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J. DE WIRE:* NOTES ON π^0 PRODUCTION BY LINEARLY POLARIZED PHOTONS.

I. General Characteristics of Two Body Reactions with Polarized Photons.

Consider a partially linearly polarized beam of photons striking a target and producing a two-body reaction. Experimently it is convenient to fix the plane of the reaction, and so we will specify all directions with respect to this plane. We define the polarization

$$P = \frac{N_\perp - N_{/\!/}}{N_\perp + N_{/\!/}}$$

where $N_\perp$ and $N_{/\!/}$ are the numbers of incident photons with electric vectors perpendicular and parallel to the reaction plane. We will call $d\sigma_\perp$ the differential cross section for the reaction by a photon whose electric vector is perpendicular to the reaction plane and $d\sigma_{/\!/}$ the corresponding cross section for a photon with electric vector parallel to the pla

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ne. Then the counting rate for the reaction can be written as

$$C = k\,(N_\perp d\sigma_\perp + N_\parallel d\sigma_\parallel)$$

where k is a factor determined by the geometry of the experiment. This can be rewritten as

$$C = k(N_\perp + N_\parallel)\left[\tfrac{1}{2}(d\sigma_\perp + d\sigma_\parallel) + \tfrac{1}{2}P(d\sigma_\perp - d\sigma_\parallel)\right] =$$

$$= k\,(N_\perp + N_\parallel)\left[d\sigma + \tfrac{1}{2}P\,(d\sigma_\perp - d\sigma_\parallel)\right]$$

where $d\sigma$ is the cross section for the reaction with unpolarized photons.

If one uses a monocrystalline target to produce the photon beam, then there exists the possibility of having a beam with a constant magnitude of polarization $|P|$ but with a variable direction of the plane of maximum (positive) polarization, the direction being determined by the orientation of the crystal axis relative to the electron beam. If we call ϕ the angle between the plane of maximum polarization and the reaction plane, then a more general expression for C can be written

$$C = k\,(N_\perp + N_\parallel)\left[d\sigma - \tfrac{1}{2}|P|\,(d\sigma_\perp - d\sigma_\parallel)\cos 2\phi\right]$$

In the case of a beam from a crystal target, it would be convenient to measure with a fixed detection geometry the ratio of counting rates for the beam polarized perpendicular to the reaction plane ($\phi = \pi/2$) and parallel to it ($\phi = 0$). If we let $R_c = C_\perp / C_\parallel$ be this ratio and let $R_\sigma = d\sigma_\perp / d\sigma_\parallel$, the we can write

$$R_c = \frac{R_\sigma + 1 + |P|(R_\sigma - 1)}{R_\sigma + 1 - |P|(R_\sigma - 1)}$$

or

$$R_\sigma = \frac{|P|(R_c + 1) + (R_c - 1)}{|P|(R_c + 1) - (R_c - 1)}$$

Some curves illustrating this relation are shown in Fig. 1.

It is interesting to see how the error in R_σ de-

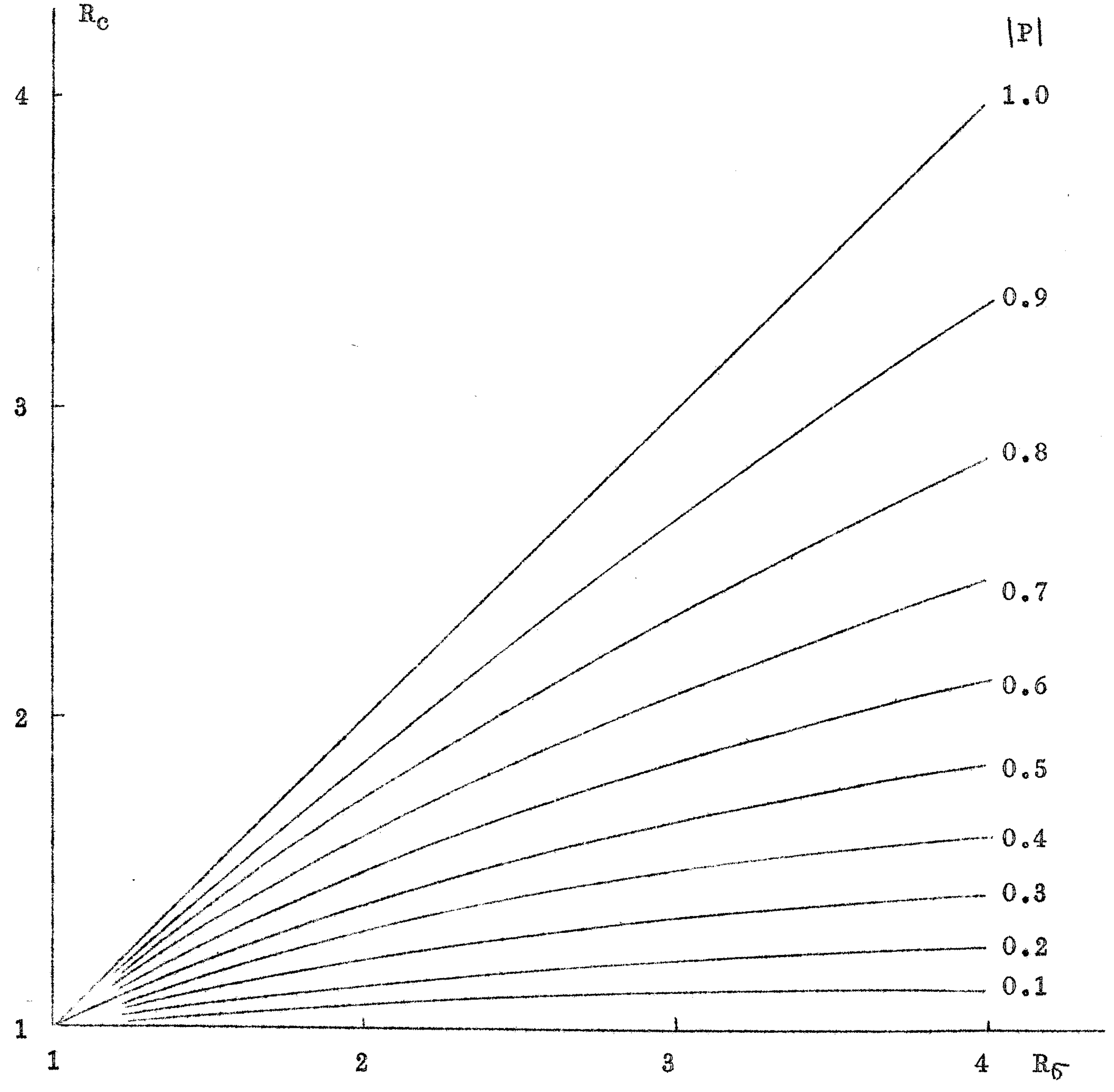


Fig. 1

pends on the errors in the measured quantities $|P|$ and R_c. We find the differential relation

$$\frac{dR_\sigma}{R_\sigma} = \frac{(R_\sigma+1)^2}{4PR_\sigma}\,\frac{dR_c}{R_c} - \frac{R_\sigma+1}{2}\,\frac{dP}{P}$$

If R_c and $|P|$ are measured independently, then we have for the relative errors

$$\frac{dR_\sigma}{R_\sigma} = \sqrt{\frac{(R_\sigma+1)^4}{16P^2R_\sigma^2}\left(\frac{dR_c}{R_c}\right)^2 + \frac{(R_\sigma+1)^2}{4}\left(\frac{dP}{P}\right)^2} = \ldots$$

$$= \sqrt{A\left(\frac{dR_c}{R_c}\right)^2 + B\left(\frac{dP}{P}\right)^2}$$

Some representative values of A and B for π^0 production are given here:

Values of A

B	R_σ	P	0.2	0.3	0.4	0.5	0.6
1	1		25	11	6.3	4.0	2.8
2.3	2		32	14	7.9	5.1	3.5
4.0	3		44	20	11	7.1	5.0
6.3	4		61	27	15	10	6.8

If we look at only the contribution of the statistical error in R_c to the error in R_σ, then since the statistical error in R_c in inversely proportional to the square root of the beam intensity I, we have

$$\frac{dR_\sigma}{R_\sigma}(stat.) \propto \frac{1}{P\sqrt{I}}$$

This shows that we can reduce the error in R_σ if we can increase the polarization of the beam by decreasing its intensity, provided that the rate of increase in polarization is at least half the rate of decrease in intensity.

II. Theoretical Predictions.

The most direct calculations on the production of π^0 mesons by polarized photons can be made from the theory of Chew, Goldberger, Low, and Nambu (CGLN) (Phys. Rev. 106, 1345 (1957)), who derive a formula for the cross section in terms of the phase shifts obtained from the analysis of the data on π, p scattering. This formula has been evaluated by Hohler and Mullensiefer (Zeit. f. Phys. 157, 30 (1959)) for unpolarized photons, giving values of the cross section in good agreement with the experimental data, at least in the region of the first resonance.

The CGLN formula can be written in the following form (all quantities in the cm system):

$$\frac{d\sigma}{d\Omega} = \sigma_0 f^2 \frac{q}{k} \Big| i\alpha\, \underline{\sigma}\cdot\underline{\varepsilon} + \beta\, \underline{q}\,(\underline{k}\times\underline{\varepsilon}) + i\gamma\, \underline{\sigma}\cdot\underline{k}\; \underline{q}\cdot\underline{\varepsilon} + i\delta\, \underline{\sigma}\cdot\underline{\varepsilon}\; \underline{q}\cdot\underline{k} + i\mu\, \underline{\sigma}\cdot\underline{q}\; \underline{q}\cdot\underline{\varepsilon} \Big|^2$$

where $\sigma_0 = \left(\frac{e^2}{\hbar c}\right)\left(\frac{\hbar}{m_{\pi^0} c}\right)^2 = 1.558 \times 10^{-28}\ cm^2$

f^2 = the pion-nucleon coupling constant ≈ 0.08 and (with $\hbar = c = m_{\pi^0} = 1$) q and k are the momenta of the pion and photon. $\underline{\sigma}$, $\underline{\varepsilon}$, $\underline{q}$, and $\underline{k}$ are vectors giving the proton spin, the photon polarization, the meson momentum, and the photon momentum.

The first term in the amplitude represents contributions from electric dipole production, leading to the S states; the second, P wave mesons produced by magnetic dipole interaction; the third and fourth contain contributions from both magnetic dipole and electric quadropole production of P wave mesons; and the fifth represents a correction coming from direct interaction with the recoil proton current, leading to S and D waves.

The amplitude coefficients can be written as follows:

$$\alpha = \frac{2}{3} i (\alpha_1 - \alpha_3) F_S + \frac{\omega}{1+\frac{\omega}{M}} (N^+ - \nu)$$

$$\beta = \lambda h^{(++)} + \frac{4}{9} i e^{i\alpha_{33}} \sin\alpha_{33} F_M$$

$$\gamma = \lambda h^{(+-)} - \frac{2}{3} i e^{i\alpha_{33}} \sin\alpha_{33} (F_Q + \frac{1}{3} F_M) - \frac{\nu}{\omega} \frac{1}{1+\frac{\omega}{M}}$$

$$\delta = -\lambda h^{(+-)} - \frac{2}{3} i e^{i\alpha_{33}} \sin\alpha_{33} (F_Q - \frac{1}{3} F_M) + \frac{\nu}{\omega} \frac{1}{1+\frac{\omega}{M}}$$

$$\mu = \frac{1}{M\omega} \frac{1}{1+\frac{\omega}{M}}$$

where α_1 and α_3 are the S wave scattering phase shifts for isospins 1/2 and 3/2

$$h^{(++)} = \frac{1}{3}(h_{11} + 2h_{13} + 2h_{31} + 4h_{33})$$

$$h^{(+-)} = \frac{1}{3}(h_{11} - h_{13} + 2h_{31} - 2h_{33})$$

$$h_{ij} = \frac{e^{i\alpha_{ij}} \sin\alpha_{ij}}{g^3}$$

α_{ij} is the P wave phase shift for isospin i/2 and angular momentum j/2

ω is the total energy minus the proton mass

M is the proton mass = 6.95

$$\nu = \frac{g_p + g_n}{2M} = \frac{2.793 - 1.913}{2 \times 6.95} = 0.063$$

$$\lambda = \frac{g_p - g_n}{4Mf^2} = \frac{0.1693}{f^2}$$

N^+ is a small undetermined real number that comes from the dispersion relation calculation of the electric dipole amplitude. We will use $N^+ = 0.03$ which gives a reasonable fit to the cos θ coefficient in the π^0 angular distribution.

$$F_S = 1 - \frac{1}{2} F$$

$$F_M = \frac{3}{4q^2} F \qquad\qquad F = 1 + \frac{1-\beta^2}{2\beta} \ln \frac{1-\beta}{1+\beta}$$

$$F_Q = \frac{1}{\omega^2}\left(1 - \frac{3}{4\beta^2}\right) F$$

where β is the pion velocity

The terms involving the F's come from the interaction of the photon with the charge of the meson-nucleon system in the static limit ($M = \infty$) and hence can be interpreted as a process involving the interaction with a charged meson which the undergoes charge exchange. The terms including the factor ν are corrections in first order in 1/M to the static limit approximation.

If we call θ the usual polar angle between $\underline{k}$ and $\underline{q}$ and ϕ the angle between $\underline{\epsilon}$ and the production plane including $\underline{k}$ and $\underline{q}$, we find by squaring the amplitude and averaging over the direction of the proton spin

$$\frac{d\sigma}{d\Omega} = \sigma_0 f^2 \frac{q}{k} \Big[|\alpha|^2 + |\beta|^2 q^2 k^2 \sin^2\theta \sin^2\phi + |\gamma|^2 q^2 k^2 \sin^2\theta \cos^2\phi +$$
$$+ |\delta|^2 q^2 k^2 \cos^2\theta + |\mu|^2 q^4 \sin^2\theta \cos^2\phi +$$
$$+ 2\,\mathrm{Re}\,|\alpha^*\delta|\, q k \cos\theta +$$
$$+ 2\,\mathrm{Re}\,|\alpha^*\mu|\, q^2 \sin^2\theta \cos^2\phi +$$
$$+ 2\,\mathrm{Re}\,|\delta^*\mu|\, q^3 k \sin^2\theta \cos\theta \cos^2\phi +$$
$$+ 2\,\mathrm{Re}\,|\gamma^*\mu|\, q^3 k \sin^2\theta \cos\theta \Big]$$

For our purposes, we can neglect all terms involving since they contribute effects no larger than several percent except for the terms involving cos θ which do not concern us at the moment. Even for the cos θ term the contribution from μ is inly about 10%.

We can now write down the differential cross section for unpolarized photons by letting $\sin^2 \phi = \cos^2\phi = \frac{1}{2}$.

If we write the cross section as

$$\left.\frac{d\sigma}{d\Omega}\right|_{unpol.} = A + B\cos\theta + C\cos^2\theta$$

then

$$A = \sigma_0 f^2 \frac{q}{k}\left[|\alpha|^2 + \frac{1}{2} q^2 k^2 \left\{|\beta|^2 + |\gamma|^2\right\}\right]$$

$$B = \sigma_0 f^2 \frac{q}{k}\, 2qk \; R_e\, |\alpha^* \delta|$$

$$C = \sigma_0 f^2 \frac{q}{k}\left[q^2 k^2 \left\{|\delta|^2 - \frac{1}{2}|\beta|^2 - \frac{1}{2}|\gamma|^2\right\}\right]$$

We can also write the ratio of the cross section for photons polarized perpendicular and parallel to the production plane. For $\theta = 90°$ we get

$$R_\sigma = \frac{d\sigma_\perp}{d\sigma_\parallel} = \frac{|\alpha|^2 + q^2k^2|\beta|^2}{|\alpha|^2 + q^2k^2|\gamma|^2}$$

It is convenient and instructive to write these expressions in approximate form, using small angle approximatiens for the small phase shifts and keeping only those terms including the 3,3 phase and interference terms between the small phases and the 3,3 term. Little accuracy is lost, and we will be able to see more clearly the effect of the small phase shifts. We have

$$|\alpha|^2 = \frac{4}{9}(\alpha_1 - \alpha_3)^2 F_S^2 + \frac{\omega^2}{(1+\frac{\omega}{m})^2}(\nu^+ - \nu)^2$$

$$|\beta^2| = \frac{16}{9}\frac{\lambda^2}{q^6}\sin^2\alpha_{33}\left[1 + \frac{1}{9}\frac{q^2}{\lambda^2}F_M^2 + \frac{1}{2}\left(\operatorname{ctn}\alpha_{33} - \frac{1}{3}\frac{q^3}{\lambda}F_M\right)(\alpha_{11} + 2\alpha_{13} + 2\alpha_{31})\right]$$

$$|\gamma|^2 = \frac{4}{9}\frac{\lambda^2}{q^6}\sin^2\alpha_{33}\left[1 + \frac{q^6}{\lambda^2}\left(F_Q + \frac{1}{3}F_M\right)^2 - \left\{\operatorname{ctn}\alpha_{33} - \frac{q^3}{\lambda}\left(F_Q + \frac{1}{3}F_M\right)\right\} \cdot \right.$$

$$\left. \cdot(\alpha_{11} - \alpha_{13} + 2\alpha_{31}) + 3\frac{q^3}{\lambda}\operatorname{ctn}\alpha_{33}\frac{\nu}{\omega}\frac{1}{1+\frac{\omega}{m}}\right]$$

$$|\delta|^2 = \frac{4}{9}\frac{\lambda}{g^6}\sin^2\alpha_{33}\Big[1+\frac{g^6}{\lambda^2}(F_Q-\tfrac{1}{3}F_M)^2-\Big\{\operatorname{ctn}\alpha_{33}+\frac{g^3}{\lambda}(F_Q-\tfrac{1}{3}F_M)\Big\}\cdot$$

$$\cdot(\alpha_{11}-\alpha_{13}+2\alpha_{31})-3\frac{g^3}{\lambda}\operatorname{ctn}\alpha_{33}\frac{\nu}{\omega}\frac{1}{1+\frac{\omega}{M}}\Big]$$

$$Re|\alpha^*\delta| = \frac{\lambda}{3}\frac{\omega}{1+\frac{\omega}{M}}(N^+-\nu)\frac{1}{g^3}\sin 2\alpha_{33}+\frac{4}{9}(\alpha_1-\alpha_3)F_S\frac{\lambda}{g^3}\sin^2\alpha_{33}$$

To evaluate these expression we chose

$$f^2 = 0.081 \quad \text{and} \quad N^+ = 0.03$$

which seem to give the best fit to the π^0 production data. Then

$$\sigma_0 f^2 = 12.6\times10^{-30}\,cm^2, \quad \lambda = 2.09, \quad N^+-\nu = -0.033$$

Various kinematic variables are given in the accompaning table for a number of laboratory photon energies $h\nu$.

In order to compute the cross sections we must find the values of the various phase shifts from the experiments on π^-,p scattering. We must use phase shifts corresponding to the same momentum of the pion in the two experiments. In general for the π,p scattering, since one deals with charged pions, one uses units in which $m_{\pi^+} = 1$. In this system we call the pion momentum q_+. Then we have

$$m_{\pi^0}\,q = m_{\pi^+}q_+ \;; \quad q = \frac{m_{\pi^+}}{m_{\pi^0}}q_+ = 1.034\,q_+$$

Probably the most recent analysis of π,p scattering in the energy interval corresponding to our region of interest has been made by Barnes (Barnes et al, Phys. Rev. 117, 226 and 238 (1960)). Other sources of experimental data are

Ashkin et al, Phys. Rev. 105, 724 (1957)

Puppi, Geneva Conference, 1958

Pontecorvo, Kiev Conference, 1959

Chiu and Lomon, Ann of Phys. 6, 50 (1959)

It is difficult to obtain meaningful values from these analyses since the variation among the results is greater than the experimental errors. We can only make some estimates based on these numbers.

For the S wave phase shifts we can use the points in the graph to find a band of possible values as shown. (Fig. 2). From this we find

$h\nu$	$\alpha_1 - \alpha_3$	$\lvert\alpha\rvert^2$
200	0.215 ± 0.015	0.0163 ± 0.0023
250	0.266 ± 0.014	0.0207 ± 0.0023
300	0.327 ± 0.023	0.0272 ± 0.0038
320	0.368 ± 0.032	0.0326 ± 0.0064
350	0.400 ± 0.040	0.0368 ± 0.0072

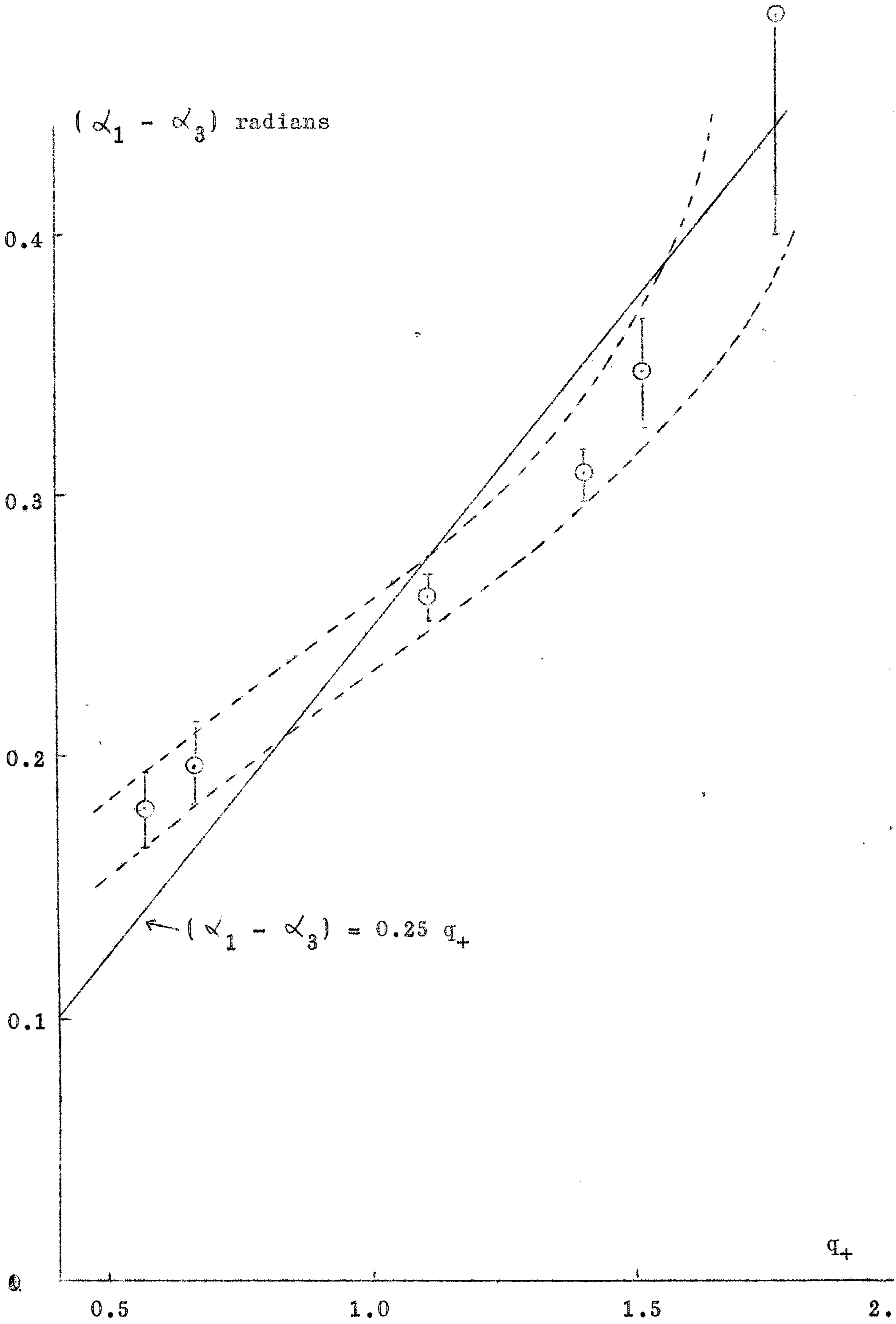
For the values of α_{33} we plot various experimental values (Fig. 3) together with some values which were computed from the effective range formula of Chew and Low, using $f^2 = 0.081$ and the value of ω at the resonance = 2.11. We get the following values for α_{33}:

$h\nu$	α_{33}	$\sin^2\alpha_{33}$	$\sin 2\alpha_{33}$
200	6.9° ± 1	0.0144 ± 0.0030	0.239 ± 0.035
250	24.0° ± 1.5	0.165 ± 0.020	0.743 ± 0.034
300	64.0° ± 2	0.784 ± 0.050	0.788 ± 0.045
330	90.0° ± 1.5	1.000 ± 0.001	0 ± 0.06
350	101.5° ± 1.5	0.960 ± 0.11	- 0.339 ± 0.047

For the small P wave phase shifts, we plot (Fig. 4) the two linear combinations that appear in the formulas. from these we make the following assignments:

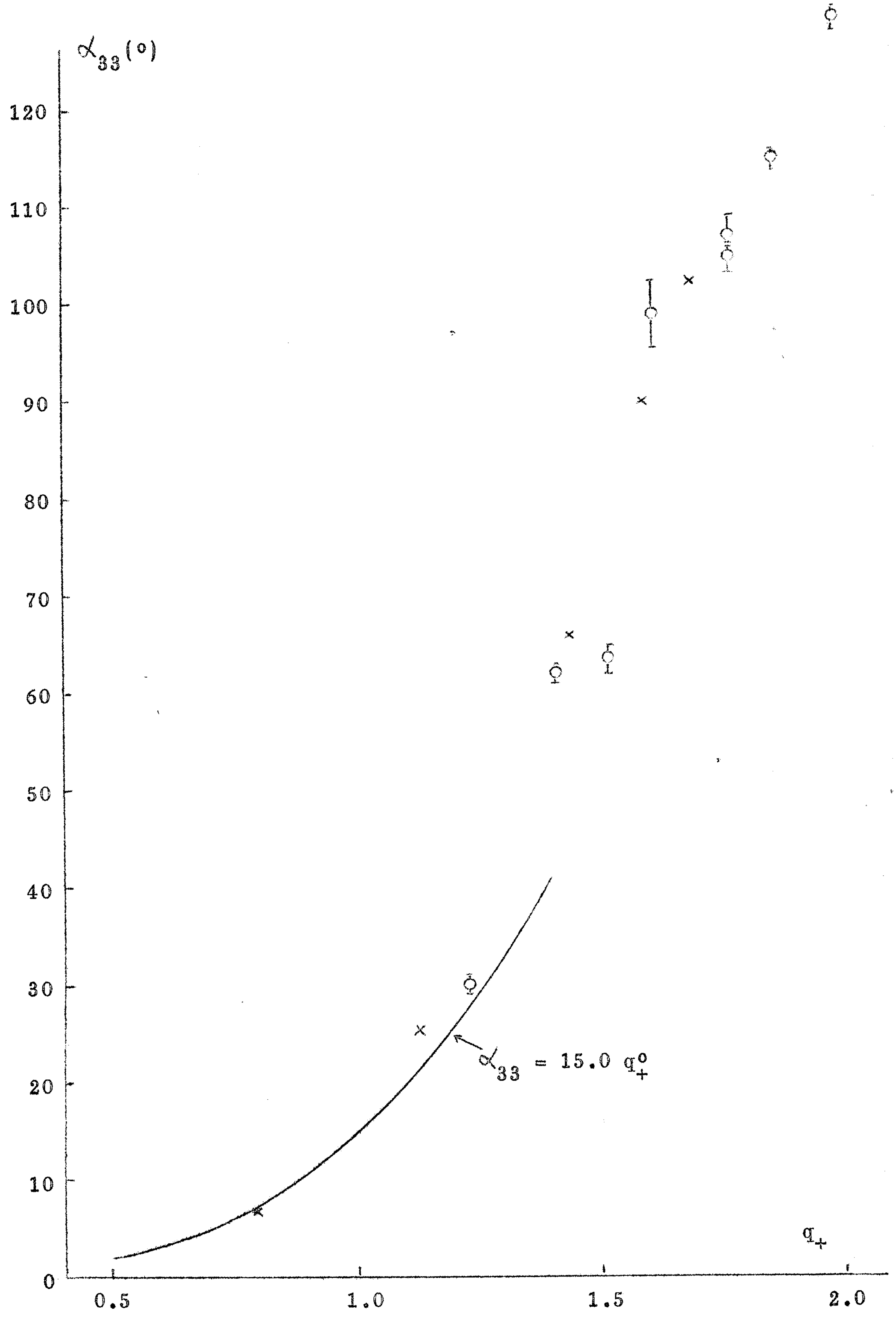
$h\nu$	$\alpha_{11} + 2\alpha_{13} + 2\alpha_{31}$	$\alpha_{11} - \alpha_{13} + 2\alpha_{31}$
200	- 0.080 ± 0.040	- 0.035 ± 0.025
250	- 0.135 ± 0.020	- 0.130 ± 0.025
300	- 0.175 ± 0.035	- 0.290 ± 0.060
330	- 0.170 ± 0.060	- 0.305 ± 0.085
350	- 0.150 ± 0.095	- 0.235 ± 0.110

We now compute values $\lvert\beta\rvert^2$, $\lvert\gamma\rvert^2$, and $\lvert\delta\rvert^2$ under the following three assumptions:



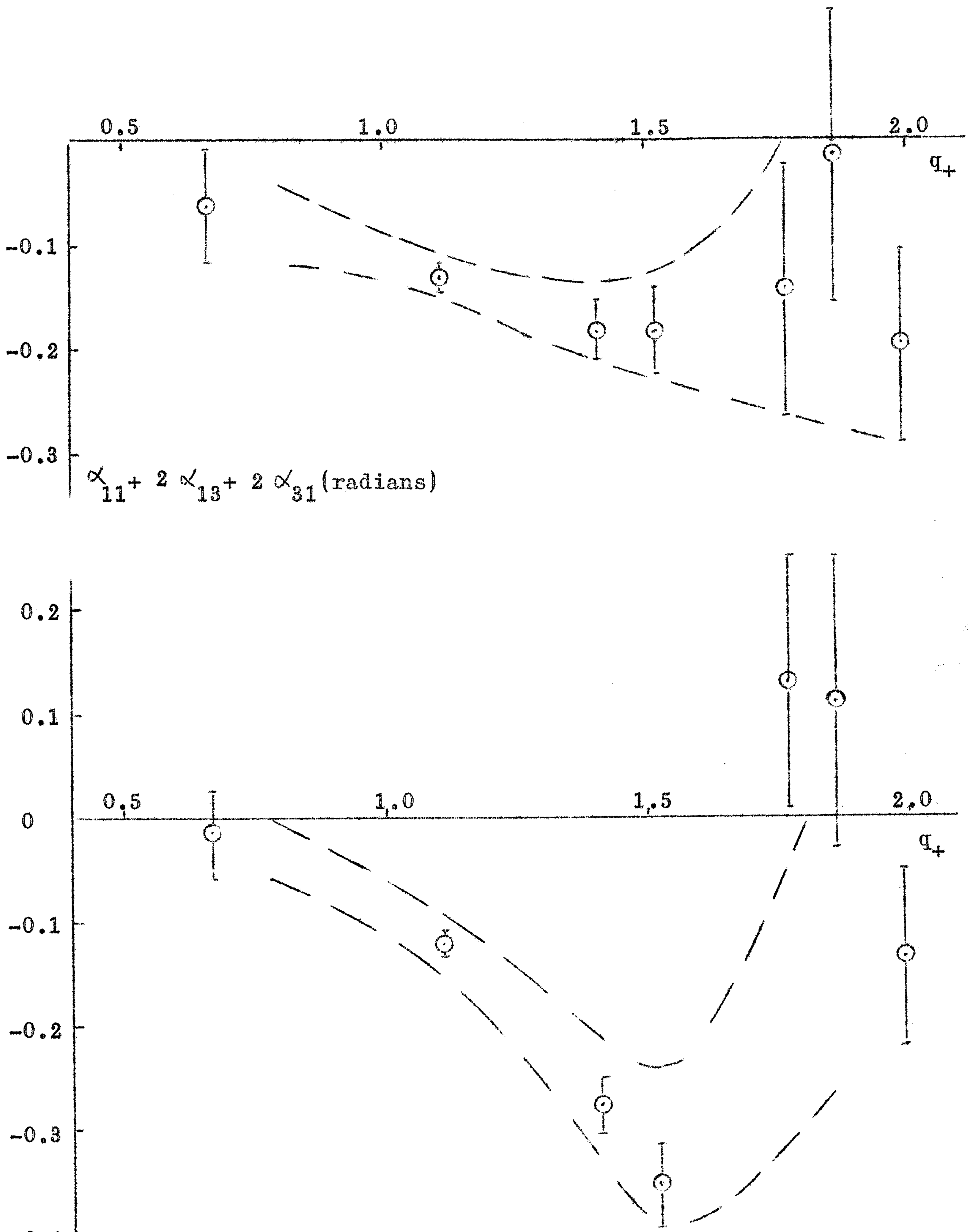
$(\alpha_1 - \alpha_3)$ radians
0.4
0.3
0.2
0.1
0
$(\alpha_1 - \alpha_3) = 0.25\ q_+$
q_+
0.5
1.0
1.5
2.

Fig. 2



$\alpha_{33}(^o)$
120
110
100
90
80
70
60
50
40
30
20
10
0
$\alpha_{33} = 15.0\ q_+^o$
q_+
0.5
1.0
1.5
2.0

Fig. 3



0.5
1.0
1.5
2.0
q_+
-0.1
-0.2
-0.3
$\alpha_{11} + 2\,\alpha_{13} + 2\,\alpha_{31}$ (radians)
0.2
0.1
0
0.5
1.0
1.5
2.0
q_+
-0.1
-0.2
-0.3
-0.4
$\alpha_{11} - 2\,\alpha_{13} + 2\,\alpha_{31}$ (radians)

Fig. 4

i. We have only the 3/2, 3/2 state produced through magnetic interaction with the magnetic moment of the pion-nucleon system and through magnetic dipole and electric interaction with the pion current.

ii. Effects included in (i) plus the proton current term, correction to the static model.

iii. Effects included in (i) and (ii) plus the effects of the small P states.

We get, ignoring the errors for the present:

$|\beta|^2$

$h\nu$	i and ii	iii
200	0.369	0.247
250	0.503	0.432
300	0.562	0.543
330	0.404	0.408
350	0.263	0.270

$|\gamma|^2$

$h\nu$	i	ii	iii
200	0.0929	0.1170	0.1434
250	0.1292	0.1485	0.1805
300	0.1490	0.1572	0.1661
330	0.1089	0.1089	0.0997
350	0.0718	0.0697	0.0616

$|\delta|^2$

200	0.0922	0.0682	0.0950
250	0.1255	0.1062	0.1421
300	0.1397	0.1315	0.1540
330	0.1003	0.1003	0.1024
350	0.0651	0.0672	0.0652

$h\nu$	Re $/\alpha^*\delta/$	B	
200	− 0.0067	− 0.11	b/ster
250	− 0.0122	− 0.42	
300	− 0.0032	− 0.18	
330	+ 0.0055	+ 0.37	
350	+ 0.0057	+ 0.58	

Now we compute A, C, and R_σ under four assumptions:

I. Same as (i)

II. (i) plus electric dipole S wave production

III. II plus proton current term.

IV. III plus small P waves

A (μb/ster)

hν	I	II	III	IV
200	1.99	2.13	2.23	1.91
250	9.43	9.64	9.94	9.64
300	25.6	25.9	26.2	26.1
330	26.6	27.0	27.0	26.6
350	22.1	22.5	22.4	22.1

C (μb/ster)

200	- 1.20	- 1.51	- 0.86
250	- 5.68	- 6.58	- 4.90
300	-15.6	-16.4	-14.5
330	-17.3	-17.3	-15.7
350	-13.5	-13.2	-13.3

R_σ

200	3.97	3.54	2.91	1.65
250	3.89	3.74	3.28	1.65
300	3.77	3.70	3.51	3.21
330	3.71	3.62	3.62	3.99
350	4.04	3.55	3.65	4.22

These are plotted in the graphs (Fig. 5 - Fig. 6). The exact shapes of the calculated curves for A and C in the neighborhood of the maximum may differ somewhat from those shown. More points should be calculated in this region.

We can make a rough estimate of the uncertainties in these predictions due to the limited accuracy of the present experimental data. The curves corresponding to case I are known as well as the 90° data on π^0 produc-

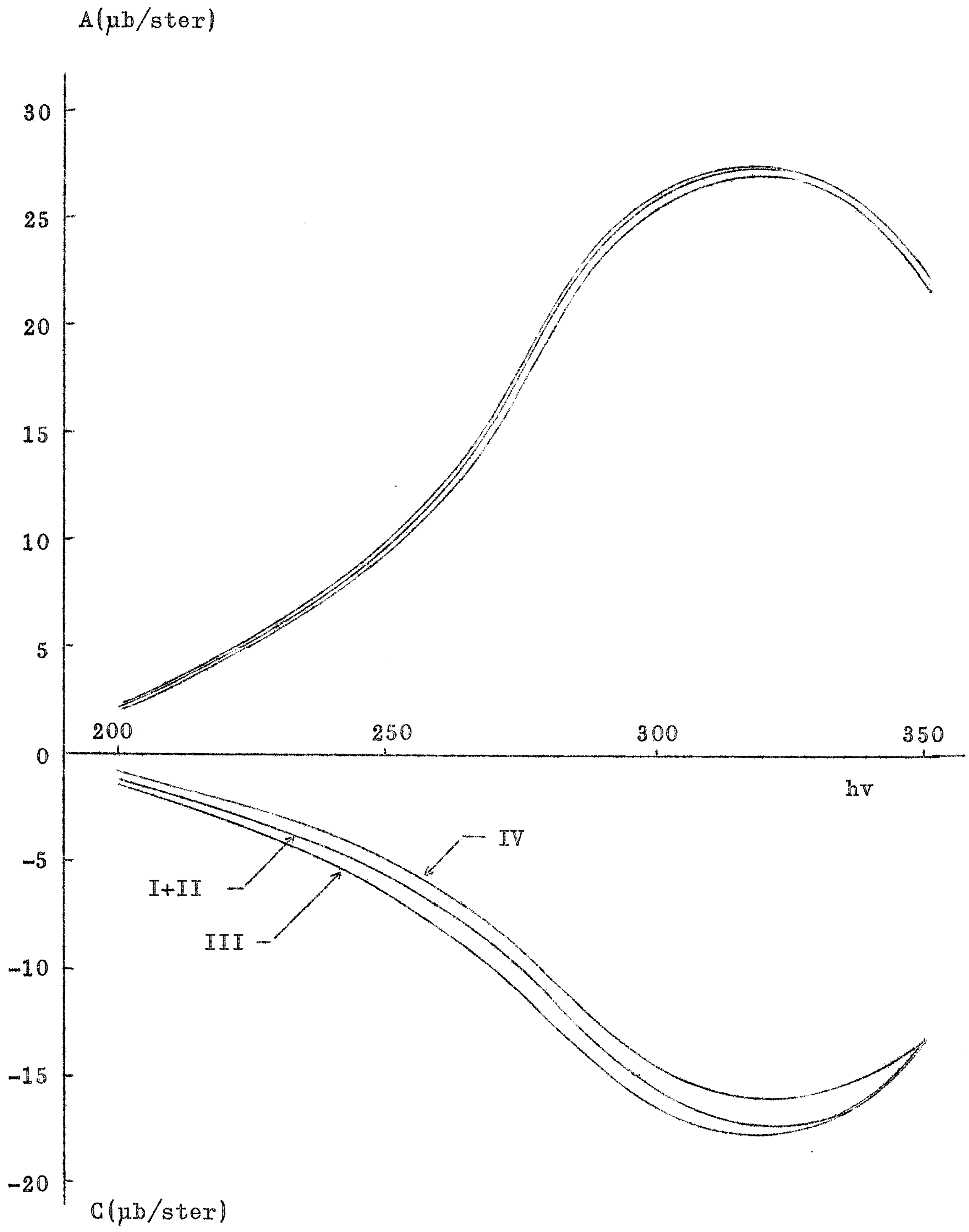

A(μb/ster)
30
25
20
15
10
5
0
-5
-10
-15
-20
200
250
300
350
hν
IV
I+II
III
C(μb/ster)

Fig. 5

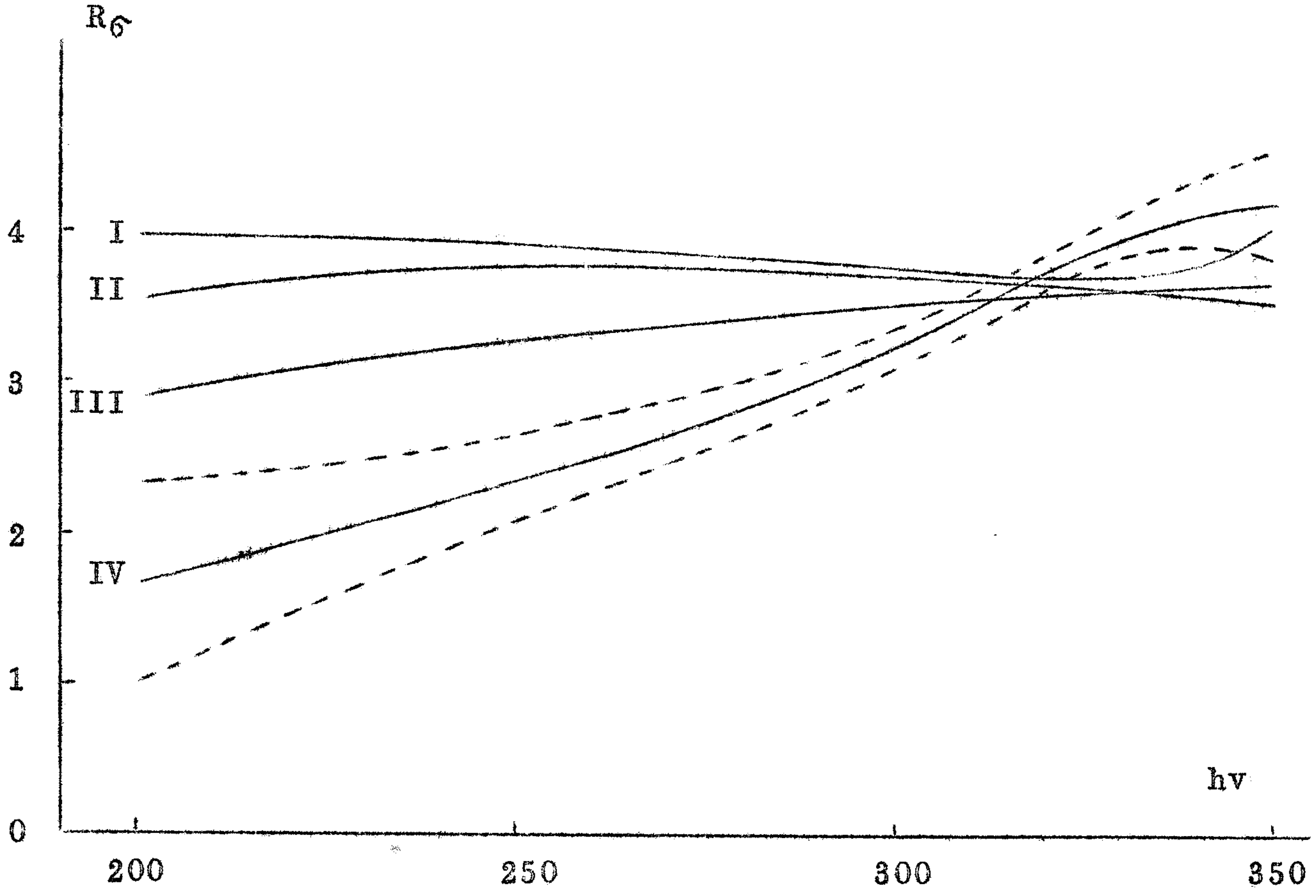


Fig. 6

tion , since we see that the other contributions to A are negligible. Since the correction added in III also depends only α_{33} (apart from limitations in the theory itself), we can say that the errors in IV come predominantly from the uncertainties in the small P wave phase shifts and are of the same relative magnitude as the errors in the phases themselves. On this basis we have drawn on the graph of R_{σ} (Fig. 6) the dashed lines corresponding to the errors in IV.

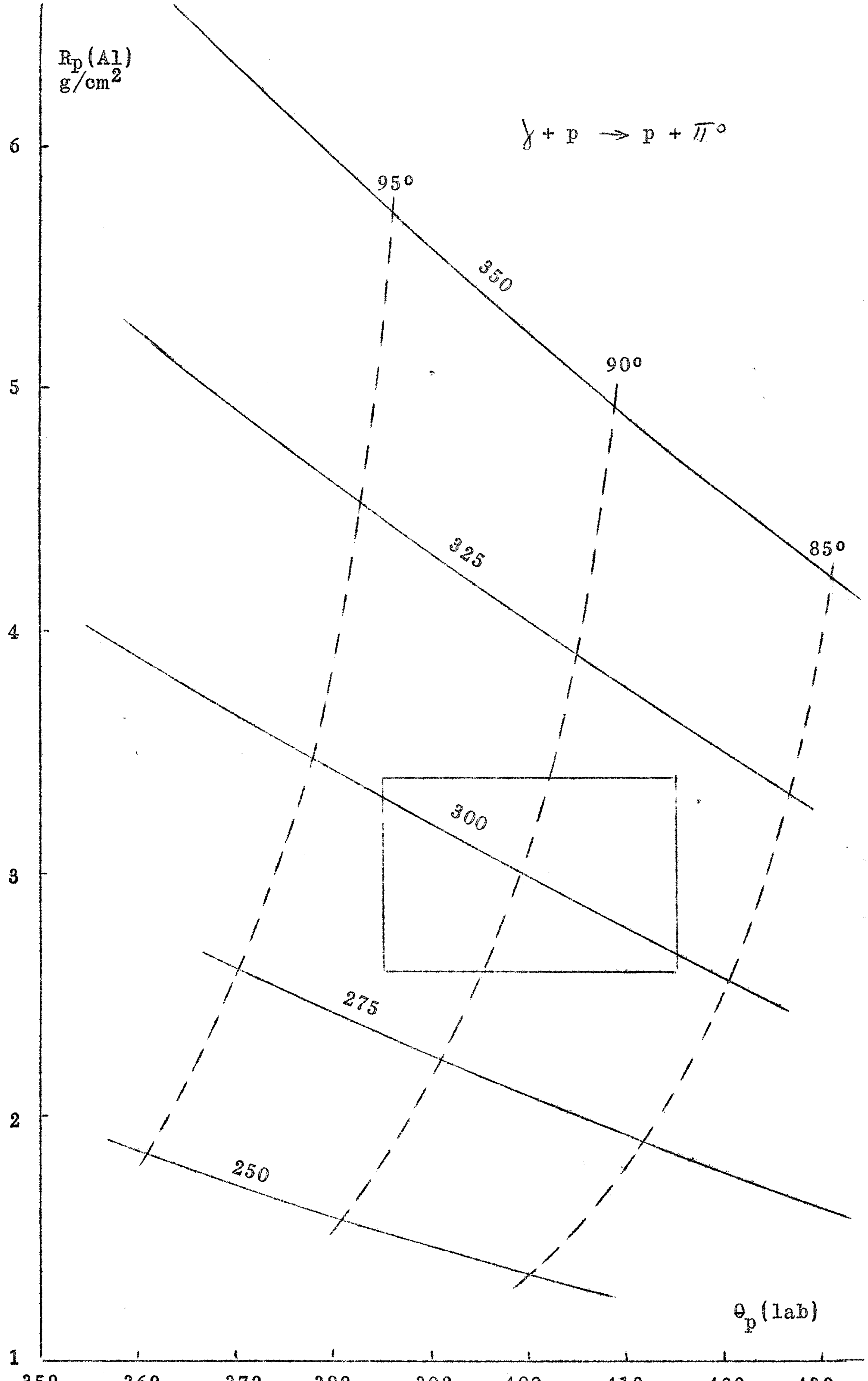
III. Experimental Considerations.

We want to use a proton range telescope in coincidence with a Cerenkov photon shower counter to detect the process

$$\gamma + p \rightarrow p + \pi^{0}$$

with polarized photons. First we want to make a first approximation for the counting rate we can expect. For sources of information on the kinematics of the rection , we have the tables and curves of Cortellessa and Reale above 300 MeV and the tables of Malmberg and Koester (Illinois, 1953) for lower energies. From these sources and Sternheimer's range-energy calculations (Sternheimer, Phys. Rev. **115**, 137, (1959)), we have made the curves of R_{p} (Al), the recoil proton range in aluminum, plotted against the proton laboratory angle for various energies of the incident photons (Fig. 7). The dashed curves correspond to three values of the π^{0} center-of-mass angle.

The proton telescope to a first approximation determines a rectangle on this graph within which it is sensitive. For our example we have chosen a width of 3° and a height of 0.8 gm/cm^2 Al, the latter determined by the distribution of absorbers in the counter telescope.



Rp(Al)
g/cm2
6
5
4
3
2
1
γ + p → p + π°
95°
90°
85°
350
325
300
275
250
θp(lab)
35°
36°
37°
38°
39°
40°
41°
42°
43°

Fig. 7

The rectangle of course is not sharply defined, its edges made diffuse by the finite dimensions of the hydrogen target and (in general to a lesser extent) by the pulse height resolution of the counters. This rectangle determines that part of the Bremsstrahlung spectrum to which the system is sensitive. We can write for the counting rate

$$C = \left(\frac{d\sigma}{d\Omega}\right)_{cm} N n \,\varepsilon (\Delta\Omega) \left.\frac{d\Omega_{cm}}{d\Omega_{lab}}\right/_p$$

where $\left(\frac{d\sigma}{d\Omega}\right)_{cm}$ is the differential cross section

- N is the number of protons/cm^2 in the hydrogen target
- n is the rate of arrival of the incident photons to which the system is sensitive
- ε is the efficiency of the Cerenkov counter for detecting the π^0.
- $(\Delta\Omega)$ is the solid angle determined by the defining proton counter
- $\left.\frac{d\Omega_{cm}}{d\Omega_{lab}}\right/_p$ is the proton solid angle transformation, given in the tables

We estimate the following values:
For N we have per centimeter of liquid hydrogen 4.3×10^{22}. For a flat target 2 cm thick set perpendicular to the direction of the recoil proton, we have a thickness of $2/\sin 40^\circ =$ $= 3.1$ cm, so $N = 1.3 \times 10^{23}$. For $(\Delta\Omega)$ we use a defining counter 3° square so

$$(\Delta\Omega) = \left(\frac{3}{57.3}\right)^2 = 0.0028$$

For n we can use the Bremsstrahlung approximation $n =$ $= Q\frac{\Delta h\nu}{h\nu}$ where Q is the beam rate in effective quanta per unit time. From the kinematics curves we have $\Delta h\nu = 20$ MeV and $h\nu = 300$ MeV, so $n = Q/15$. From the tables, $\left.\frac{d\Omega_{cm}}{d\Omega_{lab}}\right/_p =$

$= 1/0.273$ for 300 MeV and 90° c.m. For ε we can use the curves giving the probability of π^0 decay into a given cone in the direction of motion of the π^0, where we want to place the Cerenkov counter. This probability is given by

$$P = \frac{(1+\beta)(1-\cos\theta)}{(1-\beta\cos\theta)}$$

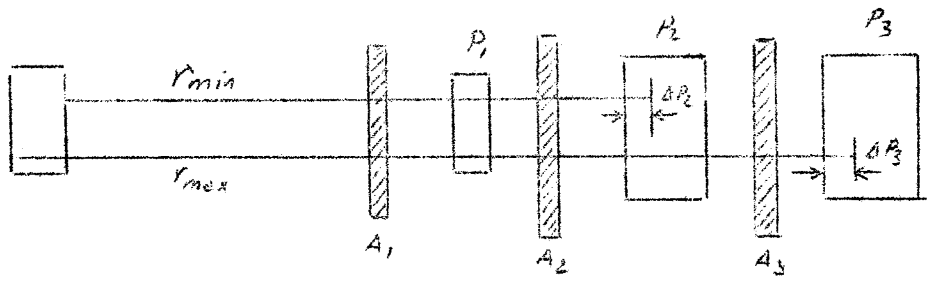
where β is the velocity of the π^0. For a counter aperture of 18 cm diameter at a distance of 40 cm we have θ = 12.7° and ε = 0.26 (Fig. 8).

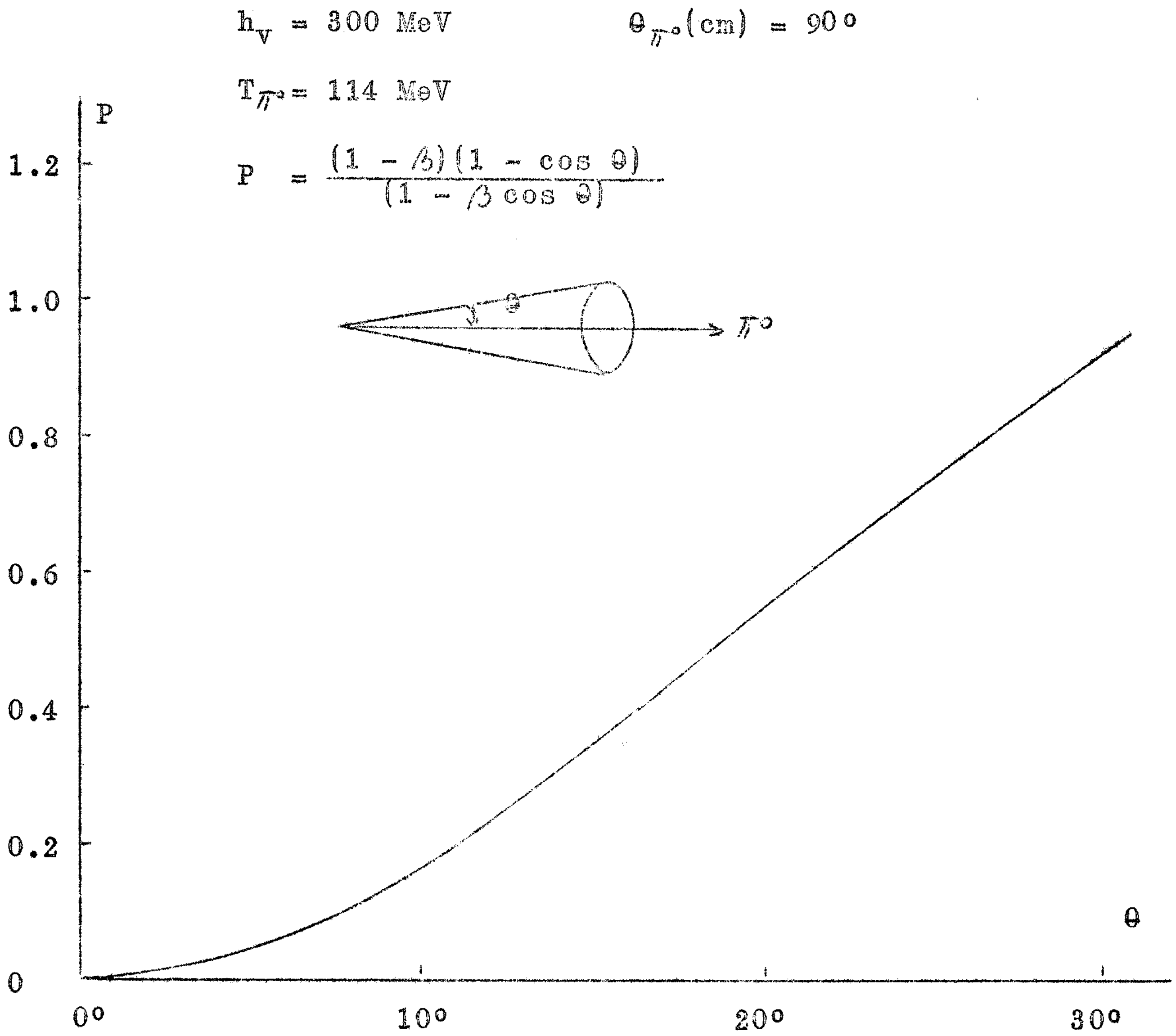
If we use the cross section for unpolarized photons (26 μb/ster, we get C = 6.0 x 10^{-10} Q. For a beam of 10^{10} effective quanta/ min we can expect an average counting rate of 16 counts/min.

For 250 MeV, the cross section and ε are lower but we have more incident photons in the same energy interval and a more favorable solid angle transformation. The net result is that we would need a 5° x 5° proton aperture to obtain a similar counting rate.

Since we operate far below the Bremsstrahlung upper limit, there is the problem of the background due to π^0 pairs. Little is known about the π^0 pair cross section and still less is known about the effectiveness of the Cerenkov counter in reducing the background . We believe, however, that we should have only several percent π^0 pair background to contend with .

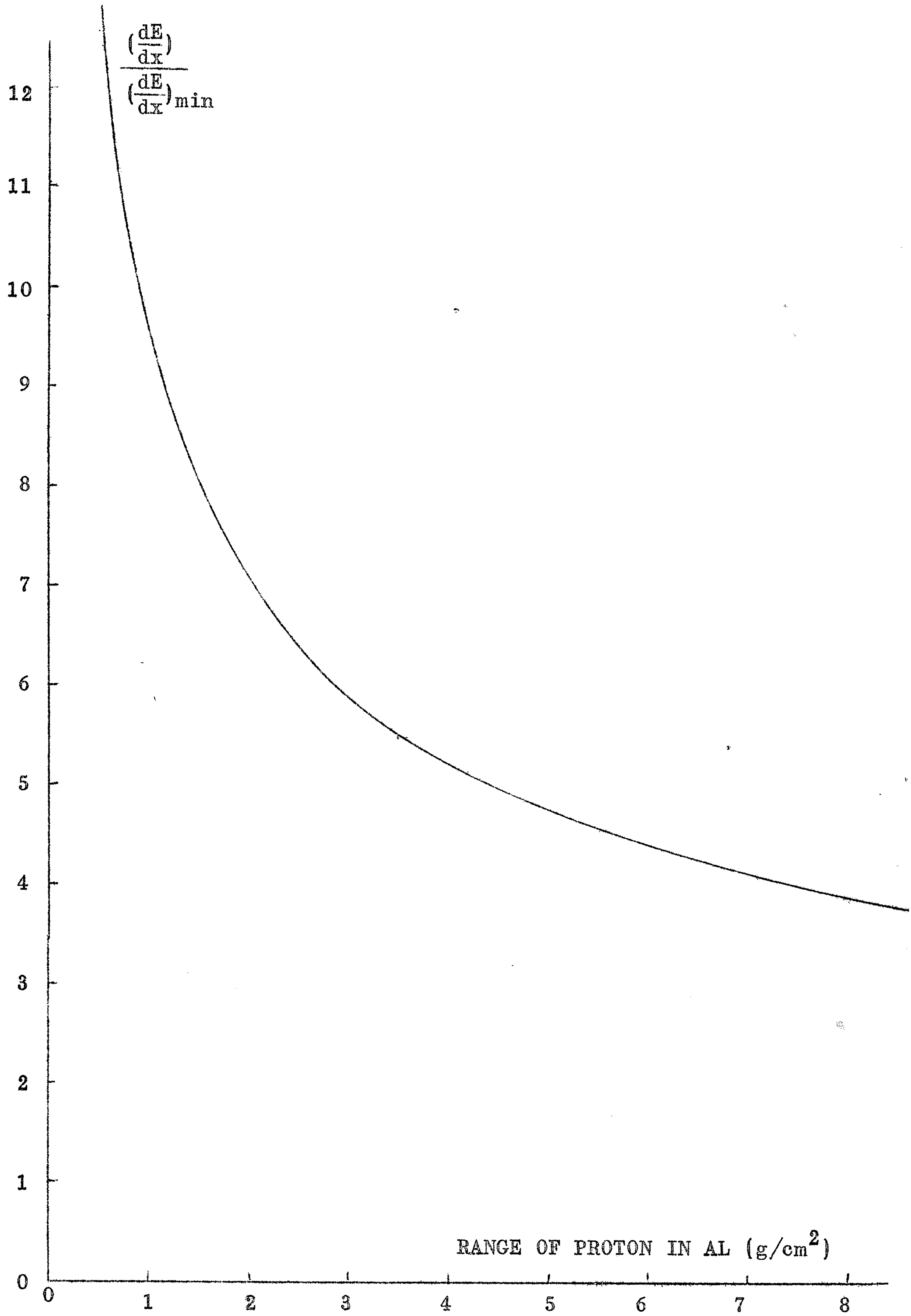
The range-energy curves, the energy loss curve for protons (Fig. 9) and the proton-meson energy loss ratio (Fig. 10) are useful for setting up the proton telescope. The following schematic diagram can illustrate some of the problems involved:





h_ν = 300 MeV
θ_{π°(cm) = 90°
T_{π° = 114 MeV
$P = \frac{(1-\beta)(1-\cos\theta)}{(1-\beta\cos\theta)}$
θ
π°
P
1.2
1.0
0.8
0.6
0.4
0.2
0
0°
10°
20°
30°
θ

Fig. 8



$\frac{(\frac{dE}{dx})}{(\frac{dE}{dx})_{min}}$
12
11
10
9
8
7
6
5
4
3
2
1
0
RANGE OF PROTON IN AL (g/cm^2)
0
1
2
3
4
5
6
7
8

Fig. 9

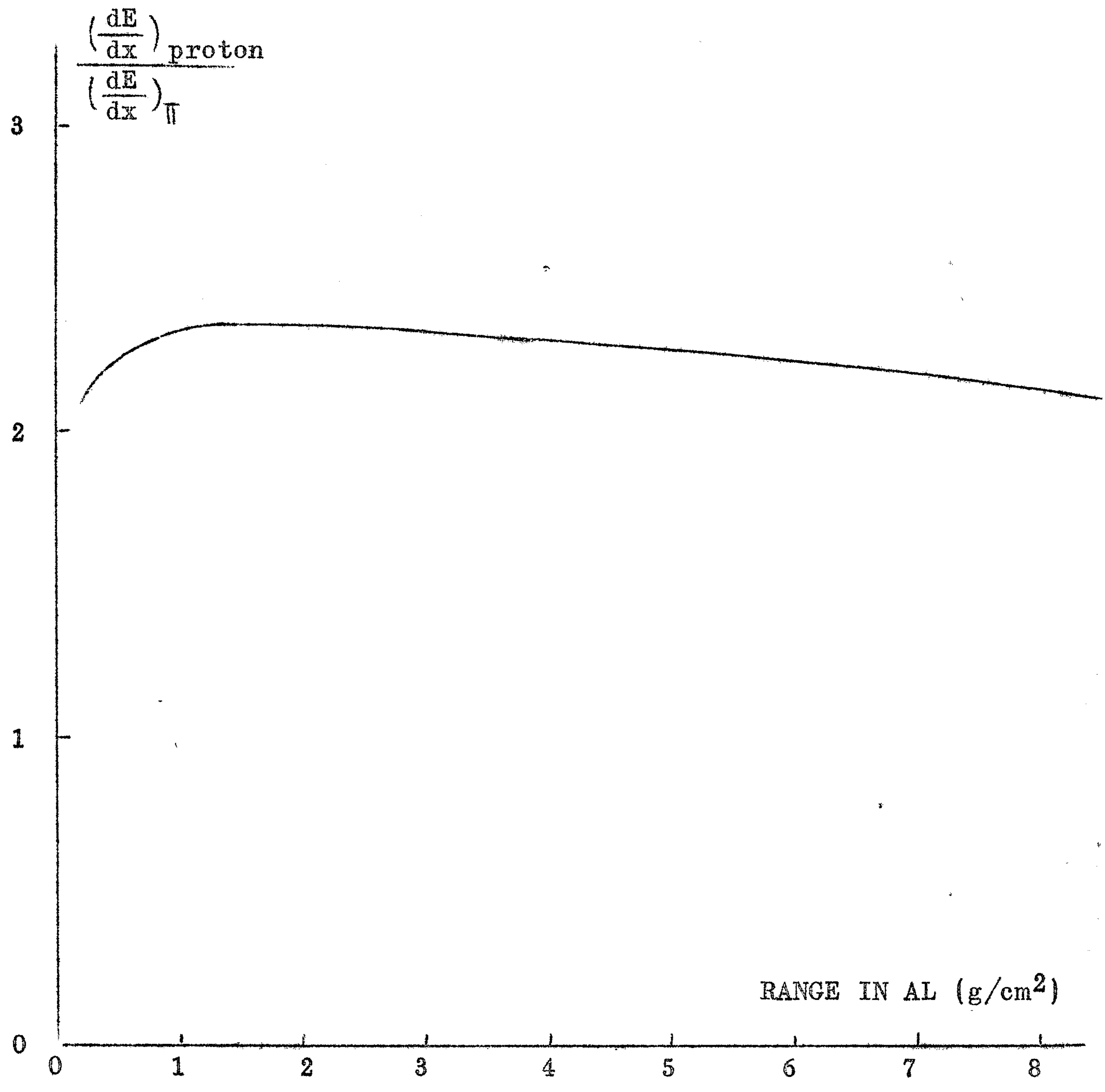
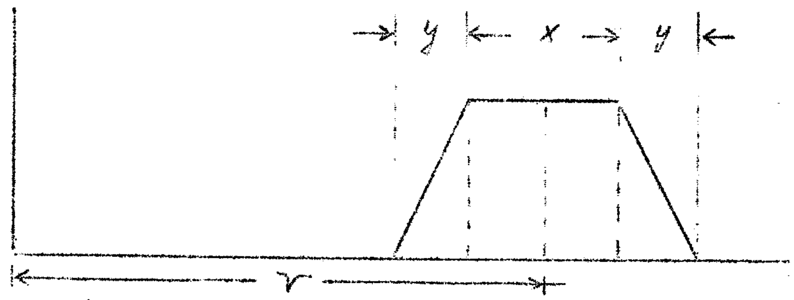

$\frac{\left(\frac{dE}{dx}\right)_{proton}}{\left(\frac{dE}{dx}\right)_{\pi}}$
3
2
1
0
RANGE IN AL (g/cm^2)
0
1
2
3
4
5
6
7
8

Fig. 10

We want to count the coincidence combination P1 + P2 - P3, where the P's represent the three proton counters. We will also use T, A_1, P_1, etc to represent the equivalent thicknesses of the target, the absorbers, and the counters in gm/cm^2 Al, computed on the basis of the curves of equivalent energy losses for various materials. In the diagram we show the protons of the maximum and minumum range, which will be counted by the telescope. The resolution in range if the telescope is a trapezoid function



where $r = T/2 + A1 + P1 + A2 + P2 + \frac{1}{2}(P2 - \Delta P2 + A3 + \Delta P3)$. $\Delta P2$ and $\Delta P3$ are the penetrations into the last two counters to give pulses above the biases. x and y are given by

$$x = P2 + A3 + \Delta P3 - \Delta P2$$

$$y = T.$$

Only the sum of A2 and A3 is determined. It is advantageous to split it into the two parts to minimize the background.

TABLE OF VARIOUS KINEMATIC VALUES

$h\nu$	κ	q	ω	β	F_S	F_m	F_Q
200	1.24	0.82	1.33	0,633	0.852	0.329	0.249
250	1.48	1.17	1.64	0.763	0.774	0.248	0.156
300	1.73	1.49	1.94	0.830	0.722	0.188	0.106
330	1.86	1.64	2.11	0.852	0.704	0.165	0.088
350	1.95	1.75	2.23	0.867	0.690	0.152	0.077

$h\nu$	q/κ	$q\kappa$	$q^2\kappa^2$	$\frac{\omega}{1+\frac{\omega}{m}}(N^{+}-\nu)$	$\frac{16}{9}\frac{\lambda^2}{q^6}$	$\frac{q^3}{\lambda}$	$\frac{q^6}{\lambda^2}$
200	0.661	0.017	1.037	- 0.037	25.58	0.2636	0.0695
250	0.790	1.732	3.00	- 0.044	3.036	0.766	0.587
300	0.860	2.578	6.64	- 0.050	0.709	1.583	2.506
330	0.882	3.05	9.30	- 0.053	0.399	2.111	4.456
350	0.896	3.41	11.65	- 0.055	0.270	2.564	6.57

$h\nu$	$\frac{1}{3}\frac{q^3}{\lambda}F_m$	$\frac{1}{9}\frac{q^6}{\lambda^2}F_m^2$	$\frac{q^3}{\lambda}(F_Q+\frac{1}{3}F_m)$	$\frac{q^6}{\lambda^2}(F_Q+\frac{1}{3}F_m)^2$
200	0.0289	0.00084	0.095	0.0090
250	0.0634	0.00402	0.1832	0.0336
300	0.0992	0.0098	0.2675	0.0716
330	0.1161	0.0135	0.3019	0.0911
350	0.1299	0.0169	0.3282	0.1078

$h\nu$	$\frac{q^3}{\lambda}(F_Q-\frac{1}{3}F_m)$	$\frac{q^6}{\lambda^2}(F_Q-\frac{1}{3}F_m)^2$	$\frac{\nu}{\omega}\ \frac{1}{1+\frac{\omega}{m}}$	q_+
200	0.0366	0.00134	0.0397	0.79
250	0.0560	0.00313	0.0311	1.13
300	0.0681	0.00463	0.0254	1.44
330	0.0697	0.00485	0.0229	1.59
350	0.0667	0.00444	0.0214	1.69

q_+ are values of the pion (π^0) momentum, refered to the mass of π^+.