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PROTON-ANTIPROTON ANNIHILATION INTO ELECTRONS MUONS AND
VECTOR BOSONS.

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Abstract.

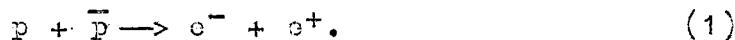
The possibility of achieving relatively high intensity anti-proton beams has prompted some considerations on the rather rare annihilation channels of the proton-anti-proton system. We propose (i) to study the two electron mode as a means of investigating the electromagnetic structure of the proton for time-like momentum transfers; (ii) to study the two muon mode and compare with the two electron mode to investigate whether the muon behaves like a heavy electron for large time-like momentum transfers; (iii) to investigate the existence of weak vector bosons by the modes $p + \bar{p} \rightarrow B + \bar{B}$ and $p + \bar{p} \rightarrow B + \pi$. Although no precise theoretical predictions can be made, crude estimates indicate that the cross-section for these four channels could be roughly of the same order of magnitude.

(°) - CERN - Geneva - Switzerland.

I. The electromagnetic annihilation $p + \bar{p} \rightarrow e^+ + e^-$, $p + \bar{p} \rightarrow \mu^+ + \mu^-$.

One of the significant programmes in high-energy physics has been the systematic study of the electromagnetic structure of nucleons carried out by Hofstadter¹⁾ and co-workers, and by Wilson²⁾ and co-workers. The theoretical explanation of these experiments has been one of the outstanding problems in the theory of strong interactions and has led to many new and interesting ideas³⁾. These experiments measure the form factors of the nucleon for space-like momentum transfers where the form factors are real and apparently decreasing with increasing momentum transfers up to highest values thus far measured of order $q^2 \approx 2(M)^2$ ($M \equiv$ nucleon mass).

The advent of antiproton beams of relatively high intensity ($\approx 10^4$ particles per pulse) allows the possibility of further investigation of the electromagnetic structure of the proton in a region thus far completely unexplored. This is accomplished by the study of the reaction



Reaction (1) is the inverse of proton anti-proton pair production from electron-positron clashing beams⁴⁾.

Figures (1a) and (1b) show the diagrams for proton-electron scattering and proton-antiproton annihilation into an electron pair respectively in the one-photon channel.

For the proton-electron scattering experiment the four momentum carried by the photon is purely space-like, i.e. $q^2 > 0$, whereas in the annihilation the photon four momentum is purely timelike, $q^2 < -4M^2$. This is clearly demonstrated in the c.m. of target and projectile, in which case the four-momentum transfer has only space components for the scattering experiment and only a time component for the annihilation.

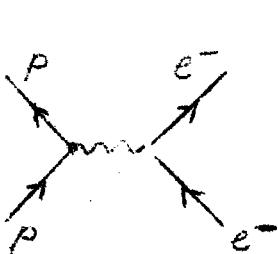


Fig. (1a)

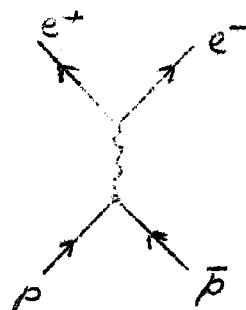


Fig. (1b)

The momentum transfer for process (1) is determined uniquely by the antiproton energy E in the laboratory system, $q^2 = -2M(E + M)$. Beginning at $q^2 = -4M^2$, when the antiproton is at rest, the momentum transfer continues to as negative a value of q^2 as can be achieved with the highest possible antiproton energy.

At the present time there exist no reliable theories for the behaviour of the form factors for time-like momentum transfers. Nevertheless we would like to propose the study of reaction (1). This programme will allow the investigation (just as in the space-like experiments) of whether the proton has a core-like structure for large momentum transfers, or whether it has a broad and complex structure.

Whereas in the space-like experiments the form factors are given the physical interpretation of the Fourier transforms of the spacial charge and magnetic structure of the proton, the time-like momentum transfers yield information about the frequency structure of the proton. For $q^2 < 0$ the 'cloud' around the proton could have various kinds of resonance structure such as the ρ and ω mesons. It would be of great interest to explore this region to see if this kind of structure is simple, i.e. one or two resonances with a more or less constant continuum, or whether more structure appears as the momentum transfer continues to larger negative values.

It appears that with existing machines such as the P.S. at CERN an antiproton beam of 3 GeV/c can be readily achieved. With an antiproton beam of this momentum it is possible to look at momentum transfers as negative as (-8.7 M^2) which is much larger in absolute value than presently possible in the space-like experiments. An experimental investigation in these directions is being undertaken at Cern⁵.

There is also the process



which occurs with the same differential cross-section, in the one-photon channel, as the electron pair, providing we neglect terms of order $(M_e/M)^2$, $(M_\mu/M)^2$ and treat the muon simply as a heavy electron. An accurate measurement of the ratio of muon pairs to electron pairs would give information on either the muon or electrodynamics in a region which has never been explored by any kind of muon experiment.

The general form for the matrix element of one photon interacting with a proton and an antiproton is written in the usual manner as

$$\bar{u}_{\bar{p}} [F_1 g_\mu + \frac{F_2}{2M} \delta_{\mu\nu} g_\nu] u_p$$

where the form factors are functions of the momentum transfer q^2 , and where $\delta_{\mu\nu} = (g_\mu g_\nu - g_\nu g_\mu)/2$; $F_1(0) = e$; $F_2(0) = e M_p$; and $q = \frac{P_{\bar{p}}}{P_p} + P_p$ [$P \equiv$ four-momentum]. In the time-like region both F_1 and F_2 can become complex, whereas they are real for space-like momentum transfers. With the above expression for the $p\bar{p}$ matrix element the differential cross-section for the two-electron annihilation channel can be written in the one-photon exchange approximation in the following forms:

a) center of mass system

$$\frac{d\sigma(p\bar{p} \rightarrow ee)}{d(\cos\theta_c)} = \frac{\pi \alpha^2}{8 EP} \left[|F_1 + F_2|^2 / (1 + \omega^2 \theta_c^2) + \frac{|M| F_1 + \frac{E}{m} F_2|^2 \sin^2 \theta_c}{E^2 + \frac{E}{m} F_2 / \sin^2 \theta_c} \right] \quad (3)$$

where E = c.m. energy of \bar{p}

P = c.m. momentum of \bar{p}

and θ_c = angle between e^- and \bar{p} in c.m.

Laboratory system -

b) Laboratory system

$$\frac{d\sigma(p\bar{p} \rightarrow ee)}{d\Omega} = \left(\frac{\alpha^2}{E(E+M)} \right) \left(\frac{1}{4} \right) \left(\frac{E_e^2}{\rho^2} \right) \frac{E_e^2}{m^2} \left[2|F_1 + F_2|^2 + \right. \\ \left. + \omega^2 (\theta/2) \left\{ |F_2|^2 \rho^2 E^2 - \frac{2M}{(E+M)} (|F_1|^2 - |F_2|^2) \right\} \right] \quad (4)$$

$$\frac{d\sigma(p\bar{p} \rightarrow ee)}{d\Omega} = \left(\frac{\alpha^2}{4mE} \right) \left(\frac{E_e}{E+M - \rho \cos \theta_0} \right) \left[2|F_1 + F_2|^2 - \right. \\ \left. - \left(\frac{\rho}{E+M} \right)^2 \delta(\theta_0) \left\{ |F_1|^2 - |F_2|^2 - \left(\frac{E-M}{2m} \right) |F_2|^2 \right\} \right] \quad (5)$$

where E = antiproton lab. energy, ρ = antiproton lab. mom., and

$$E_e = \text{electron energy} = \frac{M}{1 - \frac{\rho \cos \theta_0}{E+M}} = \frac{E+M}{2} \left[1 \pm \sqrt{1 - \frac{4M}{(E+M)(1-\cos \theta_0)}} \right]$$

$$\delta(\theta_0) = \frac{2M(E+M) \sin^2 \theta_0}{[E+M - \rho \cos \theta_0]^2}$$

The angles θ_0 and θ are shown in fig. (2).

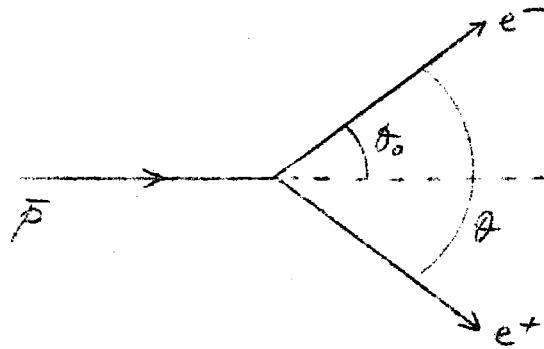


FIG. 2

c) Total cross-section

$$\begin{aligned} \sigma_T(p\bar{p} \rightarrow ee) = & \left(\frac{\pi \alpha^2}{2m^2} \right) \left(\frac{\varepsilon + M}{\varepsilon - M} \right)^{\frac{1}{2}} \left[\left(\frac{2M}{\varepsilon + M} \right) / F_1 + F_2 \right]^2 + \\ & + \frac{1}{3} \left[\left(\frac{\varepsilon - M}{\varepsilon + M} \right) \left(\frac{1}{F_2} \right)^2 - \left(\frac{2M}{\varepsilon + M} \right) / F_1 \right] \end{aligned} \quad (6)$$

where ε = antiproton lab. energy.

In the above expression for the cross-section terms of the order $(Mc/M)^2 \approx 2 \times 10^{-7}$ have been neglected. For the muon pair channel we give the exact expression (not neglecting the muon mass) which in the c.m. system takes the form
(6)

$$\begin{aligned} \frac{d\sigma(p\bar{p} \rightarrow \mu\bar{\mu})}{d(\cos\theta)} = & \frac{\pi \alpha^2}{8EP} \beta_\mu \left[\left(F_1 + F_2 \right)^2 / 2 - \beta_\mu^2 \sin^2 \theta_c \right] + \\ & + \left[\frac{m}{\varepsilon} F_1 + \frac{\varepsilon}{m} F_2 \right] \left(1 - \beta_\mu^2 \cos^2 \theta_c \right) \end{aligned} \quad (7)$$

where E and P are the c.m. energy and momentum of the antiproton and β_μ is the velocity of the muon in the c.m. system. For $\beta_\mu = 1$ (7) reduces to (3).

The total cross-section from (7) is

$$\tilde{\sigma}_T(p\bar{p} \rightarrow \mu\bar{\mu}) = \frac{1}{2} \beta_\mu (3 - \beta_\mu^2) \tilde{\sigma}_T(p\bar{p} \rightarrow ee)$$

We see from this equation that

$$\frac{\tilde{\sigma}_T(p\bar{p} \rightarrow \mu\bar{\mu})}{\tilde{\sigma}_T(p\bar{p} \rightarrow ee)} = 1 - \left(\frac{3}{8}\right) \left(\frac{m_\mu}{\varepsilon}\right)^4 + O\left[\left(\frac{m_\mu}{\varepsilon}\right)^6\right] \quad (8)$$

and that no terms of order $\frac{M^2}{\mu}$ appear in the total cross-section. We have neglected in eq. (7) and (8) the radiative corrections which could be appreciable in this case because of the large momentum transfers involved.

By plotting the differential cross-section as a function of $\omega t^{1/2}$ we see by eq (4) that one does not determine the complex form factors F_1 and F_2 separately but only the combinations

$$2/F_1 + F_2/2 \text{ and } |F_2|^2 \frac{p^2}{\varepsilon^2} - \frac{2M}{\varepsilon+M} (|F_1|^2 - |F_2|^2)$$

The fact that the form factors are complex introduces an azimuthal dependence in the differential cross-section for polarized proton target or for polarized antiproton beam. If \vec{P} is the polarization vector of the proton for polarized target, or anti-proton for polarized beam, and \vec{n} a unit vector in the direction $\vec{p} \times \vec{e}^-$ the differential cross section takes the form in the c.m. system

$$\frac{d\sigma}{d(\cos\theta_c)} = \left[\frac{d\sigma}{d(\cos\theta_c)} \right]_{\text{unpol.}} \pm \frac{e}{M} \left(\frac{p}{\varepsilon} \right)^2 I_m(F_2^* F_2) \sin 2\theta_c / (\vec{p} \cdot \vec{n}) \quad (9)$$

where the upper sign is for polarized antiprotons and the lower sign for polarized target protons.

A numerical estimate of the cross-section depends very sensitively on the values of the form factors F_1 and F_2 . Since there exists no reliable theory of these quantities in the time-like region, we can only give a very rough idea of what the cross-section might be. For example, we might choose the value

i) point proton

$$F_1 = e \quad F_2 = 1.79 e$$

ii) extrapolation of resonance fits of space-like experiments to time-like region⁷⁾

$$F_1 = \left(1 - \frac{1.18 g^2}{q^2 + 30 m_\pi^2}\right) e \quad F_2 = 1.79 \left(1 - \frac{1.54 g^2}{q^2 + 30 m_\pi^2}\right) e$$

In these examples we have assumed F_1 and F_2 real.

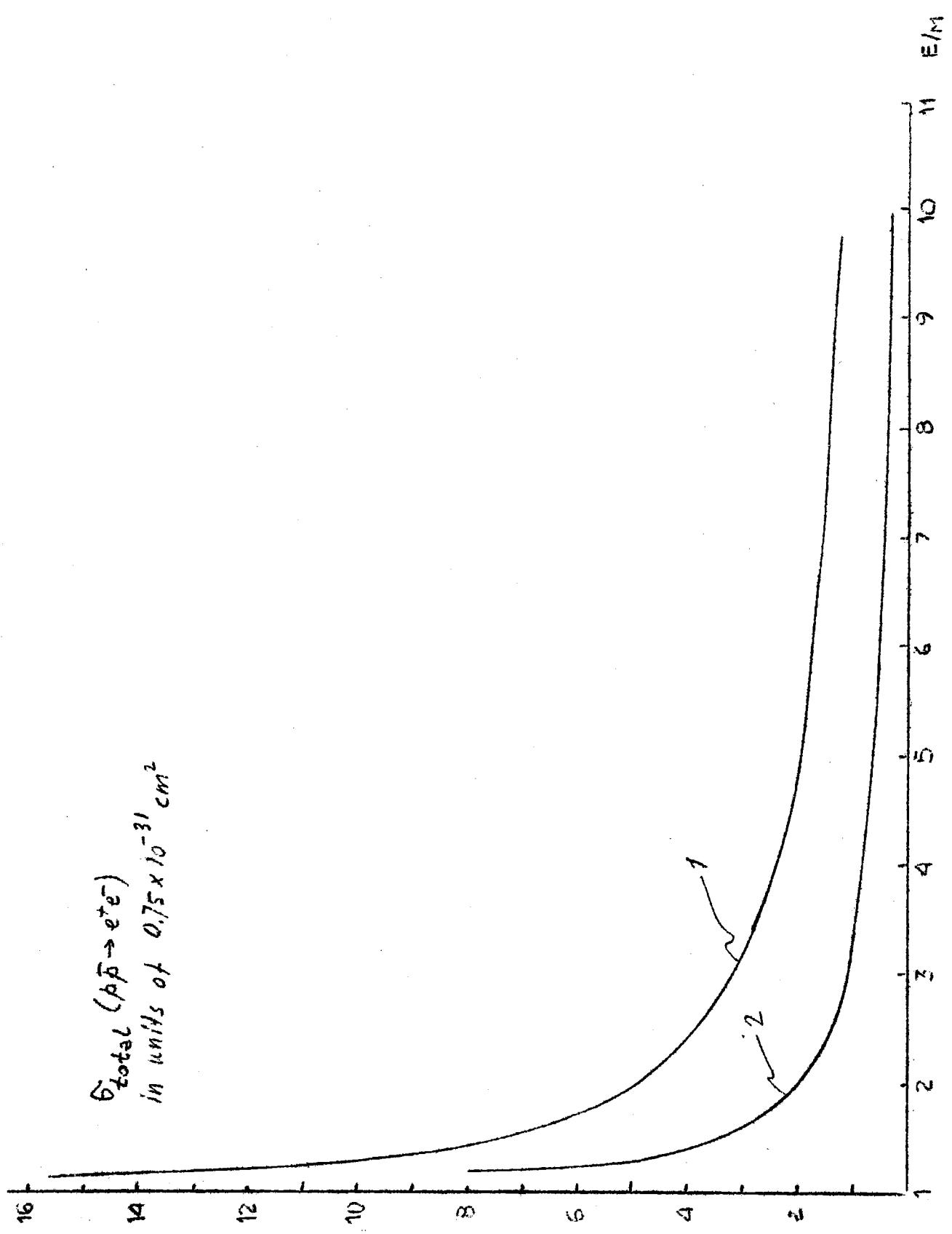
Since the peak of the pion resonance fits to the space-like form factors occurs far from the region of interest in this experiment, the imaginary parts in choice ii) give very small contributions. On the other hand, it is not known whether there are other resonances for larger time-like momentum transfers than the two-pion resonance, say, near $q^2 = -6 M^2$. Should this be the case, there could be very large contributions to the cross-section from both the real and the imaginary parts of the form factors.

If the form factors decrease fairly rapidly in the time-like region, then, just as in the space-like region, it is possible that the two-photon exchange might become important. However, if the form factors do not decrease rapidly for time-like momentum transfer, then the one-photon exchange would be dominant.

If the electron and the positron are detected in a manner which does not distinguish charge and which is symmetric under the interchange of positron and electron, then the interference term between the one and the two photon channel will not contribute to the differential cross-section⁸⁾. This symmetry between e^+ and e^- can then be used either to eliminate or detect the influence of the two photon exchange on the nucleon electromagnetic structure.

Figure 3 shows how the total cross-section varies with antiproton energy for the above two assumptions for the form factors.

FIG. (3)



We emphasize that this graph is not a theoretical prediction but a very crude guess for the cross-section which in fact could very well be ten times bigger or ten times smaller than the estimate given here.

An experiment on the annihilation at rest would involve the branching ratio for the electromagnetic modes to the total annihilation rate. In order to go from this experimental number to the evaluation of the form factors either the atomic physics of the capture must be eliminated or a separate experiment to determine the complex s-wave phase shifts in $p\bar{p}$ elastic scattering must be performed. Note that $2e$ (or 2μ) annihilation through the one-photon channel can only occur, in general, from $3S_1$ and $3D_1$. In view of these difficulties it appears that the results of the in-flight experiment can be interpreted in a much more unambiguous manner.

However, for the determination of the 2μ to $2e$ ratio, and the consequent exploration of the validity of electrodynamics, formula (8) also applies to annihilation at rest.

II) The annihilation into intermediate vector bosons.

In this section we consider the possibility of detecting the intermediate vector meson of weak interactions from proton-antiproton annihilation. Vector mesons with semiweak coupling have been suggested as intermediate agents of weak interactions^{9) 10)}. Production of such mesons from high energy neutrino beams¹¹⁾, from pion beams¹²⁾, from photon beams¹³⁾, and by electromagnetic pair production¹⁴⁾ has been recently considered. Intermediate vector mesons will decay through their semi-weak coupling in a time $\sim 10^{-17}$ sec.

We shall first discuss the annihilation mode of a proton-antiproton system into a pair of such intermediate vector mesons (that we denote by B)

$$p + \bar{p} \rightarrow B + \bar{B} \quad (10)$$

Figure (4) shows the diagram for (10) in the lowest order

of electromagnetic coupling

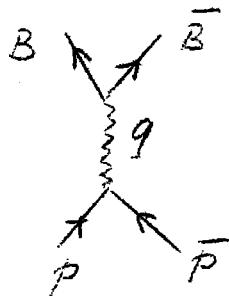


FIG. 4

The most general form of the electromagnetic vertex for a spin one boson is, on invariance grounds,

$$J_\mu = G_1(\epsilon_1, \epsilon_2) p_\mu + [G_1 + \mu G_2 + \epsilon G_3] [(q\epsilon_1) \epsilon_{2\mu} - (q\epsilon_2) \epsilon_{1\mu}] + \epsilon G_3 m_B^{-2} [(q\epsilon_1)(q\epsilon_2) - \frac{1}{2} g^2 (\epsilon_1 \epsilon_2)] p_\mu \quad (11)$$

where p is the difference of the final four-momenta of B and \bar{B} , ϵ_1 and ϵ_2 are the polarization vectors of B and \bar{B} , m_B is the mass of B , $\mu + \epsilon$ is a possible anomalous magnetic moment of B and 2ϵ a possible anomalous electric quadrupole moment. The form factors G_1 , G_2 and G_3 depend on the squared momentum transfer q^2 .

We also define the bilinear combinations

$$R = \frac{1}{2} [G_1 + \mu G_2 + \epsilon G_3]^2 / \left(\frac{\epsilon}{m_B} \right)^2$$

$$S = \frac{1}{2} [G_1 + 2 \left(\frac{\epsilon}{m_B} \right)^2 \epsilon G_3]^2 + \frac{1}{4} [G_1 + 2 \left(\frac{\epsilon}{m_B} \right)^2 \mu G_2]^2$$

The general expression for the cross section of (10) is then given in c.m. by

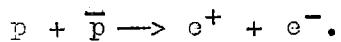
$$\frac{d\sigma(p\bar{p} \rightarrow B\bar{B})}{d(\cos\theta)} = \frac{\pi\alpha^2}{2EP} \beta_B^3 [R(A+B) + SA + (S-R)(B-A)\cos^2\theta] \quad (12)$$

$$\hat{\sigma}_T(B\bar{B}) = \frac{\pi\alpha^2}{3EP} \beta_B^3 (2A+B)(2R+S) \quad (13)$$

In (12) and (13) β_B is the velocity of B, and

$$A = \frac{1}{2} / F_1 + F_2 /^2 \quad \text{and} \quad B = \frac{1}{2} / \frac{M}{E} F_1 + \frac{E}{M} F_2 /^2$$

are exactly the same combinations of the nucleon form factors that determine the angular distribution of



Similarly, $2A + B$ in (6) also determines the total cross-section for $p + \bar{p} \rightarrow e^+ + e^-$. One thus finds for the ratio of $B\bar{B}$ annihilation to e^+e^- annihilation

$$\beta = \frac{\hat{\sigma}_T(p\bar{p} \rightarrow B\bar{B})}{\hat{\sigma}_T(p\bar{p} \rightarrow e^+e^-)} = \beta_B^3 (2R+S) \quad (14)$$

Eq. (14) holds in the most general case, and is still valid if the antiprotons are at rest.

If B has no anomalous moments and constant form factors b is simply $b = \beta_B^3 / [3/4 + (E/m_B)^2]$. In fig. (5) this branching ratio is reported versus E/m_B . Of course E must always be larger than the nucleon mass. One sees that annihilation into a pair of intermediate mesons is favored with respect to annihilation into e^+e^- or $\mu^+\mu^-$ already for c.m. energy larger than $1.5 m_B$, provided B has no anomalous electromagnetic properties. In fig. (5) we have also reported b for $\mu = +1$ and $\mu = -1$, $\varepsilon = 0$ and constant form factors.

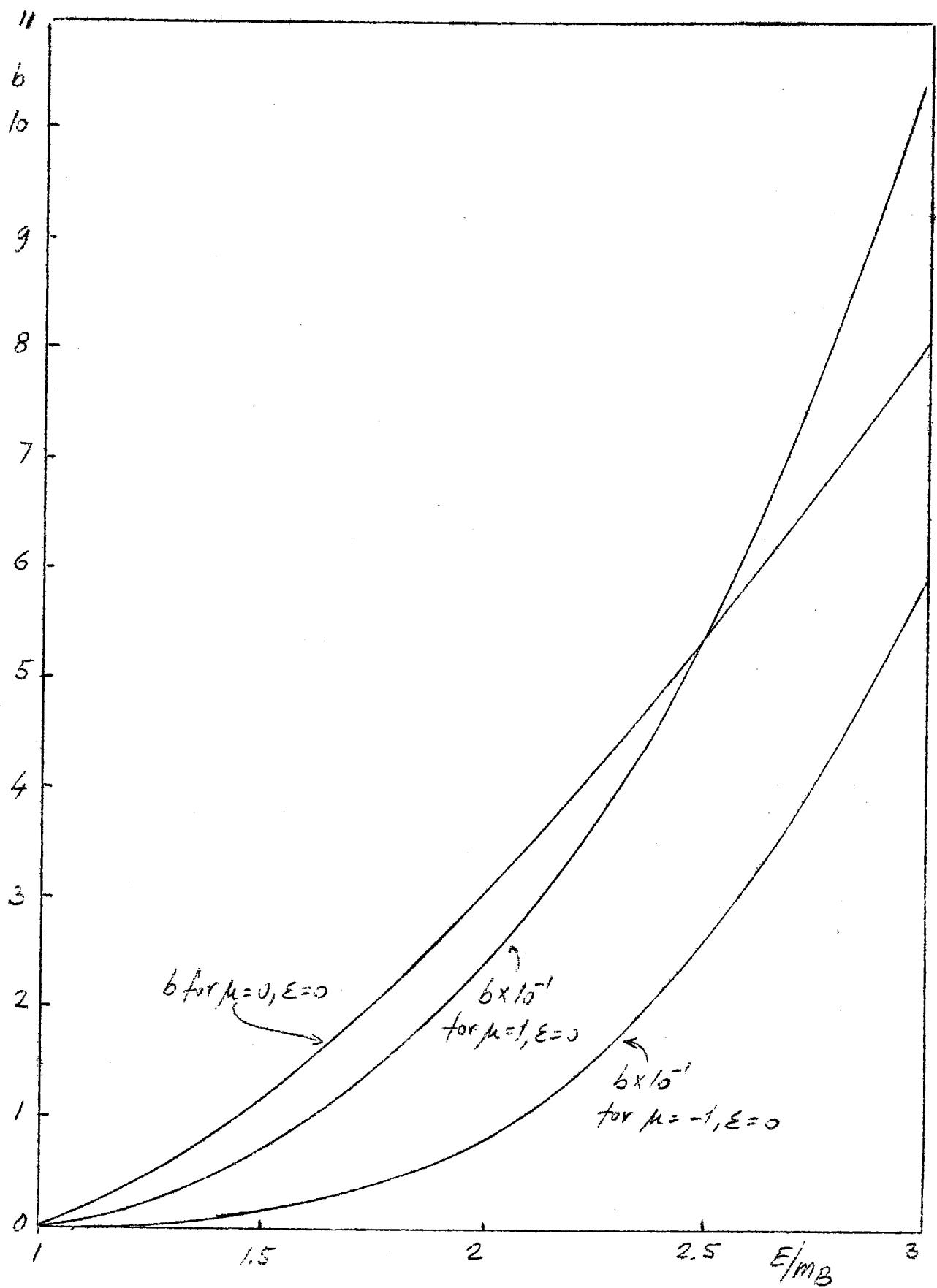


FIG. (5)

Once B is produced according to (10) it will decay rapidly (in about 10^{-17} sec) into its disintegration products (2π , 3π , $\pi + K$, $\mu + \nu$, $e + \nu$, etc.). The annihilation events will exhibit definite angular correlations and in some cases they will be of the kind

$$\begin{aligned} p + \bar{p} &\rightarrow B^+ + B^- \rightarrow (\mu + \nu) + (\pi^+ + \pi^-) \\ &\rightarrow (\mu + \nu) + (e + \nu) \\ &\rightarrow (K^0 + \pi^+) + (\pi^- + \pi^0) \\ &\rightarrow \text{etc.} \end{aligned}$$

which should allow the identification of B . Branching ratios among the various decay modes of B have recently been discussed by Bernstein and Feinberg¹⁵.

We conclude this section with the observation that vector mesons can also be produced by the reactions

$$\bar{p} + p \rightarrow B^+ + \pi^-, B^0 + \pi^0, B^- + \pi^+, B^+ + K^-, \text{etc} \quad (15)$$

$$\bar{p} + n \rightarrow B^0 + \pi^-, B^- + \pi^0, B^0 + K^-, \text{etc} \quad (16)$$

These reactions occur through the semiweak coupling of the vector meson and on dimensional grounds should have a cross-section

$$\sim G \sim 0.4 \times 10^{-32} \text{ cm}^2 \quad (17)$$

where G is the weak coupling constant. A more refined estimate than (17) would involve the complications of strong interactions at rather high energies. If the vector weak current is conserved⁹) the vector part of the amplitudes for $\bar{p}p \rightarrow B\pi$ and $\bar{p}n \rightarrow B\pi$ are related to the isovector amplitudes for

$$\bar{p} + p \rightarrow \pi^0 + \gamma$$

$$\bar{p} + n \rightarrow \pi^- + \gamma$$

with the γ off-mass-shell in the form

$$\frac{\sigma(p\bar{p} \rightarrow B\pi)}{\sigma(p\bar{p} \rightarrow \gamma\pi)} \geq \frac{\sigma_v(p\bar{p} \rightarrow B\pi)}{\sigma(p\bar{p} \rightarrow \gamma\pi)} = \frac{Gm_B^2}{4\pi/2\alpha} \frac{P_B}{P_\gamma} X = \\ = 0.77 \times 10^{-4} \frac{P_B}{P_\gamma} \left(\frac{m_B}{m_p} \right)^2 X \quad (18)$$

where σ_v is the contribution from the weak vector current (that cannot interfere with the axial contribution in the rate) and X is a number that differs from unity for two reasons: because the correspondence only holds with the γ off-shell, and also it only holds for the iso-vector electromagnetic amplitude. From angular momentum, parity, and charge conjugation one can show that $p + \bar{p} \rightarrow B^\circ + \pi^\circ$ from S states only goes through vector coupling, so that the \geq in (18) becomes an equality sign. Furthermore in the schizon's theory of Lee and Yang¹⁰⁾ $\sigma(p\bar{p} \rightarrow B^\circ\pi^\circ) \geq \sigma(p\bar{p} \rightarrow \gamma\pi^\circ)$.

In conclusion we would like to stress the fact that even though the study of these rare annihilation modes are very difficult experiments, definitive results would be of great importance in the understanding of strong, electromagnetic and weak interactions.

We would like to thank prof. S.D. Drell for informative discussions.

References.

- 1) For example: R. Hofstadter and R. Herman, Phys. Rev. Lett. 6, 293 (1961).
- 2) R.M. Littauer, H.F. Schopper and R.R. Wilson, Phys. Rev. Lett. 7, 141 (1961).
- 3) S.D. Drell and F. Zachariasen, 'Electromagnetic structure of nucleons', Oxford University Press (1961).
- 4) N. Cabibbo and R. Gatto - Phys. Rev. 124, 1577 (1961)
- 5) M. Conversi, L. di Lella, F. Farley, Th. Muller and A. Zichichi.
- 6) For a point proton with an anomalous magnetic moment Eqs. (3) and (7) reduce to the cross-sections given by L.M. Brown and M. Peshkin, Phys. Rev. 103, 756 (1956).
- 7) S. Fubini, Proceedings of the Aix-en Provence Intern. Conf. on Elementary Particles, September 1961.
- 8) S.D. Drell, Annals of Physics 4, 75, (1958)
J.D. Bjorken, S.D. Drell and S.C. Frautschi, Phys. Rev. 112, 1409, (1958) - G. Putzolu, Nu.Cim. 20, 542 (1961)
- 9) R.P. Feynman and M. Goll-Mann, Phys. Rev. 109, 193 (1958)
also S. Gerstein and J. Zeldovich, Soviet Phys. Japt 29, 576 (1957).
- 10) T.D. Lee and C.N. Yang, Phys. Rev. 119, 1410 (1960)
- 11) B. Pontecorvo (Proceedings of the 9th International Conference of high Energy Physics, Moscow, 1960 reported by Marshak, pag. 296). T.D. Lee and C.N. Yang, Phys. Rev. Letters 4, 307 (1960).
- 12) J. Zeldovich - Proceedings of the 9th International Conference of High Energy Physics, Moscow 1960, pag. 296.
N. Dompey, Phys. Rev. Letters 5, 307 (1960)
- 13) M. Bassetti - Nuovo Cimento 20, 803 (1961)
- 14) R.E. Marshak, Proceedings of the 9th International Conference of High Energy Physics, Moscow 1960, pag. 295.
S. Bludman, J.A. Young - Proc. of the 10th Rochester Conference (1960) Interscience Publishers.
- 15) J. Bernstein and G. Feinberg (Report at the Conference on elementary Particles, Aix en Provence 1961).

Caption to fig. 3

in units of $0.75 \times 10^{-31} \text{ cm}^2$.

Upper curve (1) is for point-like proton with $\mu_p = 1.79$,
lower curve (2) is obtained by extrapolating the form fac-
tors of reference (7).

Caption to fig. 5

Ratio $\frac{P + \bar{P} \rightarrow B^+ + B^-}{P + \bar{P} \rightarrow e^+ + e^-}$ for different choices of the
anomalous magnetic moment of the B mesone, and constant
form factors.