Laboratori Nazionali di Frascati

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The interference effect of bremsstrahlung in a crystal, has been investigated theoretically by Uberall[1], who calculated the cross section, $\sigma_{br}(\theta, \delta)$, of bremsstrahlung in crystals, from high energy electrons, as a function of θ, the angle between a crystal axis and primary electron beam, and, δ, the minimum momentum transferred to the nucleus.

Figure 1 shows the behaviour of the cross section $\sigma_{br}(\theta, \delta)$ for a silicon single crystal.

Various experiments have been carried out in order to see the interference effect of bremsstrahlung.

The first experiment[2], performed with the Stanford linear accelerator, did not show the expected variation of the cross section versus θ. A second experi

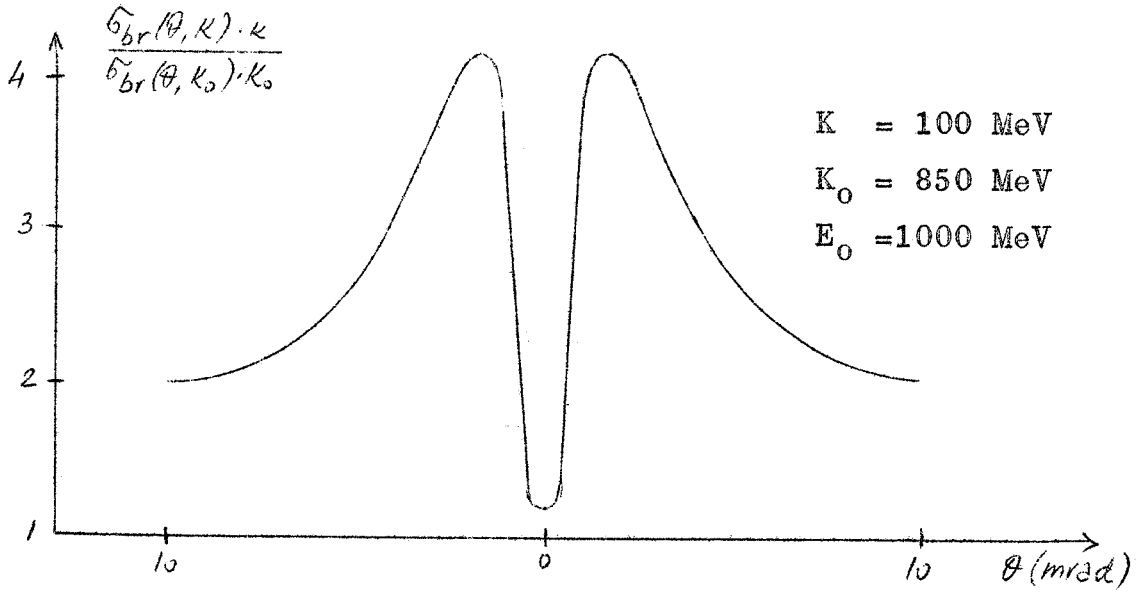


FIG. 1

ment[3], performed with the Cornell University electrosynchrotron, showed the interference effect in a qualitative fashion, but the minimum of the cross section for $\vartheta = 0$ was not found. An experiment carried out at Frascati[4] gave results in a good agreement with the Uberall's theoretical calculation.

The same Frascati experiment performed with better angular resolution[5], ($\Delta\vartheta \sim 10^{-4}$ rad) gave evidence of a fine structure, in the cross section

$$\sigma_{br}(\vartheta, \delta),$$

i.e. the behaviour of the cross section is like that of the fig. 1 but with a set of maxima and minima for some ϑ values. After an accurate analisys[6] of the ϑ values, for which were found the maxima of the cross section the conclusion was reached that: the Uberall's approximation of considering the crystal lattice planes perpendicular to the primary electron beam, were continuous planes; was inadeguate for a comparison between theoretical calculations and experimental results obtained an angular resolution like that of the Frascati experiment.

We have therefore calculated the cross section $\sigma_{br}(\vartheta, \delta)$, taking into account the discontinuous distribution of the crystal lattice points.

Fine structure of the interferential cross section.

The interferential cross section has the form:

$$(1)\quad \sigma_{br}(\theta,\delta) = \sum_{h_1 h_2 h_3} N \frac{(2\pi)^3}{\Delta} \int \sigma_{br}(\theta,\theta_3,\psi)\, \delta\left(\vec{\gamma}\frac{\vec{a}_1}{a_1} - \frac{2\pi h_1}{a_1}\right) \times$$

$$\times\, \delta\left(\vec{\gamma}\frac{\vec{a}_2}{a_2} - \frac{2\pi h_2}{a_2}\right) \delta\left(\vec{\gamma}\frac{\vec{a}_3}{a_3} - \frac{2\pi h_3}{a_3}\right) d\theta_1\, d\theta_3\, d\psi_1\, d\psi_3$$

where the simbols have the same meaning of reference (1)

and $\frac{2\pi h_1}{a_1}$, $\frac{2\pi h_2}{a_2}$, $\frac{2\pi h_3}{a_3}$, are the components of a reciprocal lattice vector.

The reciprocal lattice plane $h_1 = 0$ give the major contribution to the summ (1); so we can write down:

(2)
$$\sigma_{br}(\theta,\delta) = \sum_{h_2 h_3} N \frac{(2\pi)^3}{\Delta} \int \sigma_{br}(\theta,\theta_3,\psi)\, \delta\left(\vec{q}\,\frac{\vec{a}_1}{a_1}\right) \times$$
$$\times\, \delta\left(\vec{q}\,\frac{\vec{a}_2}{a_2} - \frac{2\pi h_2}{a_2}\right) \delta\left(\vec{q}\,\frac{\vec{a}_3}{a_3} - \frac{2\pi h_3}{a_3}\right) d\theta_1\, d\theta_3\, d\psi_1\, d\psi_3$$

where the argument of the first Dirac's delta function is:

$$\vec{q}\,\frac{\vec{a}_1}{a_1} = (\vec{p}_1 - \vec{p}_2 - \vec{k})\,\frac{\vec{a}_1}{a_1} = p_1 \cos\theta - p_2 \cos\theta_2 - k\cos\theta_3$$

that using the small angle approximation is:

$$\vec{q}\,\frac{\vec{a}_1}{a_1} = q_z - \theta\,(u\cos\psi_1 + v\cos\psi_3)$$

where

$$u = k\theta_1 \; ; \quad v = E_2\theta_3 \; ; \quad q_z = \delta + \frac{u^2}{2k} + \frac{v^2}{2E_2} \; .$$

Since the ψ_3 variable does not appear in the cross section $\sigma_{br}(\theta,\theta_3,\psi)$, we can integrate over these variable, with the first delta function. After a second integration over a variable that it is not in the two delta functions we have

(3)
$$\sigma_{br}(\theta\delta) = \sum_{h_2 h_3} \frac{4N(2\pi)^3\sigma}{\Delta E_n^2} \int dq_\perp^2 \int dq_z \, \frac{[1-F(q)]^2}{q^4} \, \frac{1}{\theta^2 q_\perp^2 - q_z^2} \times$$
$$\times \left\{ -\frac{1}{\mathcal{E}^2} - \frac{\mathcal{E} + q_\perp^2\delta}{(\eta^2 + 4q^2\delta^2)^{1/2}} + \frac{(1+k\delta)q^2+2}{\mathcal{E}} - \frac{1}{(\eta^2+4q^2\delta)^{1/2}} \right\} \delta\left(\frac{q_z}{\theta} - \frac{2\pi h_2}{a_2}\right) \times$$
$$\times\, \delta\left(\frac{1}{\theta}\sqrt{\theta^2 q_\perp^2 - q_z^2} - \frac{2\pi h_3}{\theta_3}\right)$$

where

$$\mathcal{E} = q_z - \frac{q_\perp^2}{2E_1} \; ; \quad \eta = q_z - \frac{q_\perp^2}{2E_2} \; ; \quad F(q) = \frac{1}{(1+\beta q)^2} \; .$$

In calculating (3) we have assumed that the primary electron beam lie in the $(\vec{a}_1, \vec{a}_2)$ plane (see fig. 2).

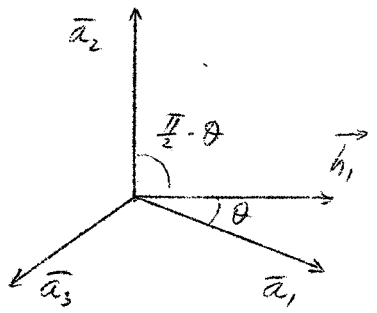


FIG. 2

When we consider a real crystal we must take into account the thermal motion of the atoms.

As shown in reference (1) thermal motion splits the cross section in to two parts, one giving a continuous contribution, and the second which apart for a factor e^{-Aq^2} is the cross section of an ideal crystal.

A is given by

$$A = \frac{3m^2c^2}{4MK\Theta}\left[1+\frac{4T}{\Theta}\Phi\left(\frac{\Theta}{T}\right)\right]$$

where $\Phi(\Theta/T)$ is the so colled Debye function, dependent on the absolute temperature T and the Debye temperature Θ .

The factor e^{-Aq^2} limits q to values less than 0,1 (mc units), so we can consider terms of the type $q^2/2E_1$ to be less than q_z and expand the roots in (3).

The cross section is now

$$(4)\qquad \sigma_{br}(\vartheta,\delta) = \sigma_{br}^{c}(\delta) + \sigma_{br}^{c}(\vartheta,\delta)$$

The continuous part of the cross section $\sigma_{br}^{c}(\delta)$ is the same of reference (1), the interferential part is:

$$(5)\qquad \sigma_{br}^{c}(\vartheta,\delta) = N\bar{\sigma}\frac{dK}{E_1'K}\frac{(2\pi)^2}{\Delta}\left[(E_1^2+E_2^2)\psi_1(\delta,\vartheta) - \frac{2}{3}E_1E_2\psi_2(\delta,\vartheta)\right]$$

with

$$\psi_1(\vartheta,\delta) = \sum_{\vec{g}} g\delta\,\frac{e^{-Ag^2}}{(\beta^{-2}+g^2)}\,\frac{g^2-g_z^2\vartheta^2}{g_z^2\vartheta^2}$$

$$\psi_2(\vartheta,\delta) = \sum_{\vec{g}} 48\delta\,\frac{e^{-Ag^2}}{(\beta^{-2}+g^2)}\,\frac{g^2-g_z^2\vartheta^2}{g_z^4\vartheta^4}\,(g_z\vartheta-\delta) \quad .$$

Where g is a reciprocal lattice vector in the plane $(\vec{g}_2, \vec{g}_3)$.

The component g_2 must be larger than δ/θ . When we consider the real points distribution in the crystal lattice. The interferential part of the cross section $\sigma_{br}(\theta, \delta)$ becomes a function also of the angle $\propto$ between the $(\vec{p}_1\ \vec{a}_1)$ and $(\vec{a}_1, \vec{a}_2)$ planes.

In (5) $\propto$ is set egual to zero.

Actually the behaviour of the cross section depends on the type of crystal and on the choice of the axis, at small angles with primary electron beam. For a silicon single crystal (axis [111] with the assumption that the electron momentum $\vec{p}_1$ is in the $(\vec{a}_1\ \vec{a}_2)$ plane the $\psi_{1,2}(\theta, \delta)$ are.

$$\psi_1 = 4 \sum_{n,m} 8\delta \frac{e^{-Aa^2(n^2+3m^2)}}{(\beta^{-2}+a^2(n^2+3m^2)} \frac{n^2+3m^2-n^2\theta^2}{n^2\theta^2}$$

$$\psi_2 = 4 \sum_{nm} 48\delta \frac{e^{-Aa^2(n^2+3m^2)}}{(\beta^{-2}+a^2(n^2+3m^2)} \frac{n^2+3m^2-n^2\theta^2}{a^2n^4\theta^4} (an\theta-\delta) .$$

We have taken into account the diamond lattice type, of the silicon crystal, by a factor 4 in the $\psi_{1,2}(\theta, \delta)$ and taking a suitable value of a. The figures (4, 5, 6 , 7,) show the theoretical cross section $\sigma_{br}(\theta, \delta)$ for bremsstrahlung in a silicon single crystal (axis [111]) and in a diamond single crystal (axis [110] at room temperature (T = 293°K), by 1 GeV electrons.

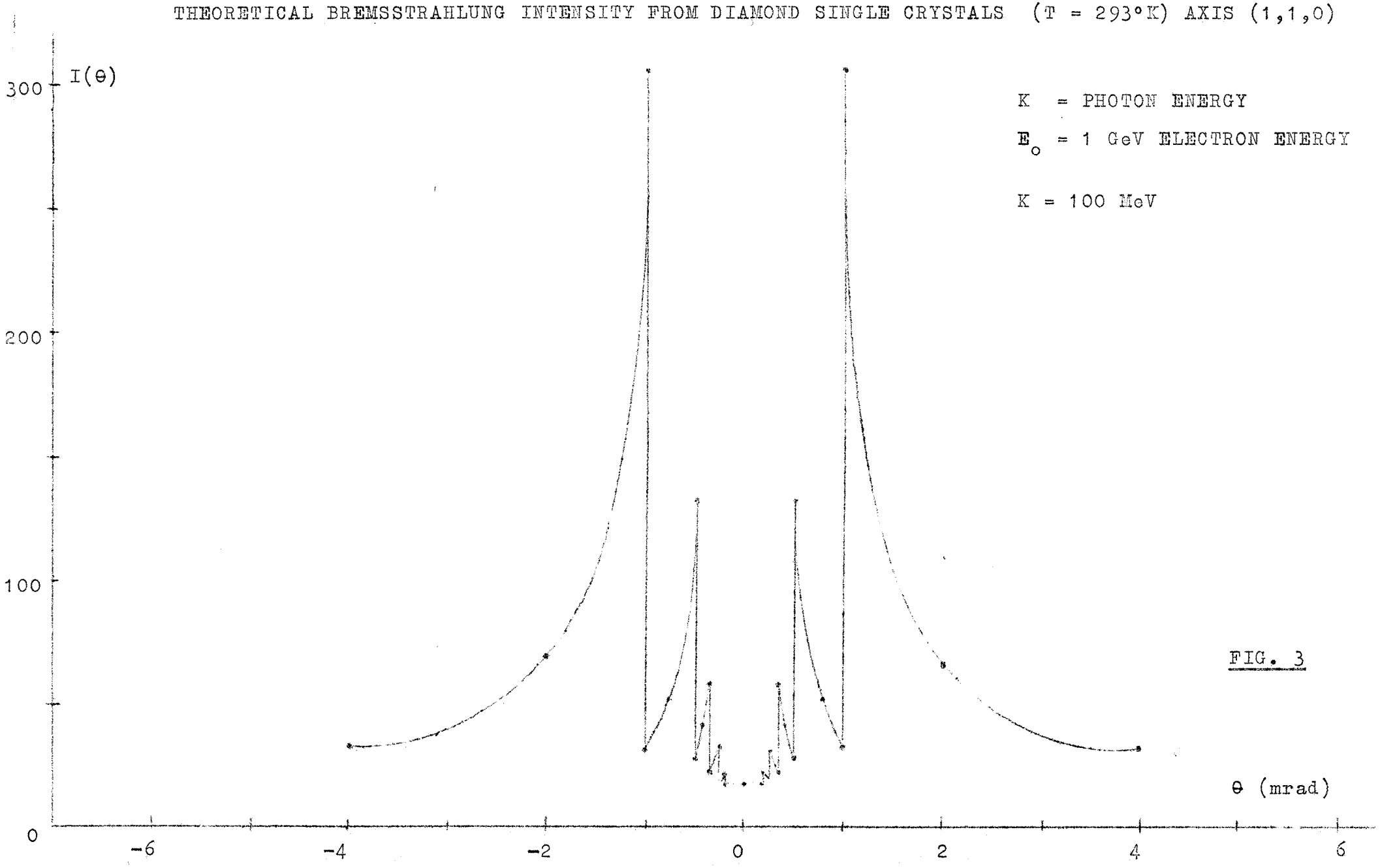

THEORETICAL BREMSSTRAHLUNG INTENSITY FROM DIAMOND SINGLE CRYSTALS (T = 293°K) AXIS (1,1,0)
I(θ)
300
200
100
0
-6
-4
-2
0
2
4
6
θ (mrad)
K = PHOTON ENERGY
E0 = 1 GeV ELECTRON ENERGY
K = 100 MeV

FIG. 3

THEORETICAL BREMSSTRAHLUNG INTENSITY FROM DIAMOND SINGLE CRYSTALS (T = 293°K) AXIS (1,1,0)

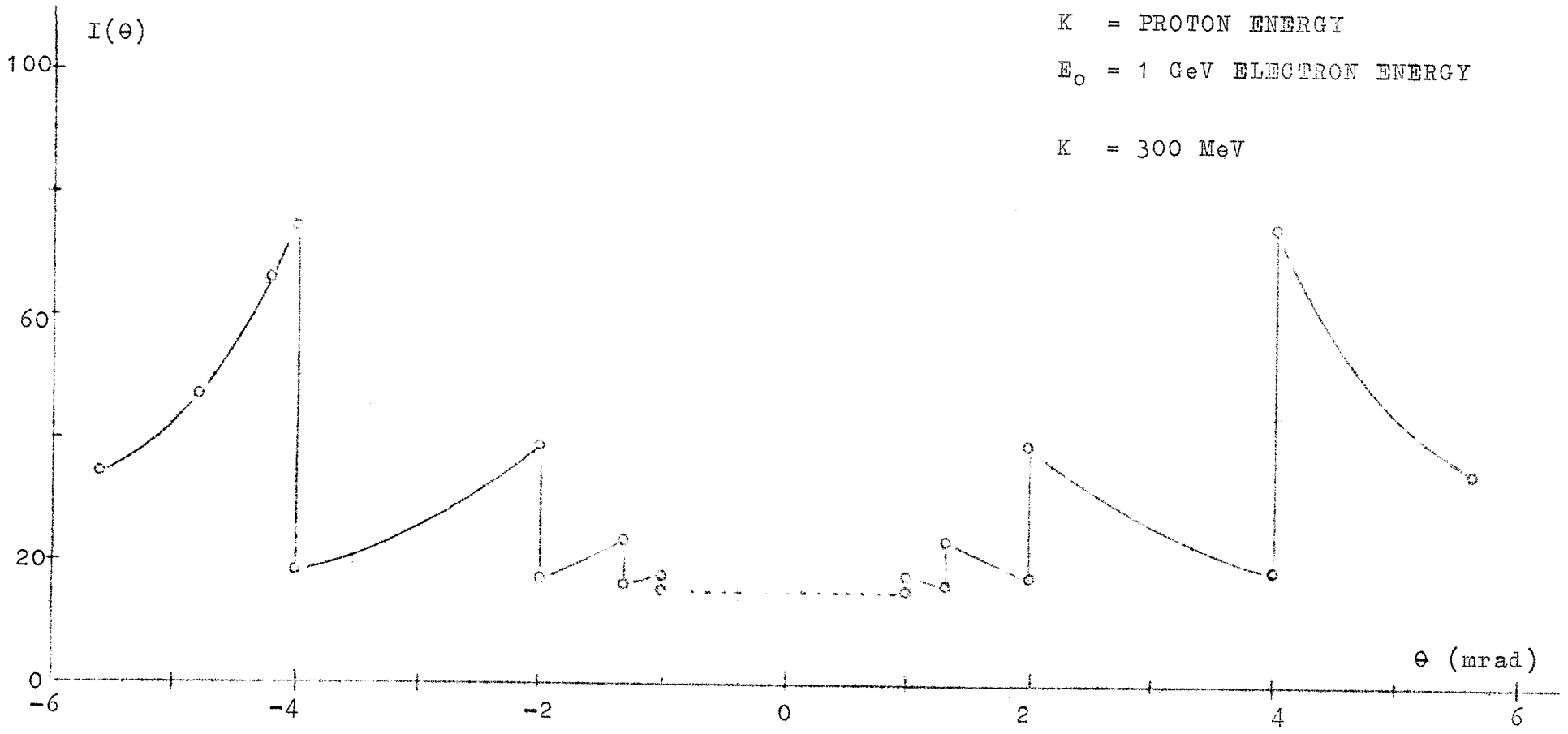


FIG. 4

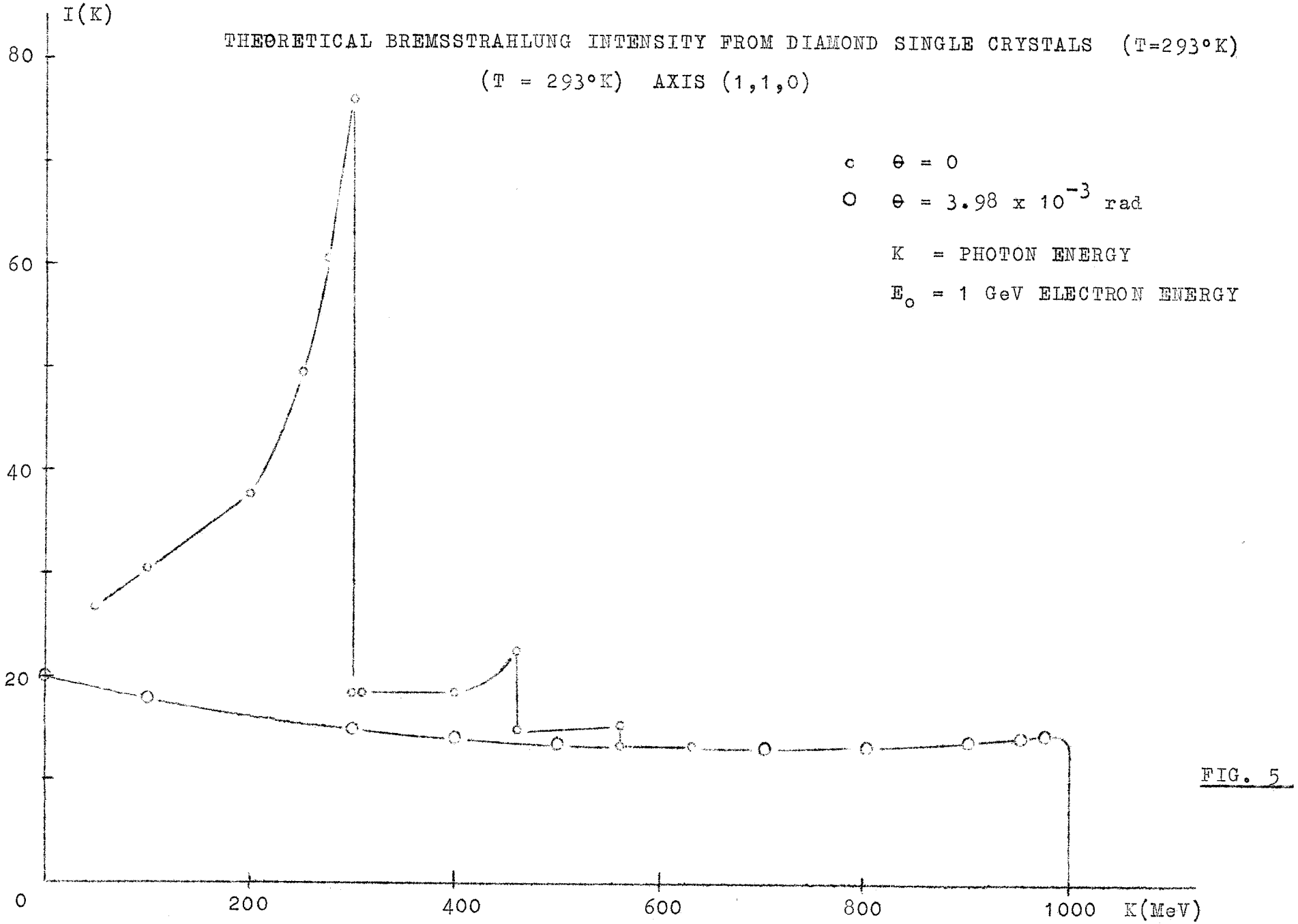

I(K)
THEORETICAL BREMSSTRAHLUNG INTENSITY FROM DIAMOND SINGLE CRYSTALS (T=293°K)
(T = 293°K) AXIS (1,1,0)
c θ = 0
O θ = 3.98 x 10^-3 rad
K = PHOTON ENERGY
E0 = 1 GeV ELECTRON ENERGY
80
60
40
20
0
200
400
600
800
1000
K(MeV)

FIG. 5

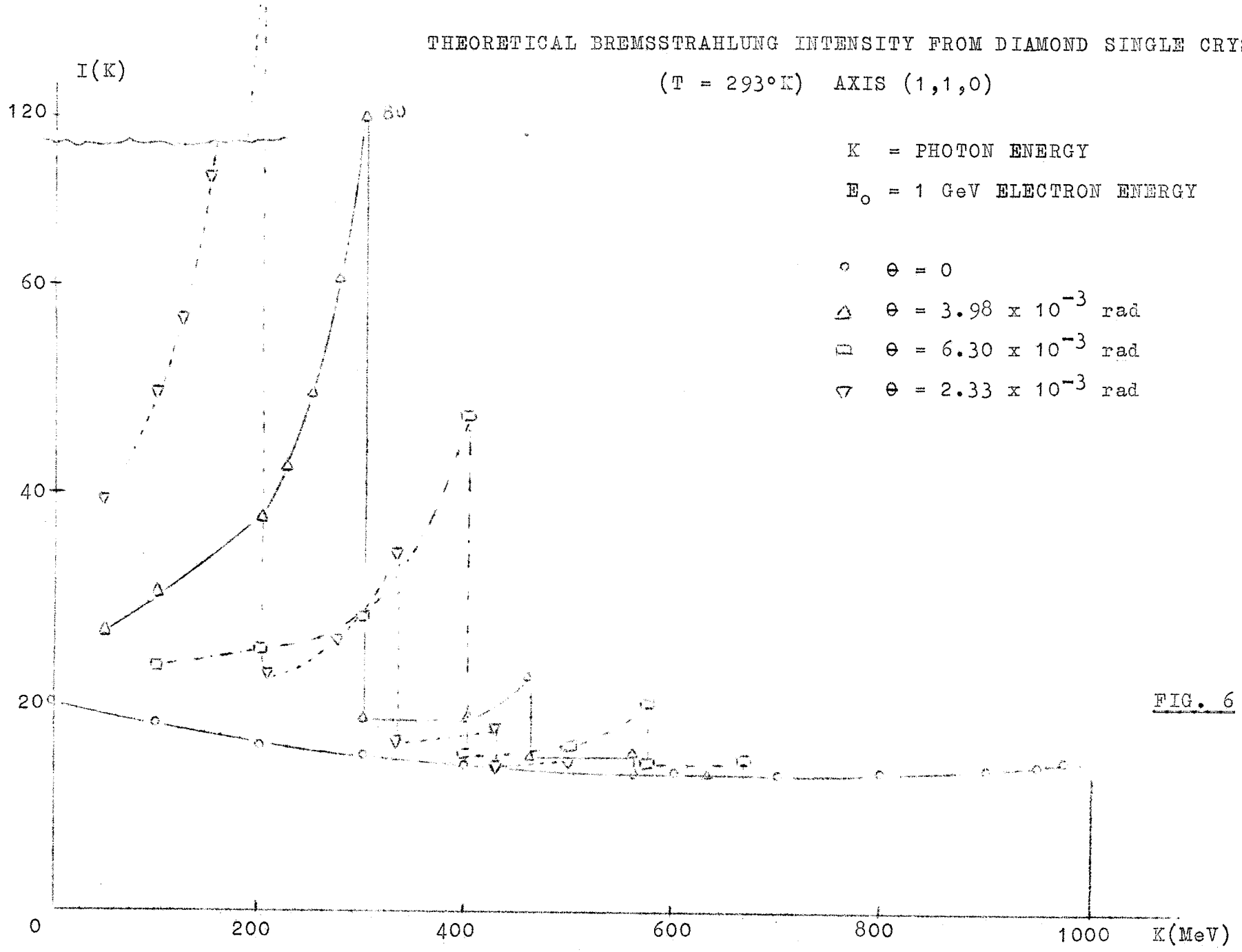
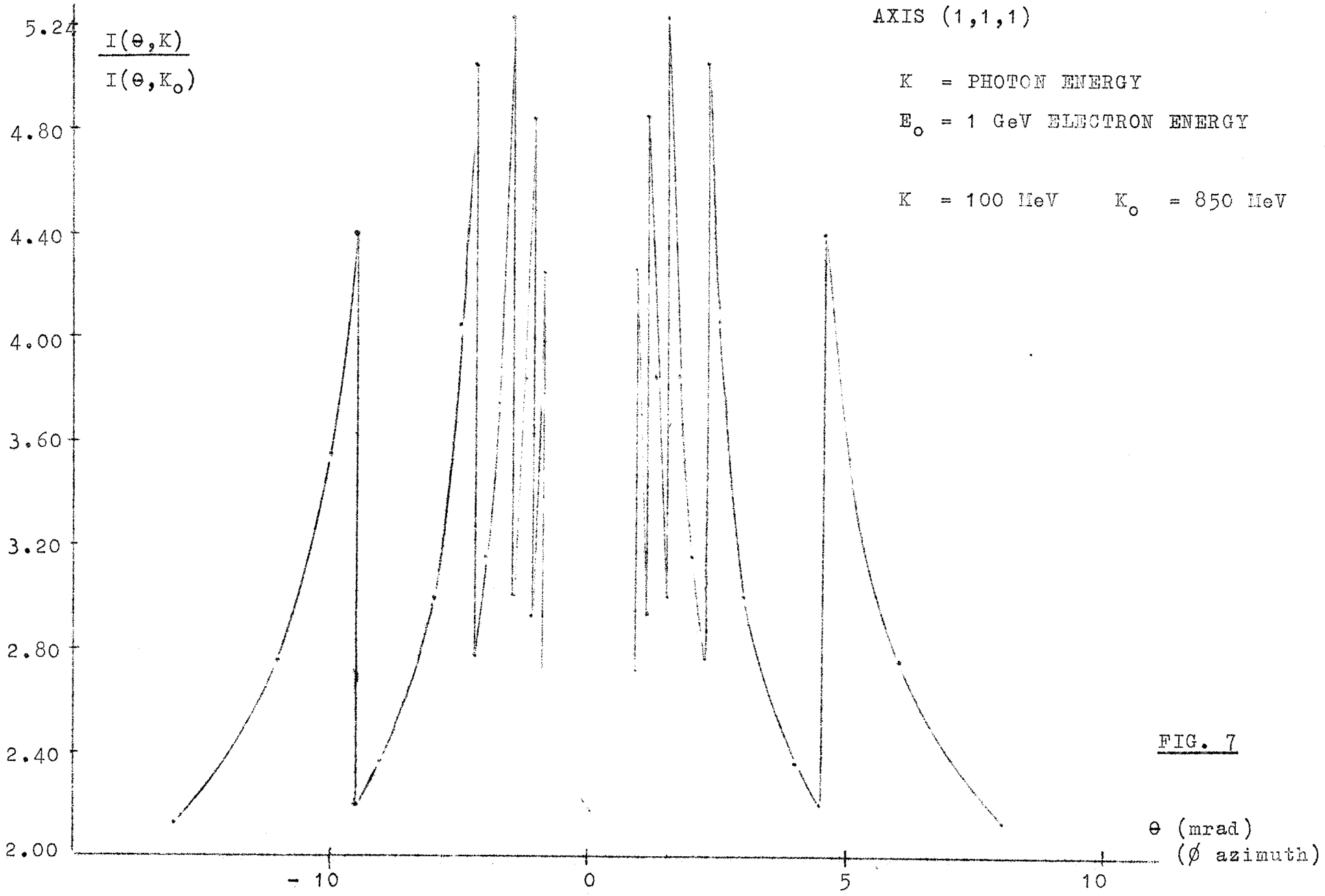

THEORETICAL BREMSSTRAHLUNG INTENSITY FROM DIAMOND SINGLE CRYSTALS
(T = 293°K) AXIS (1,1,0)
I(K)
K = PHOTON ENERGY
E_o = 1 GeV ELECTRON ENERGY
θ = 0
θ = 3.98 x 10^{-3} rad
θ = 6.30 x 10^{-3} rad
θ = 2.33 x 10^{-3} rad
120
60
40
20
0
200
400
600
800
1000
K(MeV)

FIG. 6

I(θ,K) = THEORETICAL BREMSSTRAHLUNG INTENSITY FROM SILICON SINGLE CRYSTALS

AXIS (1,1,1)



K = PHOTON ENERGY

E_o = 1 GeV ELECTRON ENERGY

K = 100 MeV K_o = 850 MeV

FIG. 7

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(1) Uberall, Phys. Rev. 103, 1055 (1956)

(2) Panofsky, Saxena, Phys. Rev. Lett. 2, 219 (1959)

(3) Frisch, Olson, Phys. Rev. Lett. 3, 141 (1959)

(4) Bologna, Diambrini, Murtas, Phys. Rev. Lett. 4, 572 (1960

(5) Barbiellini, Bologna, Diambrini, Murtas, to be published.

(6) This conclusions are obtained by Diambrini.