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ELECTRON-POSITRON COLLIDING BEAM EXPERIMENTS

R. Gatto,

Laboratori Nazionali del CNEN, Frascati, Rome.

Electron-electron colliding beams were proposed by Barker, Gittelmann, O'Neill, Panofsky and Richter at Stanford[1]. Electron-positron colliding beams are under development at Stanford[1] and at Frascati[2,3]. The Frascati project, called Adone, is intended to produce high-energy (2-3 GeV c.m. energy) colliding beams of electrons and positrons.

I shall report on work done in Frascati in collaboration with Dr. Cabibbo in continuation of some previously published work[4]. We shall be concerned with:

I. The "one-photon" channel: general considerations, radiative corrections.

II. $e^+ + e^- \to$ pions; $e^+ + e^- \to 2\pi, 3\pi$; K mesons.

III. $e^+ + e^- \to \pi^0 + \gamma$: the 2π state; the 3π state.

IV. Resonances: general discussion.

V. Baryon pairs: general considerations; cross-sections and polarization; hopes; $\Sigma\Lambda$.

VI. Vector mesons. Pair production. Cross-section, angular correlations, violation of unitarity.

VII. Unitarity in the "one-photon" channel.

VIII. Electrodynamic vacuum polarization and cross-sections.

IX. Weak interactions: neutral B^0; pair production of B; local weak interaction.

As a matter of introduction let me recall a few relevant points concerning the electron-positron colliding beam experiments.

1) Tests of validity of quantum electrodynamics.

2) Most of the annihilation processes go through the "one-photon" channel. The reactions proceed through a state of well-defined quantum numbers.

3) Systematic exploration of form factors of strong interacting particles for time-like values of their argument.

4) Panofsky programme: systematic exploration of the spectrum of elementary particles through their interaction with photons.

5) Some unstable particles would be directly observed through resonances in the cross-sections.

6) At a given energy the angular distributions of the simplest reactions are uniquely predicted by theory or they depend on a few parameters.

7) Other typical features will appear from the following.

High-energy electron-positron colliding beam experiments may become a field of spectacular development in high-energy physics.

I. THE "ONE-PHOTON" CHANNEL

1. One annihilation

$$e^+ + e^- \rightarrow a + b + \dots c$$

is assumed to go through the "one-photon" channel, as described by graph A (Fig. 1). q_+ and q_- are the momenta of e^+ and e^-, K is the virtual photon momentum.

The matrix element is

$$< a,b,\dots c|S|e^+e^- > = \frac{2\pi e}{K^2} (\bar{v}\gamma_\nu u) < a,b,\dots c|j_\gamma(0)|0> \delta(K-a-b-c). \quad (1)$$

u and v are Dirac spinors, $j_\gamma(x)$ is the e.m. current operator.

We define

$$J_\gamma = (2\pi)^{\frac{3n}{2}} < a,b,\dots c|j_\gamma(0)|0 > \quad (2)$$

where n is the number of final particles. From gauge invariance

$$K_\gamma J_\gamma = 0 \ . \quad (3)$$

In the c.m. system

$$K^2 = (q_+ + q_-)^2 = -4E^2 \quad (4)$$

where E is the energy of e^+ (or e^-). Note that K is time-like. In the c.m. system $K \equiv (0, i\,2E)$. From Eq. (3)

$$K_4 J_4 = 0$$

or $J_4 = 0$. Thus $J \equiv (\vec{J},0)$. The cross-section is (we always neglect the electron mass)

$$\sigma = \frac{(2\pi)^{5-3n}\alpha}{16\ E^4} \int d^3\vec{a}\ d^3\vec{b}\ldots d^3\vec{c}\ \delta(E_a + E_b + \ldots E_c - 2E)\delta(\vec{a} + \vec{b} + \ldots \vec{c}) T_{mn} \sum_{a,b,..c} R_{mn} \tag{5}$$

where: $\alpha = 1/137$; $\vec{a}$, E are the momentum and energy of a;

$$T_{mn} = \tfrac{1}{2}(i_m i_n - \delta_{mn}) \qquad \vec{i} = \frac{\vec{q}_+}{|\vec{q}_+|} = - \frac{\vec{q}_-}{|\vec{q}_-|} \tag{6}$$

$$R_{mn} = - J_m J_n^* \tag{7}$$

and $\sum_{a,b,..c}$ refers to the final polarizations. For annihilation into two particles

$$d\sigma = \frac{\alpha}{32} \left(\frac{p}{E}\right) \frac{1}{E^2} \frac{E_a E_b}{E^2} \left(T_{mn} \sum_{a,b} R_{mn}\right) d(\cos \Theta) \tag{8}$$

where p is the final momentum of a or b in c.m. and Θ the c.m. angle.

2. Remark about radiative corrections

Inclusion of radiative corrections[5)] brings about graph B (Fig. 2) with two photons exchanged. Their interference with (A) vanishes, however, in experiments which treat the charges symmetrically (such as total cross-section or, say, $e^+ + e^- \to \pi^+ + \pi^-$ at definite angles but without distinguishing the charges, etc.). In fact the interference is essentially

$$\mathrm{Re} < i|S_A P_f S_B|i > \tag{9}$$

where S_A and S_B are the contributions to the S matrix from the one-photon channels and two-photon channels and P_f projects into the final states selected in the experiment. But $S_B|i> = S_B|i_+>$, $S_A|i> = S_A|i_->$, where $|i_+>$ and $|i_->$ are even and odd under charge conjugation, and $C\, S_A P_f S_B C^{-1} = S_A P_f S_B$ for the symmetrical experiments considered. Thus

$$(9) = \mathrm{Re} < i_- C^{-1}|S_A P_f S_B|C i_+ > = - (9) = 0 .$$

Therefore the "one-channel" picture is valid including terms $O(e^6)$. Of course radiative corrections in S_A must be considered. The electron-photon part of them is a known universal energy dependent term.

II. $e^+ + e^- \rightarrow$ n PIONS[4,6,7]

The final n-pion state, for the "one-photon" channel, has:

$$\begin{aligned} P &= -1 \\ C &= -1 \\ J &= 1 \\ T &= 1 \text{ for n even} \\ &= 0 \text{ for n odd.} \end{aligned} \tag{10}$$

P,C,J,T are parity, charge conjugation, angular momentum, and isotopic spin. From Eqs. (6) and (7)

$$R_{mn}T_{mn} = \tfrac{1}{2}|\vec{J}|^2 \sin^2\Theta \tag{11}$$

where Θ is the angle between $\vec{J}$ and $\vec{i}$. $\vec{J}$ is a vector (pseudovector) formed out of the independent final momenta for n even (odd). For n = 2 $\vec{J} \propto (\vec{p}_1 - \vec{p}_2)$; for n = 3 $\vec{J} \propto (\vec{p}_1 \times \vec{p}_2)$, normal to the production plane.

1. $e^+ + e^- \rightarrow \pi^+ + \pi^-$

The two final pions must be produced in a p-state. The cross-section is

$$\frac{d\sigma}{d(\cos\Theta)} = \frac{\pi}{16}\,\alpha^2\,\frac{1}{E^2}\,\beta^3\,|F(K^2)|^2\,\sin^2\Theta \tag{12}$$

The total cross-section

$$\sigma = \frac{1}{m^2}\,(0.53\times 10^{-32}\ \text{cm}^2)\,\frac{1}{x^2}\left(1 - \frac{1}{x^2}\right)^{3/2} |F(-4E^2)|^2 \tag{13}$$

where m is the boson mass in GeV, $x = E/m$, and $F(K^2)$ is a form factor. On the K^2 axis

$K^2 < 0$, time-like K	$K^2 > 0$, space-like K
absorptive region	
physical region for $e^+ + e^- \to \pi^+ + \pi^-$ $\quad K^2 = -(2m_\pi)^2$	physical region for $K^2 = 0 \quad e + \pi \to e + \pi$

An interpretation of $e^+ + e^- \to \pi^+ + \pi^-$ near threshold should be possible in terms of two-pion intermediate states only. Frazer and Fulco[8] proposed $|F(K^2)|^2$ approximately of the form

$$|F(K^2)|^2 = \frac{\beta^2 + (K_0^2)^2}{\beta^2 + (K^2 - K_0^2)^2} \tag{14}$$

with $\beta \cong 2.65\ m_\pi^2$ and $K_0^2 = 10.4\ m_\pi^2$. According to Bowcock, Cottingham and Lurié[9])

$$F_\pi(K^2) = \frac{t_r + \gamma}{t_r + K^2 - i\gamma\left(-\frac{K^2}{4} - 1\right)^{3/2}} \tag{15}$$

with $t_r = 22.4\ m_\pi^2$ and $\gamma = 0.4\ m_\pi^{-1}$. With Eq. (14) at E = 230 MeV, $\sigma = 8.35 \cdot 10^{-31}$ cm^2 (17 times bigger than with F = 1); with Eq. (15) at $E \cong 330$ MeV, $\sigma = 6.6 \cdot 10^{-31}$ cm^2 (33 times bigger than with F = 1), $e^+ + e^- \to \pi^0 + \pi^0$ requires the exchange of two photons.

2. $e^+ + e^- \to \pi^+ + \pi^- + \pi^0$

Call ℓ the relative $\pi^+ - \pi^-$ angular momentum and L the π^0 angular momentum relative to $\pi^+ - \pi^-$. Then from Eq. (10) ℓ = L = 1,2,3,5, etc.

$$\frac{d^2\sigma}{d\omega_+ d\omega_- d(\cos\Theta)} = \frac{\alpha}{(2\pi)^2}\frac{1}{64E^2}|H|^2 \sin^2\Theta\ (\vec{p}^{(+)} \times \vec{p}^{(-)})^2 \tag{16}$$

where ω_+, ω_- are the energies of π^+, π^- in c.m. and $\vec{p}^{(+)}, \vec{p}^{(-)}$ their momenta; H is a form factor depending on E, ω_+ and ω_-; and Θ is the angle between $\vec{i}$ and the normal to the production plane. If there is a T = 0, J = 1 bound state near the threshold

$$H \cong (\text{constant}) \times \frac{1}{K^2 + M^2}$$

should be a good approximation (M mass of the bound state). $e^+ + e^- \to 3\pi^0$ and, in general, $\to n\pi^0$ is simply forbidden by C. Production of many pions would perhaps exhibit patterns reflecting the T = 1, J = 1 2π resonance [also a 3π resonance or bound state can be thought of as occurring through the action of the T = 1, J = 1 force[10]].

3. K mesons

Eq. (12) applies to

$$e^+ + e^- \to K^+ + K^-$$
$$\to K^0 + \bar{K}^0$$

with F interpreted as the relevant form factor. The final $K^0\bar{K}^0$ pair must have C = -1 and therefore the final amplitude is

$$K^0\bar{K}^0 - \bar{K}^0K^0 . \qquad (17)$$

In terms of K_1^0, K_2^0

$$(M) = K_1^0K_2^0 - K_2^0K_1^0 . \qquad (18)$$

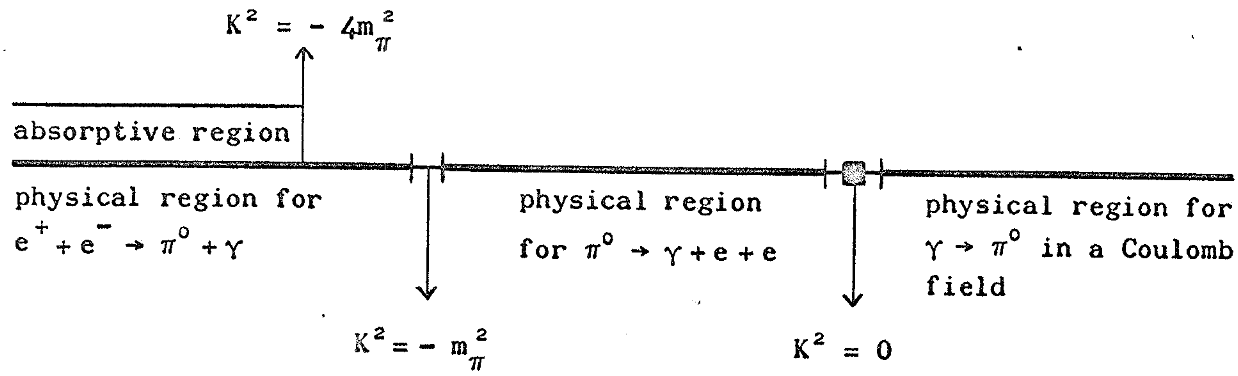
Therefore only $K_1^0K_2^0$ pairs are produced. Furthermore if K_1^0, K_2^0 propagate through vacuum

$$K_1^0 \to K_1^0 \, e^{-(\lambda_1 + i\, m_1)t}$$
$$K_2^0 \to K_2^0 \, e^{-(\lambda_2 + i\, m_2)t}$$

and thus Eq. (18), or equivalently Eq. (17), conserves its form at any time before an interaction occurs.

III. $e^+ + e^- \to \pi^0 + \gamma$

This reaction measures the vertex of Fig. 3 where p is the π^0 momentum, K the virtual photon momentum, and q the momentum of the final photon. On the real K^2 axis



The decay $\pi^0 \to 2\gamma$ occurs at $K^2 = 0$. The absorptive cut starts at $K^2 = -4m_\pi^2$ with two pion states. The T = 1, J = 1 resonance will contribute. Next, at $K^2 = -9m_\pi^2$, starts the 3π cut. If there is a T = 0, J = 1 3π bound state there would be a pole at $K^2 = -M^2$ where M is the mass ($M < 3m_\pi$). Decay of the bound state (into $\pi^0 + \gamma$, $2\pi + \gamma$) is slow ($\tau \sim 10^{-20} - 10^{-21}$ sec) and the associated width correspondingly very small ($\Gamma \sim 0.06 - 0.6$ MeV). Therefore it is safe to approximate its contribution with a pole at a real K^2, except near the pole itself. The cross-section would have a sharp narrow peak at $E = \frac{1}{2}M$. The measured cross-section is approximately

$$\bar{\sigma} = \frac{1}{2\Delta E} \int_{2E = M - \Delta E}^{2E = M + \Delta E} \sigma d(2E) = \frac{1}{\Delta E} \int_{E = \frac{M}{2} - \frac{\Delta E}{2}}^{E = \frac{M}{2} + \frac{\Delta E}{2}} \sigma dE \qquad (19)$$

where ΔE is the experimental energy resolution. Let us use a Breit-Wigner formula

$$\sigma = \frac{3}{4} \pi \lambda^2 \frac{\Gamma_i \Gamma_f}{(2E - M)^2 + \frac{\Gamma^2}{4}} \qquad (20)$$

with: $\lambda = 1/E$; Γ_i, Γ_f partial rates for decay into the initial $(e^+ + e^-)$ channel, final $(\pi^0 + \gamma)$ channel; Γ total disintegration rate. Since $\Gamma \ll \Delta E$

$$\bar{\sigma} \cong \frac{3}{2} \pi^2 \lambda^2 B_i B_f \left(\frac{\Gamma}{2\Delta E}\right) \qquad (21)$$

with $B_i = \Gamma_i/\Gamma$, $B_f = \Gamma_f/\Gamma$. For $M \lesssim 3m_\pi$, $B_i \cong 10^{-3}$

$$\bar{\sigma} = 1.3\times 10^{-28} \left(\frac{\Gamma}{2\Delta E}\right) \text{cm}^2 . \qquad (22)$$

For $\Gamma \sim 10^{20}\ \text{sec}^{-1}$, $\bar{\sigma} = 0.8\times 10^{-29}\ (2\Delta E \text{ in MeV})^{-1}\ \text{cm}^2$.

We shall discuss $e^+ + e^- \rightarrow \pi^0 + \gamma$ on the hypothesis of the dominance of the 2π resonance and of the 3π bound state. Experiments on internal conversion of gammas in π^0 decay indicate a negative value for the derivative of the π^0 form factor at the origin. Berman and Geffen[11] explain the negative coefficient by the coherent contribution of the 2π resonant state and of nucleon-antinucleon states. Our point of view is closer to that proposed by Wong[12]. The $\pi^0\gamma\gamma$ vertex can in general be written as

$$\frac{1}{4}\, G(-K^2, -q^2, -p^2)\ \epsilon_{\mu\gamma\lambda\rho}\ F_{\mu\gamma}(q) F_{\lambda\rho}(K) \qquad (23)$$

where G is a form factor, ϵ the isotropic antisymmetric tensor, and $F_{\mu\gamma}(q)$, $F_{\lambda\rho}(K)$ are Fourier transform of the e.m. tensor. We write $G(-K^2) = G(-K^2, 0, m_\pi^2)$. The π^0 lifetime is given by

$$\frac{1}{\tau} = \frac{m_\pi^3}{64\pi} \left| G(0) \right|^2 \qquad (24)$$

The cross-section for $e^+ + e^- \rightarrow \pi^0 + \gamma$ is given by

$$d\sigma = \frac{\pi\alpha}{m_\pi^3} \frac{1}{\tau}\, \beta^3 (1 + \cos^2\Theta) \left| \frac{G(-K^2)}{G(0)} \right|^2 d(\cos\Theta) . \qquad (25)$$

The total cross-section is

$$\sigma = \frac{8\pi}{3} \frac{\alpha}{m_\pi^3}\ \frac{1}{\tau}\, \beta^3 \left| \frac{G(-K^2)}{G(0)} \right|^2 . \qquad (26)$$

For $\tau = 2.2\times 10^{-16}$ sec

$$\sigma = 2.75\times 10^{-35}\ \text{cm}^2 \left| \frac{G(-K^2)}{G(0)} \right|^2 (1 - x^{-2})^3 \qquad (27)$$

with $x = 2E/m_\pi$. We assume a dispersion relation ($m_\pi = 1$)

$$G(-K^2) = \frac{1}{\pi}\int_4^\infty \frac{\text{Im } G(t)dt}{t+K^2-i\epsilon} . \qquad (28)$$

The contribution from the 2π state can be shown to be given by

$$G_2(-K^2) \propto \frac{1}{\pi}\int_4^\infty \frac{(t-4)^{3/2}|F_\pi(t)|^2 dt}{t^{1/2}(t+a)(t+K^2-i\epsilon)} \propto \frac{1}{\pi}\int_4^\infty \frac{(t-4)^{3/2}|F_\pi(t)|^2 dt}{t+K^2-i\epsilon} \qquad (29)$$

where we have neglected the t dependence of slowly varying factors. The Bowcock form factor

$$F_\pi(t) = \frac{t_2+\gamma}{t_2 - t - i\gamma\left(\frac{t}{4}-1\right)^{3/2}} \qquad (30)$$

satisfies Eq. (28) with

$$\text{Im } F_\pi(t) = \frac{\gamma}{4(t_2+\gamma)} (t-4)^{3/2}|F_\pi(t)|^2 \Theta(t-4) . \qquad (31)$$

Therefore

$$F_\pi(t) \propto \frac{1}{\pi}\int_4^\infty \frac{(t-4)^{3/2}|F_\pi(t)|^2 dt}{t^2+K^2-i\epsilon} \propto G_2(-K^2). \qquad (32)$$

Thus we write

$$G(-K^2) = G_2(-K^2) + G_3(-K^2) = \frac{c_2}{t_2+K^2-i\gamma\left(-\frac{K^2}{4}-1\right)^{3/2}} + \frac{c_3}{t_3+K^2-i\Gamma} \qquad (33)$$

where we have used Eq. (32) and added a pole term at $K^2 = -t_3 + i\Gamma$ for the 3π bound states. The constants c_2 and c_3 are determined from Eq. (24) and the value α measured from $\pi^0 \to \gamma + e^+ + e^-$. α is given by

$$-\frac{1}{G(0)}\left(\frac{\partial G(-K^2)}{\partial K^2}\right)_0 = \alpha m_\pi^2 . \qquad (34)$$

The experimental value for α is -0.24 ± 0.16 [13]. One finds

$$|G(-K^2)|^2 = \frac{64\pi}{m_\pi^3}\tau\frac{1}{(t_2-t_3)^2}\left|\frac{t_2^2(1-\alpha t_3)}{t_2+K^2-i\gamma\left(-\frac{K^2}{4}-1\right)^{3/2}} - \frac{t_3^2(1-\alpha t_2)}{t_3+K^2-i\Gamma}\right|^2 . \qquad (35)$$

We take $t_2 = 22.4\ m_\pi$ and $t_3 = 5\ m_\pi$. The last assumption comes from identifying the bound state with the resonance observed by Abashian, Booth and Crowe[14]. Furthermore $\gamma \cong 0.4\ m_\pi^{-1}$ and $\Gamma = 10^{20} - 10^{21}\ \sec^{-1}$. According to Eq. (35) there is a peak near $K^2 = -t_2$, with an enhancement factor (for $\alpha = -0.24$)

$$\left|\frac{G(t_2)}{G(0)}\right|^2 \cong 250 \; . \tag{36}$$

The cross-section near t_2 is $\sim 6 \cdot 10^{-33}$ cm^2 in the above enhancement factor. There is a peak also at $K^2 = -t_3$ and the average (19) around the peak is

$$\bar{\sigma} \cong 3.5 \cdot 10^{-29} \left(\frac{t_3 - \alpha t_2\, t_3}{t_3 - t_2}\right)^2 \frac{1}{2\Delta E}\ \mathrm{cm}^2 \tag{37}$$

with ΔE in MeV. With the above values for the parameters, $\bar{\sigma} \cong 10^{-28}(2\Delta E)^{-1}\mathrm{cm}^2$, not very far from the previous independent estimate (22). The energy resolution ΔE could reasonably be assumed ~ 5 MeV.

One of the concurrent reactions to $e^+ + e^- \to \pi^0 + \gamma$ against which one has to discriminate is $e^+ + e^- \to \gamma + \gamma + \gamma$. We also should mention that $e^+ + e^- \to e^+ + e^- + \pi^0$ [15,16] has a large cross-section $\sim 2.2 \cdot 10^{-35}\ \mathrm{cm}^2 (E/m_\pi)$ for $\tau = 2.2 \cdot 10^{-16}$ sec.

IV. GENERAL DISCUSSION OF THE POSSIBLE RESONANCES

Suppose the annihilation goes through a resonant channel

$$e^+ + e^- \to B_J \to (\text{final state}) \; .$$

B_J is a boson state with spin J and mass M. Assume a Breit-Wigner description for the resonance cross-section near the maximum

$$\sigma\ (E) = \pi \lambda\!\!\!^{-2}\ \frac{2J+1}{4}\ \frac{\Gamma_i \Gamma_f}{(2E - M)^2 + \frac{\Gamma}{4}} \; . \tag{38}$$

An experiment with energy resolution (in c.m.) $2\Delta E$ measures

$$\bar{\sigma}_R = \frac{1}{\Delta E} \int_{\frac{1}{2}(M - \Delta E)}^{\frac{1}{2}(M + \Delta E)} \sigma(E) dE \; . \qquad (39)$$

ΔE is not expected to be smaller than ~ 1 MeV.
We distinguish among three cases:

i) total width $\Gamma << 2\Delta E$;
ii) $\Gamma >> 2\Delta E$;
iii) $\Gamma \sim 2\Delta E$.

In case i)

$$\bar{\sigma}_R \cong 2\pi \lambda\!\!\!^{-2} \frac{\pi}{4} (2J + 1) B_i B_f \frac{\Gamma}{2\Delta E} . \qquad (40)$$

In case ii)

$$\bar{\sigma}_R \cong \sigma(2M) = \pi \lambda\!\!\!^{-2} (2J + 1) B_i B_f . \qquad (41)$$

In case iii), one can use either Eq. (40) or Eq. (41) since they differ by a factor $\pi/2$ for $\Gamma \cong 2\Delta E$.
Here $B_i = \Gamma_i/\Gamma$, $B_f = \Gamma_f/\Gamma$ are the branching ratios for $B_J \to e^+ + e^-$, $B_J \to$ (final state). We can limit the discussion to final states such that $B_f \cong 1$. Then the relevant quantities are

in case i) $\quad \frac{\Gamma_i}{\Delta E}$,

in case ii) $\quad \frac{\Gamma_i}{\Gamma}$,

and any of these two quantities in case iii).

Now there are some factors in Γ_i determined from charge conjugation, gauge invariance and angular momentum. If B_J has

$J = 0$, $C = 1$ then $\Gamma_i \propto \alpha^4 m_e^2$
$J = 0$, $C = -1$ $\quad \Gamma_i \propto \alpha^6 m_e^2$
$J = 1$, $C = 1$ $\quad \Gamma_i \propto \alpha^4$
$J = 1$, $C = -1$ $\quad \Gamma_i \propto \alpha^2$
$J = 2$, $C = 1$ $\quad \Gamma_i \propto \alpha^4$
$J = 2$, $C = -1$ $\quad \Gamma_i \propto \alpha^6$, etc.

Therefore in case i) only $(J=1, C=-1)$ will produce large effects. The same applies to ii) since $\Gamma >> \Delta E$, and also to iii). Case i), $\Gamma << 2\Delta E \cong$ $\cong$ (a few MeV), implies $\Gamma << \sim 10^{21}$ sec^{-1}, or $\tau = \Gamma^{-1} >> \cong 10^{-21}$ sec, and it occurs if decay through strong interaction is inhibited in some way. If $\Gamma_i/\Delta E >> \alpha^2$ the resonance effect is indeed very big (compare with $e^+ + e^- \to \mu^+ + \mu^-$, for instance, whose $\sigma \cong \frac{\pi}{3}\alpha^2\lambda\!\!\!^-{}^2$ for $E > m_\mu$). The same applies to case iii). Case ii) occurs whenever B_J decays by strong interactions, $\Gamma \cong 10^{23} - 10^{22}$ sec^{-1}. For instance, the 2π $T = 1$, $J = 1$ resonance belongs to case ii); the 3π bound state, decaying into $\pi^0 + \gamma$, to i) or iii).

V. BARYON PAIRS

1. In

$$
\begin{aligned}
e^+ + e^- &\to p + \bar{p},\ n + \bar{n} \\
&\to \Lambda + \bar{\Lambda} \\
&\to \Sigma + \bar{\Sigma} \\
&\to \Xi + \bar{\Xi}
\end{aligned}
$$

the final pair is produced in

$$^3S_1 \quad \text{and} \quad ^3D_1$$

(always limiting to one-photon channel). Therefore near threshold the cross-section is isotropic and $\propto \beta$. The differential cross-section is in general

$$\frac{d\sigma}{d(\cos\Theta)} = \frac{\pi}{8}\alpha^2\lambda\!\!\!^-{}^2\beta\left[\left|F_1(K^2) + \mu F_2(K^2)\right|^2(1+\cos^2\Theta) + \left|\frac{m}{E}F_1(K^2) + \frac{E}{m}\mu F_2(K^2)\right|^2\sin^2\Theta\right] \tag{42}$$

where $F_1(K^2), F_2(K^2)$ are the analytic continuations of the electric and magnetic form factors of the fermion for large negative $K^2 < -4m^2$, where m is the mass of the fermion. μ is the static anomalous magnetic moment of the fermion (the normalization is $F_1(0) = 1$, $F_2(0) = 1$ for a charged fermion; $F_1(0) = 0$, $F_2(0) = 1$ for a neutral fermion).

2. The form factors are in general complex along the absorptive cut, in particular for $K^2 < -4m^2$. Thus in $e^+ + e^- \to$ (fermion) + (antifermion), there can be a polarization of the fermion normal to the production plane,

already at the lowest electromagnetic order. This is opposite to scattering, e + (fermion) → e + (fermion), where there is no polarization at lowest order. The polarization $P(\Theta)$ of the fermion is given by

$$\frac{d\sigma}{d(\cos\Theta)} P(\Theta) = -\frac{\pi}{8}\, \alpha^2 \lambda^2 \beta^3 \mu \frac{E}{m}\, \mathrm{Im}\left[F_2(K^2)F_2(K^2)\right] \sin(2\Theta) \tag{43}$$

and it is along $\vec{p}\times\vec{q}_+$, where $\vec{p}$ is the momentum of the final fermion and $\vec{q}_+$ that of the positron. For the antifermion the polarization is $-P(\Theta)$, by use of TCP.

3. With $F_1 = 1$, $F_2 = 0$

$$\sigma = m^{-2}(2.1 \cdot 10^{-32}\ \mathrm{cm}^2)u(1-u)^{1/2}(1+\tfrac{1}{2}u) \tag{44}$$

with m in GeV and $u = (m/E)^2$. The actual cross-section, for instance, for $e^+ + e^- \to p + \bar{p}$, or $n + \bar{n}$, may be well above or below Eq. (44). To show a typical argument: Suppose that the core terms[17] in the form factors given by Hofstadter and Herman[17] comes from an absorptive contribution around, say, $K^2 = -(3m)^2$. For $K^2 \cong -(3m)^2$ the nucleon form factors could then be approximated, using Hofstadter's result, as $F_{1p} \cong 1.2/D$, $F_{2p} \cong -3.4/D$, $F_{1n} \cong 3.2/D$, and $F_{2n} = 0$, with $D = 20 - 2E - i(\Gamma/2)$ (all in units of m_π). Inserting into Eq. (42) we find for $e^+ + e^- \to n + \bar{n}$, for instance, a value 50 times bigger than Eq. (44), near $K^2 = -(3m)^2$, by taking Γ, say, $\cong m_\pi$. All this is meant only to illustrate that the cross-sections can be bigger than Eq. (44).

4. The reactions

$$e^+ + e^- \to \Sigma^0 + \bar{\Lambda}^0,\ \Lambda^0 + \bar{\Sigma}^0$$

proceed to $^3S_1$, $^3D_1$ for relative Σ–Λ parity, $P_{\Lambda\Sigma} = +1$; to $^1P_1$, $^3P_1$ for $P_{\Lambda\Sigma} = -1$. Therefore, near threshold σ increases $\propto p$ (the final c.m. momentum) and is isotropic for $P_{\Lambda\Sigma} = +1$, is $\propto p^3$, and contains also a $\cos^2\Theta$ term for $P_{\Lambda\Sigma} = -1$. The $\gamma\Sigma\Lambda$ vertex is

$$J_\gamma = \bar{u}_\Lambda\left[f_1(K^2)\gamma_\gamma + f_2(K^2)\sigma_{\gamma\mu}K_\mu + f_2(K^2)K_\gamma\right]\Gamma\, V_\Sigma \tag{45}$$

where $\Gamma = 1\ (\gamma_5)$ for $P_{\Sigma\Lambda} = +1\ (-1)$.

$$\frac{d\sigma}{d(\cos\Theta)} = \frac{\pi}{8}\alpha^2\lambda^2\beta\left\{\beta^2\cos^2\Theta\left[|f_1(K^2)|^2 + K^2|f_2(K^2)|^2\right] + \left[|f_1(K^2)|^2 - \right.\right.$$

$$\left.\left. - K^2|f_2(K^2)|^2\right]\frac{E_\Lambda E_\Sigma \pm m_\Lambda m_\Sigma}{E^2} + -4\ \frac{m_\Lambda E_\Sigma \pm m_\Sigma E_\Lambda}{E}\,\mathrm{Re}\left[f_1(K^2)f_2^*(K^2)\right]\right\} \tag{46}$$

where $\beta = p/E$; E_Λ, m_Λ, E_Σ, m_Σ are the energies and masses of Λ and Σ. Relative $\Lambda\Sigma$ parity could be measured in this way.

VI. VECTOR MESONS

1. In the reaction

$$e^+ + e^- \rightarrow B + \bar{B}$$

where B is a spin one meson, the possible final states are $^1P_1$, $^5P_1$ and $^5F_1$ from angular momentum, parity, and charge conjugation. The reaction measures the vertex shown in Fig. 4. p_1, ϵ_1, p_2, ϵ_2 are the four-momenta and polarization vectors of $B,\bar{B}$. They satisfy, for physical particles, $p_1^2 = p_2^2 = -m_B^2$, $(p_1\epsilon_1) = (p_2\epsilon_2)$. To form J_μ take as independent vectors: $K = p_1 + p_2$, $p = p_1 - p_2$, ϵ_1, ϵ_2. The independent scalars are K^2, $(\epsilon_1 K)$, $(\epsilon_2 K)$ and $(\epsilon_1\epsilon_2)$. J_μ must be linear in each ϵ and transform like a vector. We write thus in general

$$J_\mu = K_\mu\left[(\epsilon_1\epsilon_2)a + (\epsilon_1 K)(\epsilon_2 K)b\right] + p_\mu\left[(\epsilon_1\epsilon_2)c + (\epsilon_1 K)(\epsilon_2 K)d\right] +$$

$$+ \epsilon_{1\mu}(e_2 K)e + \epsilon_{2\mu}(\epsilon_1 K)f \tag{47}$$

where a,b...f are functions of K^2. From KJ = 0

$$a = 0 \quad \text{and} \quad -K^2 b = e + f. \tag{48}$$

Because of the odd behaviour of the e.m. current under charge conjugation $J_\mu \rightarrow -J_\mu$ when $K \rightarrow K$, $p \rightarrow -p$, $\epsilon_1 \leftrightarrow \epsilon_2$: it follows $e = -f$. By convenient redefinition of the form factors we then write

$$J_\mu = \frac{e}{\sqrt{4\omega_1\omega_2}}\Big\{G_1(\epsilon_1\epsilon_2)p_\mu + \Big[G_1+\mu G_2+\epsilon G_3\Big]\Big[(\epsilon_1 K)\epsilon_{2\mu} - (\epsilon_2 K)\epsilon_{1\mu}\Big] + \epsilon\, G_3 m_B^{-2}\Big[(K\epsilon_1)(K\epsilon_2) - \tfrac{1}{2} K^2(\epsilon_1\epsilon_2)\Big]p_\mu\Big\} \qquad (49)$$

where ω_1 and ω_2 are the c.m. energies of $B, \bar{B}$; eG_1, μG_2, ϵG_3 describe (for small space-like K^2) the charge distribution, the magnetic moment distribution, and the electrical dipole moment distribution of B. G_1, G_2, G_3 are functions of K^2.

2. The cross-section is

$$\frac{d\sigma}{d(\cos\Theta)} = \frac{\pi}{16}\alpha^2\lambda\!\!\!^{-2}\beta^3\Big\{2\left(\frac{E}{m_B}\right)^2\left|G_1(K^2)+\mu G_2(K^2)+\epsilon G_3(K^2)\right|^2(1+\cos^2\Theta) + \sin^2\Theta\Big[2\left|G_1(K^2)+2\left(\frac{E}{m_B}\right)^2\epsilon G_3(K^2)\right|^2 + \left|G_1(K^2)+2\left(\frac{E}{m_B}\right)^2\mu G_2(K^2)\right|^2\Big]\Big\}. \qquad (50)$$

The β^3 dependence (β = final velocity) near threshold is typical of P-state production.
With $G_1 = 1$, $G_2 = G_3 = 0$ the total cross-section is

$$\sigma = m_B^{-2}(2.1\cdot 10^{-32}\ \mathrm{cm}^2)\tfrac{3}{4}(1-u)^{3/2}\left(\tfrac{4}{3}+u\right) \qquad (51)$$

where m_B is expressed in GeV and $u = (m/E)^2$. Therefore $e^+ - e^-$ collisions may turn out to be very efficient for detecting possible unstable vector mesons.

3. Specific angular correlations will be evident after B decays. For instance, if B is produced near threshold in $e^+ + e^- \to B + \bar{B}$, and then decays according to $B \to \pi + \pi$ the angular correlation is

$$2-\cos^2\Theta-\cos^2\varphi + 2\cos\Theta\cos\varphi\cos\chi$$

where Θ is the production angle, φ the angle between the incoming momentum at production and the final relative momentum at the decay, and χ the angle between the outgoing production momentum and the decay relative momentum.

If it decays according to $B \to \mu + \nu$ or $B \to e + \nu$ the correlation is

$$3 + \cos^2\varphi - 2 \cos\Theta \cos\varphi \cos\chi . \tag{53}$$

We have assumed $\mu = \epsilon = 0$ and neglected the lepton masses.

4. The cross-section (51) violates unitarity at high energies. In fact it goes to a constant at high energy, whereas it can be shown, on the basis of unitarity arguments, that the total reaction cross-section must decrease proportional to $\lambda\!\!\!^-{}^2$. The unitarity upper limit to the reaction cross-section is derived in the next section.

VII. LIMITATIONS FROM UNITARITY FOR THE "ONE-PHOTON" CHANNEL

Consider

$$a + b \to (\text{final state}) .$$

We use Jacob-Wick notation. The initial state is defined for a given c.m. momentum of a,b by the helicities λ_a, λ_b

$$| i \gg = |\lambda_a, \lambda_b > . \tag{54}$$

The total cross-section from Eq. (54) into a set of final states F is

$$\sigma(\lambda_a, \lambda_b; F) = (2\pi)^2 \lambda\!\!\!^-{}^2 < \lambda_a \lambda_b | T(E)^\dagger P_F(E) T(E) | \lambda_a \lambda_b > \tag{55}$$

Assume F rotational-invariant. Then we can define

$$\sigma_J(\lambda_a, \lambda_b; F) = \pi \lambda^2 (2J + 1) < J; \lambda_a \lambda_b | T_J^\dagger P_F^{(3)} T_J | J; \lambda_a, \lambda_b > \tag{56}$$

For reactions (as opposed to scattering) we substitute $T \to S$, with $SS^\dagger = 1$. Then, from Eq. (56)

$$\sigma_J(\lambda_a \lambda_b; F) \leq \pi \lambda^2 (2J + 1). \tag{57}$$

Now consider

$$e^+ + e^- \to (\gamma) \to \text{final state} .$$

In the limit $m_e = 0$ only the superposition

$$\frac{1}{\sqrt{2}}\Big(|1, -1 > + |-1, +1 > \Big) \tag{58}$$

in the notation (54) takes part to the reaction. In fact

$$\bar{v}\gamma_\mu u = \bar{v}(\bar{a}\gamma_\mu a + a\gamma_\mu \bar{a})u \tag{59}$$

where $a = \frac{1}{2}(1+\gamma_5)$, $\bar{a} = \frac{1}{2}(1-\gamma_5)$. Thus averaging Eq. (57), for J = 1

$$\sigma < \tfrac{3}{4}\pi\lambda^2 . \tag{60}$$

Using Eq. (60) we see that Eq. (51) violates unitarity at high energies ($\cong 10^2\ m_B$). At these energies, however, other neglected effects are important.

VIII. RELATION BETWEEN THE ANNIHILATION CROSS-SECTION INTO STRONG INTERACTING PARTICLES AND MODIFICATIONS OF THE PHOTON PROPAGATOR

The quantity

$$\Pi(K^2) = -\frac{(2\pi)^3}{3K^2} \sum_{p^{(z)}=K} < 0|j_r(0)|z > < z|j_r(0)|0 > \tag{61}$$

is known to be of fundamental importance in quantum electrodynamics[18]. For instance the Fourier transform of the photon propagator can be written as

$$D^{F\prime}_{\mu\gamma}(K) = \frac{\delta_{\mu\gamma}}{K^2 - i\epsilon} + \frac{K^2\delta_{\mu\gamma} - K_\mu K_\gamma}{K^2} \; \frac{\bar{\Pi}(0) - \bar{\Pi}(K^2) - i\pi\,\Pi(K^2)}{K^2 - i\epsilon} \tag{62}$$

$$\bar{\Pi}(K^2) = P\int_0^\infty \frac{\Pi(-a)}{K^2 + a}\, da . \tag{63}$$

We call $\sigma_F(E)$ the cross-section (5) summed over a set F of final states

$$\sigma_F(E) = -\frac{(2\pi)^5\alpha}{16\ E^4}\, T_{mn} \sum_{p_z=K} < 0|j_m(0)|z > < z|j_n(0)|0 > . \tag{64}$$

In general, from gauge invariance

$$(2\pi)^3 \sum_{\substack{F \\ p_z = K}} < 0|j_\mu(0)|z > < z|j_\gamma(0)|0 > = \Pi_F(K^2)(K_\mu K_\gamma - K^2\delta_{\mu\gamma}). \qquad (65)$$

Substituting Eq. (65) into Eq. (64) and using Eq. (6)

$$\sigma_F(E) = \frac{\pi^2\alpha}{E^2}\,\Pi_F(-4E^2) \qquad (66)$$

which is the desired relation[19]. Note that in order $\bar{\Pi}(K^2) - \bar{\Pi}(0)$ to be convergent (it must be convergent from Eq. 62)

$$\int^{\infty} \frac{\sigma_F(E)}{E}\, dE \qquad (67)$$

must converge (which we know to happen from unitarity). However, $\bar{\Pi}(0)$, connected to charge renormalization, is convergent only if

$$\int^{\infty} E\, \sigma_F(E) dE \qquad (68)$$

is finite. This does not happen if σ_F decreases $\propto \lambda^2$. Note that $\sigma_F(F)$ is the cross-section in the approximation where only one photon is exchanged.

IX. REMARKS ABOUT WEAK INTERACTIONS

Intermediate charged vector mesons, as suggested in theories of weak interactions[20], can be produced according to

$$e^+ + e^- \rightarrow B + \bar{B}$$

and the formulae and the considerations of Section VII apply to this process.

A neutral B^0 cannot be coupled to the leptons if it is also coupled to the weak neutral strangeness non-conserving current. If B^0 exists and is coupled to the leptons in

$$e^+ + e^- \rightarrow B^0 \rightarrow \mu^+ + \mu^-$$

it would lead to a resonant contribution. Assuming a width $\Gamma \cong 5 \cdot 10^{17}$ sec^{-1}, a branching ratio for $B^0 \to e^+ + e^-$ of the order of $1/5$, a mass m_B of the order of the K mass, we find for the resonant contribution to the above reaction a value $\bar{\sigma}_R$ about three times bigger than the value for the electromagnetic cross-section. Local weak interaction, with the known strength, would be felt only at higher energies. For instance at E $\cong$ 30 GeV the muons from $e^+ + e^- \to \mu^+ + \mu^-$ would have a strong longitudinal polarization if there is a parity non-conserving coupling $(\mu^+\mu^-)(e^+e^-)$ of typical weak interaction strength.

* * *

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Fig.1. The one photon channel.

(A)

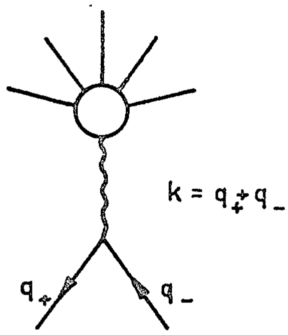

Fig.2. Radiative correction.

(B)

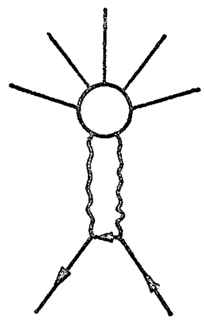

Fig.3. $\gamma \rightarrow \pi^0 + \gamma$.

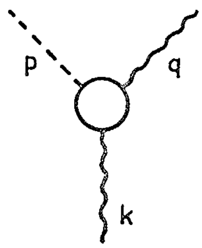

Fig.4. Vector meson production