^2)|^2$$

for reaction (1).

The interesting point of this note is that in order to measure $G(q^2)$ for time-like values of the four-momentum transfer it seems necessary to have both electron-electron and electron-positron storage rings. Furthermore the electron-electron machine should provide higher energy than the electron-positron one. It looks also very important that in designing the interaction region of the $e^+ e^-$ colliding beams experiments particular care should be devoted to the possibilities of detecting particles scattered at 180^o.

A measurement of the differential cross section for electron--positron scattering at 180^o seems very reasonable with the storage rings which are now under construction. Focusing our attention on the proposed italian storage ring[(3)] (ADONE) and assuming a solid angle of 10^{-2} steradiant for the detection of the scattered particles and a circulating current of 100 mA we obtain 6 events per hour at the highest energy (1.5 GeV each beam). This number, calculated under the hypothesis that all form factors and corrective terms are unity, decreases in inverse proportion to the energy of the colliding particles.

If we assume that all discrepancies from the first Born approximation come from a breakdown of QED of the form:

$$G(q^2) = \frac{1}{1 + \frac{q^2}{Q^2}}$$

it is easy to see that such an experiment with a statistical accuracy of 0.1 will set a lower limit on Q of the order of 10 GeV.

It is a pleasure to thank Prof. R. Gatto and Prof. C. Bernardini for many useful discussions.

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