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N. Cabibbo and G. Da Prato: PION PRODUCTION BY HYGH ENERGY NEUTRINOS.

Nota interna: n° 119
19 Febbraio 1962

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1. It is known that the cross sections of weak processes like

$$(1) \qquad \bar{\nu} + p \rightarrow n + e^{+}$$

increase for low energies like the square of the center of mass energy of the incoming neutrino. At higher energies, owing to the extended structure of the nucleons (which manifests itself by the appearence of form factors) the cross sections approach an asymptotic constant value(1).

The opening of new channels could however still produce an increase of the total cross section of neutrinos on nucleons.

In this present work we give a rough evaluation for single or multiple pion production events like(2)

$$(2) \qquad \nu + N \rightarrow N' + \pi + e^{-} (\mu^{-})$$

$$(3) \qquad \nu + N \rightarrow N' + n\pi + e^{-} (\mu^{-})$$

and similar for antineutrinos. We make use of the peripheral model, in which the relevant graphs for processes (2) and (3) are given respectively in fig. 1 and 2.

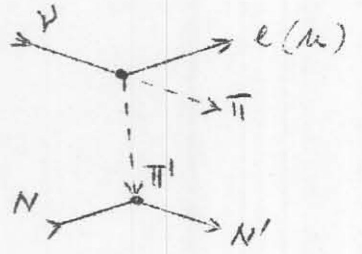

FIG. I

Peripheral graph for reaction (2)

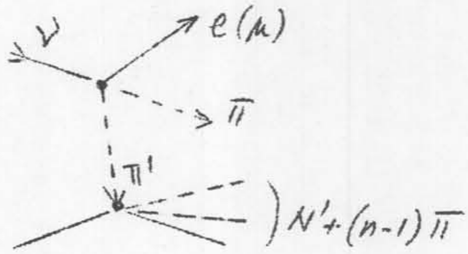

FIG. 2

Peripheral graph for reaction (3)

In both cases the weak vertex involve only vector interactions. The contributions of the axial current through graphs like the one in fig. 3, were not considered. In fact the matrix element derived by this graphs is proportional to the mass of the charged lepton and its contribution is smaller than the contribution of graphs 1 and 2.

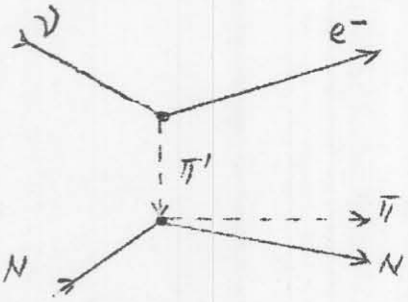

FIG. 3: An axial contribution to process (2)

The top vertex in graps I and 2 represents the weak process

$$\nu + \pi \rightarrow e^- + \pi$$

which will be examined in the next section. The lower vertex represents in fig. 1. the pion nucleon vertex, in fig. 2 a pion nucleon elastic or inelastic collision.

The cross section for process (2) is found to increase logarithmically with the c.m. neutrino energy, while that for process (3) increases quadratically. These results for the asymptotic behaviour of the cross sections are however not reliable. In fact for high energies the main contributions to the total cross section come from highly virtual intermediate pions, a situation in which the peripheral model is expected to break down.

Our numerical results should however give a realistic evaluation for neutrino energies in the laboratory up to a few GeV, the region which is relevant for the present experimental situation.

In this region the cross sections for the considered inelastic processes result much smaller than those for "elastic" processes like (1).

2. In this section we evaluate the cross section for the process

$$(4) \qquad \nu + \pi^0 \rightarrow e^- + \pi^+$$

which appears as upper vertex in our graphs 1 and 2. The matrix element for this process is given by

$$(5) \qquad iG2\pi\sqrt{2}\left(\bar{u}(e)\gamma^\mu \frac{1+\gamma_5}{2} u(\nu)\right)\langle\pi^+/j_\mu^{(V)}/\pi^0\rangle$$

Under the conserved vector current hypothesis the matrix element $\langle\pi^+/j_\mu^{(V)}/\pi^0\rangle$ can be expressed by means of the electromagnetic form factor of the pion.

$$(6) \qquad (\pi^+/j_\mu^{(V)}/\pi^0) = \frac{1}{\sqrt{2}}(\pi_\mu^0 + \pi_\mu^-)\, F_\pi(k^2)$$

where k^2 is the momentum transfer to the pion: $k_\mu = \pi_\mu^+ - \pi_\mu^0$

The cross section is then given (neglecting the electron mass) by

$$(7) \qquad \sigma_s(u^2) = \frac{G^2}{4\pi\chi^2 u^2}\int_0^{4\chi^2} |F_\pi(k^2)|^2\left[(u^2-m_\pi^2)^2 - u^2k^2\right] dk^2$$

Where u is the total energy in the center of mass system, χ is the common value of the momentum of the incoming particles, m_π is the pion mass.

We have evaluated numerically (7) using for $F_\pi(k^2)$ the formula suggested by Bowcock, Cottingham and Lurie[3])

$$(8) \qquad F_\pi(k^2) = \frac{\Gamma_r + \gamma\, m_\pi^3}{\Gamma_r + k^2 + \gamma q^3}$$

where $$q = \frac{1}{2}\left(+k^2 + 4m_\pi^2\right)^{\frac{1}{2}}$$

the parameters where assumed to be: tr = 22.4 m_π^2 ,

$$\gamma = 3{,}2\cdot 10^{-3}\ m_\pi^{-1}$$

We can safely neglect the small width γ , and integrating directly (4) we obtain

$$(9) \qquad \sigma_1/u^2/ = \frac{G^2 t_r}{\pi}\left(1 - \frac{t_r}{4\chi^2}\log\left(1 + \frac{4\chi^2}{t_r}\right)\right)$$

The cross section tends at high energy to $\sigma(\infty) = G^2 t_r/\pi \cong$ $\cong 7 \times 10^{-39}$ cm^2.

Under the hypothesis of the conserved vector current the following equalities hold:

$$(9') \quad \sigma_{\nu+\pi^0\to e^-+\pi^+} = \sigma_{\nu+\pi^-\to e^-+\pi^0} = \sigma_{\bar\nu+\pi^+\to e^++\pi^0} = \sigma_{\bar\nu+\pi^0\to e^++\pi^-}$$

In the case of muons one gets the same results, neglecting contributions proportional to the muon mass. This is a good approximation in the high energy region.

3. Possible processes of kind 2 are:

2a) $\nu + p \to e^- + \pi^+ + p$	2d) $\bar\nu + p \to e^+ + \pi^0 + n$
2b) $\nu + n \to e^- + \pi^+ + n$	2e) $\bar\nu + p \to e^+ + \pi^- + p$
2c) $\nu + n \to e^- + \pi^0 + p$	2f) $\bar\nu + n \to e^+ + \pi^- + n$

The cross sections for these processes as derived from graph 1 are related:

$$\sigma_a = \sigma_b = \frac{1}{2}\sigma_c = \frac{1}{2}\sigma_d = \sigma_e = \sigma_f .$$

so that we can give the results for the first

$$\sigma_a(W) = \frac{g_1^2}{8(2\pi)^2 k^2 W^2}\int_{m_\pi^2}^{(W-M_p)^2} du^2\,\sigma_1(u^2)\chi u \int_{(\Delta_1)^2}^{(\Delta_2)^2} d\Delta^2 \frac{\Delta^2}{(\Delta^2+\mu^2)^2}$$

Where the integration limits for Δ^2, which represents the square of the momentum exchanged through the pion, are:

$$(\Delta_{1,2})^2 = 2K_1^0 K_2^0 - 2M_p^2 \mp 2K/K_2/$$

g_1 is the renormalized pion nucleon coupling constant, W the total energy in the c.m. system, k_1^0 and k_2^0 are the energies of the incoming and outcoming protons k and $|k_2|$ their momenta all in the center of mass system. $\sigma_1(\mu^2), \chi, \mu$ have been defined in section 2. Numerical results are given in fig. 4.

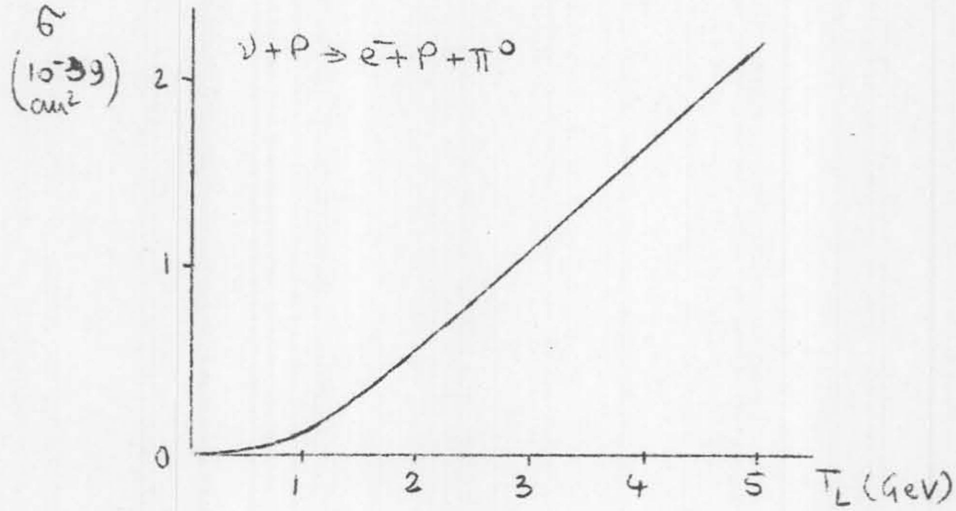

FIG. 4

The asymptotic limit for very high energy is formed to be

$$\sigma(W) \underset{W\to\infty}{\rightarrow} \frac{\sigma_1(\infty)}{4\pi}\left(\frac{g}{4\pi}\right)\left[\log\left(\frac{W}{\mu}\right) - 2{,}7\right]$$

This formula is in good agreement with the numerical results above $\sim$ IO GeV.

4. In this section we consider the total cross section for the ν + N processes in which more then one pion is produced. We will evaluate this cross section by using graph n° 2; since in the lower vertex we introduce the total pion nucleon cross section, our procedure takes into account a whole set of reactions like

$$\begin{aligned} \nu + N &\rightarrow \pi + e^- + N' + \pi \\ &\rightarrow \pi + e^- + N' + \pi \\ &\rightarrow \pi + e^- + \Sigma + K \end{aligned}$$

etc.

For each possible initial state there are two possibilities for the pion emitted in the higher vertex and the virtual pion, as shown in table I.

TABLE I

INITIAL STATE	(weak) higher vertex emit. pion	virt. pion	Lower vertex	σ_2
$\nu + p$	π^+	π^0	$\pi^0 + p$	$\frac{5}{3}\sigma_{\frac{3}{2}} + \frac{1}{3}\sigma_{\frac{1}{2}}$
	π^0	π^+	$\pi^+ + p$	
$\nu + n$	π^+	π^0	$\pi^0 + n$	$\sigma_{\frac{3}{2}} + \sigma_{\frac{1}{2}}$
	π^0	π^+	$\pi^+ + n$	
$\bar{\nu} + p$	π^-	π^0	$\pi^0 + p$	$\sigma_{\frac{3}{2}} + \sigma_{\frac{1}{2}}$
	π^0	π^-	$\pi^- - p$	
$\bar{\nu} + n$	π^-	π^0	$\pi^0 + n$	$\frac{5}{3}\sigma_{\frac{3}{2}} + \frac{1}{3}\sigma_{\frac{1}{2}}$
	π^0	π^-	$\pi^- + n$	

We sum over the two cases, disregarding interference terms. The total cross section is given by

$$\sigma(W) = \frac{\pi}{(2\pi)^4 k^2 W^2} \int \frac{\rho\,\omega\,\chi\,u\,\sigma_1(u^2)\,\sigma_2(\omega^2)}{(\Delta^2+\mu^2)^2}\, d\omega\, du^2\, d\Delta^2$$

$\sigma_1(u^2)$ is the $\nu + \pi$ cross section defined in section 2, χ, u, k, W and Δ^2 have the same meaning that in section 3. The integration limits $\Delta_{1,2}$ are $\Delta_{1,2} = 2k_1^0 k_2^0 - w^2 - M^2 \mp 2K|K_2|$. $\sigma_2(\omega)$ is a combination of the $T = \frac{1}{2}$ and $T = \frac{3}{2}$ total pion nucleon cross sections, defined in table I for the different initial states. K_{10} is the c.m. energy of the target nucleon $\vec{K}_2$ and K_{20} are the momentum and energy in the c.m. system of the group of particles coming out of the lower vertex in fig. 2.

We have performed analitically integration oner Δ^2 and numerically the other two(4). The results are given in fig. 5 sin-

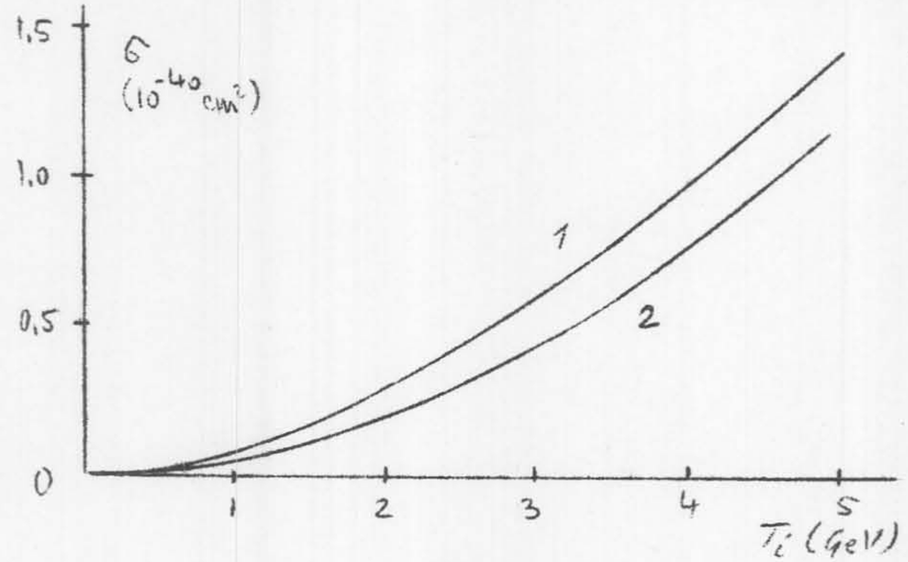

FIG. 5

ce both σ_1 and σ_2 go to constant values for high energy we can get the following asymptotic formula:

$$\sigma(W^2) \underset{W \to \infty}{\to} \frac{\sigma_1(\infty)\,\sigma_2(\infty)}{24(2\pi)^3} W^2$$

This formula is however not reliable for very high energies where very high momentum transfers are involved and the peripheral model is expected to break down.

The calculation presented here suggests that in the energy range up to a few GeV, inelastic processes of the kind considered here should be less frequent (by at least a factor ten) than elastic ones like 1.

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(2) - See also S.M. Berman, in the International Conference on Theoretical Aspects of very high energy phenomena (1961)

(3) - Bowcock, Cottingham and Soriè: Nuovo Cimento 16, 918 (1960)

(4) - $\sigma_2(\omega^2)$, was taken from P. Falk-Vairant and G. Vallardas: Centre d'Etudes Nucleaires de Saclay, rapport à la Conference de Rochester (1960).