Laboratori Nazionali di Frascati

LNF-61/27 (6. 6. 61)

F. Amman: PRELIMINARY CONSIDERATIONS ON A PROPOSAL FOR A $e^+ e^-$ COLLIDING BEAM ACCELERATOR IN THE GeV REGION (ADONE).

Laboratori Nazionali di Frascati del CNEN
Servizio Documentazione

Nota interna: n° 79
6 Giugno 1961.

F. Amman: PRELIMINARY CONSIDERATION ON A PROPOSAL FOR A e^+e^- COLLIDING BEAM ACCELERATOR IN THE GeV REGION (Adone).-

(Report of the Adone Group - Laboratori Nazionali di Frascati)

1) - Introduction.

In the Frascati meeting, last December 1960, two papers, presented by Touschek and Gatto, pointed out the interest for the reactions produced in the e^+e^- annihilation with energies, in the CM system, in the GeV region.

It has been also pointed out that, at least for the time being, this type of experiment seems feasible only if the CM system coincides with the Lab. system; otherwise the kinetic energy T required for electrons colliding with positrons at rest, to get an energy E^*, in the CM system, useful for the reaction, is

$$T \cong \frac{E^{*2}}{2\, m\, c^2} \qquad (1)$$

Besides this, which an optimist can consider only a temporary limitation, the coincidence of the Lab. and CM systems makes the experimental set up much simpler, avoiding the angle transformation due to the CM momentum.

Together with the two theoretical papers, during the Frascati meeting has been also presented a draft proposal for the construction of a colliding beam accelerator; its purpose was a break-down on the problems involved, more than a display of possible solutions.

On the ground of the physical interest for a colliding beam experiment, the Directors' Committee of the National Institute of Nuclear Physics decided, in February 1961, to support such a program, and proposed, as a first step, the establishment in Frascati of a group for the study of the feasibility of such an accelerator; in the same time appointed a Commission, headed by Touschek, in charge of the study of the experimentation with the accelerator.

In the following it is presented a short review of the special features of a colliding beam accelerator for electrons and positrons, together with some of the technical problems involved.

No attempt has been made to derive the formulas presented here; they are the results of the work of a number of people; we want to remember the Stanford storage ring group[1] and the AdA group (and, for this part, mainly C. Bernardini and B. Touschek).

2) - The accumulation process and the injector.

If in a cyclic accelerator we have two beams, one of electrons and the other of positrons, running in opposite directions, their trajectories will be the same, if the forces acting on them are the same.

The TPC theorem tells us that a magnetic field act on two beams, of opposite charge and opposite velocities, with the same force; an electric field, on the contrary, gives on the two beams, opposite forces. If there were only magnetic fields, the two trajectories would then be coincident; for the time being we can consider negligeable the effects of electric fields and ignore them.

The two beams will collide in a fixed number of points (their number and location depend on the RF acceleration characteristics), where the annihilation reactions, which we are interested in, will take place.

If the circulating electrons and positrons are N_- and N_+, or the equivalent currents I_- and I_+, the rotation period is t_s, the number of bunches per turn (equivalent to the harmonic number of the RF accelerating cavities) is k, the transverse section of the two beams in the collision region is S, the cross section for a certain reaction is σ, the interaction rate $\dot{n}$ in each of the 2K collision sections is given by:

$$\dot{n} = \frac{1}{kt_s} \frac{N_+ N_-}{S} \sigma \text{ (ev/sec)} = \frac{4 \times 10^{37} t_s}{K} \frac{I_+ I_-}{S} \sigma \text{ (ev/s)} \qquad (2)$$

It is quite easy to see that, to get a convenient value for $\dot{n}$, let say 1 ev/1' corresponding to 1/60 ev/sec, it is necessary an accumulation process of charges into the accelerator; the cross sections for the annihilation reactions range from 10^{-30} to 10^{-32} cm^2, the rotation frequency can be of the order of 5 Mc/sec, K = 2, S of the order of 10^{-3} cm^2, which give:

$$I_+ I_- = \dot{n} \frac{S\ k}{4 \times 10^{37}\ \sigma t_s} \approx 4 \times 10^{-4}\ (A^2) \qquad (3)$$

for $S = 10^{-3}$ cm^2 and $\sigma = 10^{-32}$ cm^2

Assuming one turn injection, the values of the currents given by the (3) are also the currents which must be delivered by the injector; if the conversion efficiency is 5×10^{-5}, this gives

$$I_- = 2{,}8\ A\ ; \qquad I_+ = 1{,}4 \times 10^{-4}\ A \qquad (4)$$

A linac delivering 2,8 A at an energy around 50 ÷ 100

MeV, within an energy spread of 1 ÷ 2 %, is not impossible in principle, but certainly is very difficult and very expensive to build. Moreover there are many reasons which suggest to have in the accelerator two beams not very different in intensity; with the same luminosity of the beams (we use this term for the quantity N_+N_-/S), tha case of $I_+ = I_-$ is the most convenient for all the effects which depend on the sum of the two intensities or on one of them, like the power transferred from the RF to the beams and the power radiated on the donut, the space charge effects, the background.

Let us then consider, as more realistic figures for the currents, something like

$$I_- = 0{,}1 \text{ A} ; \qquad I_+ = 4 \times 10^{-3} \text{ A} \tag{5}$$

This value of I_+ is now beyond the possibilities of a linac; we have then to accumulate into the donut many different pulses to get the final intensity we need.

As we are dealing with electrons and positrons, the radiation losses make the phase volume occupied by the injected beam, after a certain characteristic time which depends on the energy and on the parameters of the accelerator, indipendent from the phase volume occupied at the time of injection; in other words, the Liouville's theorem does not hold.

Not considering its dependence on the structure of the accelerator (we shall come back on this point), the damping constant is given by:

$$\tau = 2 \frac{E}{\Delta E_\gamma} t_s \tag{6}$$

where E is the energy of the particle, ΔE_γ is the energy radiated in one turn and t_s is the rotation period; the (6) holds exactly for the vertical oscillations, if they are completely decoupled from the horizontal ones.

In the case of a isomagnetic machine, the (6) can be rewritten in terms of energy E, radius of curvature of the equilibrium orbit ρ, and the circumference factor λ (ratio between the length of the equilibrium orbit and the length of the magnetic sectors):

$$\tau = 4{,}7 \times 10^{-4} \frac{\lambda \; \rho^2_{(m)}}{E^3_{(GeV)}} \quad (sec) \tag{7}$$

This gives an idea of the required time separation between two consecutive pulses; each pulse will be injected outside of the volume occupied by the accumulated beam, in the phase space representation, because for time short compared to the damping time the Liouville's theorem approximately holds, but in a damping constant it will move toward the regime configuration enough to allow a new pulse to be injected.

At this point we have to consider in detail which is the most convenient injector; from the technical standpoint the higher are the energy and the current, the easier are the problems of accumulation; this is certainly true up to energies around 500 ÷ 700 MeV, where the injection into the ring may present some difficulty.

The choice is then mainly a compromise between cost and probability of reliable operation.

Let us consider the case of a linac with a maximum positron current $i_+ = 10\ \mu A$; if we want to inject one pulse per second in an accelerator with $\rho = 6{,}7$ m and $\lambda = 1{,}5$ from (7) we get, for the injection energy, 315 MeV; the accumulation time will be 400 sec, or, roughly, 7 minutes.

There can be another approach, namely to find a way to decrease the damping constant for a certain value of the injection energy.

Suppose we inject at, say, 100 MeV, then we make a cycle of acceleration up to a certain energy E_{max} and a deceleration down to the injection energy E_{inj}, with sinusoidal dependence of the energy with time of the type:

$$E = E_o(1 - a \cos 2 \pi_{ft})$$

$$E_{max} = E_o (1 + a) \qquad (8)$$

$$E_{inj} = E_o (1 - a)$$

If τ is the damping constant at the injection energy, we can find an equivalent damping constant τ^*, which gives the exponential decay between two consecutive passages at the injection energy:

$$\tau^* = \frac{(1 - a)^3}{1 + 1.5 a^2} \tau \qquad (9)$$

If we finally put $\tau^* = 1/f$, we can find the dependence of the injection repetition rate as a function of E_{inj} and E_{max}, through the parameter a.

With the values of ρ and λ already considered, we find that we can inject one pulse per second at 100 MeV, with a cycling up to 440 MeV.

This system seems to allow the choice of a very low injection energy; but a limitation is set by the oscillation induced by the radiation losses, which depend on the maximum energy and at the injection energy are amplified by the factors $(E_{max}/E_{inj})^{1/2}$ for the betatron and $(E_{max}/E_{inj})^{3/4}$ for the synchrotron oscillations; another limitation is due to the lifetime for scattering in the residual gas, which goes with E^2, and becomes too short for energies lower than 50 ÷ 100 MeV.

With the adiabatic injection we need a linac of 315 MeV, for 1 pulse/sec (or 630 MeV, for 8 pulses/sec); cycling the energy, up to 440 MeV for 1 pulses/sec (or up to 900 MeV for 8 pulses/sec), we need a linac for only 100 MeV, but the power supply for the magnet, the magnet itself and the donut are much more troublesome and expensive; in the second case considered (cycling up to 900 MeV) the radial aperture of the donut must be about twice larger than is the case of adiabatic injection.

As far as the lifetime for gas scattering is concerned, in the case of the cycling it can be calculated at the equivalent energy:

$$E_{eq} = \sqrt{E_{inj}\, E_{max}}\ \sqrt[4]{1 - a^2} \tag{10}$$

This lifetime is then certainly always shorter than in the case of adiabatic injection by a factor ranging from 3 to 7 for the cases considered.

The value of the peak positron current delivered by the linac is of paramount importance in the choice of the injector energy and injection system. The higher is this current, the lower is the number of injection pulse required for a certain total intensity, therefore the longer can be the damping constant, to allow the accumulation to take place in a fixed time, which has been chosen of the order of 10^3 sec.

An accurate calculation has not yet been made, but probably if the required damping constant is longer than 3 ÷ 5 sec the adiabatic injection is more convenient, if it is shorter the cycling becomes more convenient.

The linac will be made of different types of sections: there will be a low energy, high current part, up to about 40 MeV with electron current of the order of 1 A, followed by a high energy, low current (order of 10 mA) part.

Using a fixed magnetic focusing system, to focus the positrons produced in the converter into the high energy sections of the linac, it seems that the most convenient conver sion energy is about 45 MeV; if the solid angle accepted is of the order of 0.2 ster and the energy band 1 MeV, for 15 MeV positron energy, the conversion efficiency in a thick target can be as high as 5×10^{-4}(2).

This figure has been calculated in a very rough approximation; we hope to get for the conversion efficiency, in 1 MeV band, at least 5×10^{-5}.

If the adiabatic injection is chosen, a figure of merit for the linac can be defined, given by the product of the peak positron current times the cubic power of the final energy; it is proportional to the number of positrons which can be injected per second.

This figure must be maximized at a given cost.

3) - The radiation damping.

Many calculations have been done on the effects of the radiation on the motion of electrons in cyclic accelerators(3)(4)(5).

They all agree on the value of the damping constants for a constant gradient, isomagnetic accelerator; they agree also on the existance of an antidamping effect on the radial betatron oscillations in a isomagnetic strong focusing accelerator; where they do not agree anymore is on the changes to be made in a strong focusing to get damping for the radial betatron oscillations.

Robinson and Tarasov find that is necessary to change the magnetic field along the orbit, without any change in the focussing properties of the structure, while Kolomenski finds that is necessary a change in the focussing parameters.

C. Bernardini and C. Pellegrini are now working on this subject, which of course is a focal point in a storage ring; for the time being they have not yet arrived at a clear cut conclusion.

The calculation starts from the equation of motion, in which is included the classical reaction force due to the radiation, as the time derivative of the recoil momentum due to the emitted radiation; another approach is possible, which is more difficult, and it is the calculation of the motion equations including the discrete emission of photons; it should give as a result directly the regime configuration of the beam.

Anyway, as a starting point for what follows, we assume that it is possible in a strong focussing accelerator, in some way, to share the damping due to the radiation in two equal parts, for the vertical and radial betatron modes, and in a third part, a factor of two bigger than the preceding ones, for the synchrotron mode; this means that we assume the following damping constants for the three modes

$$\tau_v = \frac{1}{J_v}\frac{2E}{\Delta E_\gamma} t_s ; \quad J_v = 1 \qquad \text{betatron-vertical}$$

$$\tau_r = \frac{1}{J_r}\frac{2E}{\Delta E_\gamma} t_s ; \quad J_r = 1 \qquad \text{betatron-radial} \qquad (11.)$$

$$\tau_s = \frac{1}{J_s}\frac{2E}{\Delta E_\gamma} t_s ; \quad J_s = 2 \qquad \text{synchrotron}$$

The damping coefficients in weak focussing have the following values:

$$J_v = 1$$

$$J_r = \frac{n}{1-n} \simeq 1.5 \qquad (12)$$

$$J_s = \frac{3-4n}{1-n} \simeq 1.5$$

4) - Lifetime of the beam in the accelerator.

The lifetime of the beam in the accelerator depends on the three following effects:

a) scattering in the residual gas;

b) bremsstrahlung in the residual gas;

c) diffusion due to the radiation fluctuations.

For each one of them can be defined a time constant τ ; the lifetime of the beam will be given by:

$$\frac{1}{\tau} = \frac{1}{\tau_{sc}} + \frac{1}{\tau_{br}} + \frac{1}{\tau_{rad}} \qquad (13)$$

The first two effects depend on the pressure of the residual gas, the last one on the radiofrequency voltage; the time constants are:

$$\tau_{sc} = \frac{A}{F} \frac{Q^2}{R^2} \left(\frac{E}{mc^2}\right)^2 \frac{1}{7.35\text{x}10^4 P} \frac{b/a}{1+(b/a)^2} \text{ (sec)} \qquad (14)$$

$$\tau_{br} \cong \frac{10^{-4}}{P} \text{ (sec)} \qquad (15)$$

$$\tau_{rad} = \tau_s \frac{\exp\delta}{\delta} \qquad (16)$$

where: $$\delta = \frac{E}{E_1} \frac{J_s}{\alpha} \frac{1}{k} \left\{2\sqrt{s^2-1} - \pi + 2\text{arcsen}\frac{1}{S}\right\} \qquad (17)$$

and: A is the useful cross section of the donut;

a, b are the axis of the elliptical cross section of the donut;

F is a form factor governing the acceptance: $F \geq 1$ (= 1 for constant gradient; ranging from 1.5 to 3 in strong focussing);

Q is the number of betatron oscillations per turn;

R average radius of the machine: $R = \lambda\rho$;

E energy of the electrons;

P pressure of the residual gas in mm of Hg;

τ_s damping constant for the synchrotron oscillations (see the (11));

$\alpha \cong 1/Q^2$ momentum compaction;

K harmonic number of the radiofrequency cavities;

$\rho = 1/\mathrm{sen}\, \varphi_s = V_{RF}/\Delta E_\gamma$ ratio between the peak RF voltage and the average radiation loss per turn;

E_1 = 104 MeV.

Usually, in a storage ring design, the time constant which is more easily made longer than the other two, is τ_{rad}; if we then write the ratio:

$$\frac{\tau_{sc}}{\tau_{br}} \simeq 4.8 \, \frac{A(\mathrm{cm}^2)}{\lambda^2 E} \; Q^2 \; B^2_{(\mathrm{wb/m}^2)} \; \frac{b/a}{1+(b/a)^2} \tag{18}$$

we see that at a certain value of the field B, which is usually around 0.1 ÷ 0.3 Wb/m^2 for the values of A we are dealing with, τ_{sc} becomes equal to τ_{br}; at higher fields the dominant term for the lifetime of the beam is τ_{br}.

As this quantity is very approximately indipendent from the energy and the structure of the machine (the dependence is through a term $\ln \frac{E}{\Delta E}$ where ΔE is the energy acceptance of the machine; this term does not very more than a few percent with the type of accelerator and the energy), we can say that the maximum value of lifetime attainable in a storage ring, whichever it is, depends only on the pressure of the residual gas.

The design figure for the pressure in our case is 10^{-9} mm Hg, which gives

$$\tau_{br} \simeq 10^5 \text{ sec} \simeq 28 \text{ hrs} \tag{19}$$

The lifetime at the injection will be probably determined by τ_{sc}; if we have an energy cycling, the lifetime τ_{sc} must be calculated for the equivalent energy given in (10).

5) - Equilibrium dimensions of the beam.

As we already said, the classical radiation gives a damping term, which would tend to decrease the dimensions of the beams to a very small value, limited only by the multiple scattering in the gas.

If the discrete emission of photons is taken into account, we find that the equilibrium dimensions of the beam are governed by the radiation in the radial plane (radial width and length of the bunches), while the vertical dimension depends mainly on the multiple scattering.

The root mean square of the maximum amplitudes of the beam, due to the radiation fluctuations, with the assumption that there is no coupling between the vertical and radial modes, are given by:

$$\sqrt{<A_r^2>} = 0.17\ F^{1/2} \alpha\ E_{(GeV)} \sqrt{\frac{R(m)}{J_r}\lambda} \quad (cm) \qquad (20)$$

betatron-radial

$$\sqrt{<A_s^2>} = 0.17\ F^{1/2} \alpha\ E_{(GeV)} \sqrt{\frac{R(m)}{J_s}\lambda} \quad (cm) \qquad (21)$$

synchr.-radial

$$\sqrt{<A_v^2>} = 3.03\times10^{-5} \frac{F^{1/2}}{Q_v} \sqrt{\frac{R(m)}{J_v}\lambda} \quad (cm) \qquad (22)$$

betatron-vertical

The formula (20) holds in strong focussing if the envelope of the betatron oscillations has the same shape as the closed orbits; usually this condition is fulfilled within a good approximation (a).

(a) More generally, $F^{1/2} \alpha R^{1/2}$ must be replaced with:

$$R\, \chi^{1/2} = R < \frac{\beta_{max}}{\beta \rho^3} \Big[(\alpha \rho)^2 \Big\{ 1 + \frac{\beta'^2}{4} \Big\} + (\alpha \rho)^2 \beta^2 - \beta \beta' (\alpha \rho)(\alpha \rho)' \Big] >^{1/2}$$

with the usual notations (see Green and Courant. The proton synchrotron: Handbuch der Physik - Vol.XLIV - pag.218).

This hypothesis means also that:

$$\alpha \cong 1/Q_r^2 \qquad (23)$$

The root mean square of the maximum vertical amplitudes of the beam, due to the multiple scattering in the residual gas, is:

$$\sqrt{<A_v^2>} = 1.2 \frac{R^2_{(m)} \; P^{1/2}}{Q_v E^{5/2}_{(GeV)}} \frac{F^{1/2}}{\lambda^{1/2} J_v^{1/2}} \quad (cm) \qquad (24)$$

To get the total root mean square demensions of the beam at a certain azimuth of the machine, the preceding quantities must be multiplied by $\sqrt{2}$, and the form factor F must be replaced with another coefficient, whose maximum value is F, which depends on the azimuth and can be smaller than 1 (the average of its inverse, over one turn, is 1).

The root mean square length of the bunches is:

$$\sqrt{<\xi^2>} = 0.64 \, R_{(m)} \sqrt{\frac{\alpha}{K \, J_s E_{(GeV)} \sqrt{s^2-1}}} \quad (m) \qquad (25)$$

The root mean square cross section of the beam in a collision section, where the form factor of the betatron oscillations is close to 1, is remembering the (23) and letting $Q_r = Q_v$:

$$S = 0.4 \sqrt{\frac{P}{J_v J_r} \left(1 + \frac{J_r}{J_s}\right)} \; \frac{R^{5/2}_{(m)}}{Q^3 E^{3/2}_{(GeV)}} \quad (cm^2) \qquad (26)$$

With this formula we can calculate the cross section of the beam, with the parameters used for the formula (3), which are:

$R = 10$ m; $\lambda = 1.5$; $E = 1.5$ GeV; $K = 2$; $P = 10^{-9}$mmHg

Strong focussing: $J_v = J_r = 1$; $J_s = 2$; $Q = 2.25$; $s = 1.7$

Weak focusing : $J_v = 1$; $J_r = J_s = 1.5$; $Q = 0.8$; $s = 3.3$

From these we get the values $S = 2,4 \times 10^{-4}$ cm^2 for strong focussing and $S = 5 \times 10^{-3}$ cm^2 for weak focussing; the radial widths in the two cases are 0,33 cm and 2,5 cm, while the vertical dimensions are 7.1×10^{-4} cm and 2×10^{-3}; the lengths of the bunches are 1,0 m and 2.1 m.

6) - The space charge effects.

Under the headline we consider the effects on the beam of electromagnetic forces due to the beams themselves or to the ions produced in the residual gas. They can be divided into three classes:

a) the forces on one beam due to its charge density, which will be called self space charge;

b) the forces due to the ions, or ion space charge;

c) the forces due to the interaction between the two beams, or coupled space charge.

They all depend an the dimensions of the beams; the energy dependence goes like the inverse of the cubic power for the first and the inverse of the first power for the others.

Usually one calculates the effects as a change δQ of the betatron frequency Q, and finds an intensity limitation when $Q + \delta Q$ becomes an integral number, because in this case there is a resonance, and, in a linear theory, the closed orbit displacements due to field errors, go to infinity.

For the δQ due to the self space charge we can write:

$$\delta Q_s \simeq - \frac{N r_e R}{2Q\eta S} \left(\frac{mc^2}{E} \right)^3 = - 1.9\times10^{-21} \frac{N R_{(m)}}{\eta Q S_{(cm^2)} E^3_{(GeV)}} \qquad (27)$$

where N is the number of electrons (or positrons) in the ring;

η is the ratio between the total length of the K bunches and the circumference;

$r_e = 2.82 \times 10^{-13}$ cm is the classical electron radius.

In (27) we can replace S with its expression given in (26); we find:

$$\delta Q_s \cong -4.7 \times 10^{-21} \frac{1}{\eta} \left\{ \frac{J_v J_r}{P(1+\frac{J_r}{J_s})} \right\}^{1/2} \frac{Q^2}{E^{3/2}_{(GeV)} R^{3/2}_{(m)}} N \qquad (28)$$

With the same parameters used for the calculation of S (see (26)), and for $N = 10^{12}$, (which is considered the upper limit for the electron intensity), the δQ_s is very small, between 3×10^{-4} for strong focussing and $2,5 \; 10^{-5}$ for weak focussing.

The energy dependence of the (28) tells us that the self space charge effect is completely negligeable also for lower energies.

The effect of the ions is much more difficult to deal with; the positive ions produced in the gas are 'trapped' by the electron beam, if its intensity is higher than the intensity of the positron beam (condition which in most cases will be fulfilled); the electrostatic forces act on the electron beam, increasing its betatron frequency (or on the positron beam, decreasing its betatron frequency); as soon as the $(Q + \delta Q)$ approaches an integral number, the closed orbit begins to move. As the ion mobility is quite low, the δQ changes, because the beam does not see any more the same number of ions.

The non-linearity of this effect and its dependence on the ion motion, makes very difficult a reliable calculation.

The δQ_i given by the ions is:

$$\delta Q_i \simeq \frac{N_i r_e R}{2QS} \left(\frac{mc^2}{E} \right) = 7.2 \times 10^{-15} \frac{N_i R_{(m)}}{QS_{(cm^2)} E_{(GeV)}} \qquad (29)$$

or, replacing S with the (26):

$$\delta Q_i \simeq 1.8\times10^{-14} \left\{ \frac{J_v \; J_r}{P(1+\frac{J_r}{J_s})} \right\}^{1/2} \frac{Q^2 E^{1/2}_{(GeV)}}{R^{3/2}_{(m)}} N_i \qquad (30)$$

The number of ions N_i corresponding to the limit values of δQ(0.25 in strong focussing and 0.1 in weak focussing) is 3×10^9 for strong focussing and 7.5×10^9 in weak focussing, at 1.5 GeV

As we said, the positive ions are trapped by the elec tron beam; the trapping condition, in our case, can be found to be, approximately:

$$N_- - N_i \geq 3 \times 10^9 \qquad (31)$$

which means, for $N_e \geq 10^{10}$, that the number of ions is prati cally the same as the number of electrons.

The ionization rate, assuming for the residual gas Z = 10, is given by:

$$\frac{d(N_i/N_-)}{dt} = \sigma_i c N = 2 \times 10^9 \; P \quad \text{(ions/electron-sec)} \qquad (32)$$

where: $\sigma_i = 0.2\ Z \times 10^{-18}$ cm^2 ionization cross section

$N = 3.2 \times 10^{16} \times P\ cm^{-3}$ atoms per cm^3.

If the number of ions is more or less the same as the number of electrons, according to (30) something must happen to the beam when the number of electrons is close to 3×10^9, for a strong focussing, or $7,5\times 10^9$, for a weak focussing; these intensities correspond to currents of 2,4 and 6 mA, a factor of 40 to 16 lower than the value of 100 mA assumed at the beginning.

To avoid the ions effect, whichever might it be, one can use D.C. electric fields, directed along the magnetic lines of forces, which sweep the ions off the electron beam; this has been done at Stanford for their electron storage ring. In our case the electric fields change the electron and positron orbits in an opposite direction; in order to make as small as possible this effect, we should alternate, along the machine, electric fields of opposite directions, with a periodicity as high as possible.

Probably the ion space charge is not a catastrophic effect, but it only increases the cross-section of the beam; in this case, being the ionization rate very slow at 10^{-9}mm Hg, we could pulse the electric fields 200 to 300 times a second for a time of the order of 10 μsec (the ion velocities are of the order of 10^3 m/sec) and clear the beams of the ions formed, not caring whether the two beams, during this time, do not collide.

On this problem we hope anyway to have some experimental data from the Stanford electron storage ring, which is supposed to work sometime this year.

The coupled space charge effect is due to the focussing force exerted by one beam on the other; let us assume that $N_- >> N_+$, so that the electron beam behaves as it were alone, and that the force on a positron depends only on the distance from the center of gravity of the cross section of the electron beam.

If $f(\theta)$ represents the field inhomogeneities, and $g(Z_+ - \bar{Z}_-)$ represents the force of the electron beam on the positron beam, we can write the equations of motion for the vertical oscillations, for electrons and positrons:

$$\ddot{Z}_-'' + Q^2 Z_- = f(\theta) \qquad (33)$$

$$\ddot{Z}_+'' + Q^2 Z_+ + g(Z_+ - \bar{Z}_-) = f(\theta) \qquad (33)$$

Subtracting from (33) the (32), written for $\bar{Z}_-$, we get:

$$(Z_+ - \bar{Z}_-)'' + Q^2 (Z_+ - \bar{Z}_-) + g(Z_+ - \bar{Z}_-) = 0 \qquad (34)$$

This equations means that resonances due to field-type errors do not set a limit in intensity for the coupled space charge.

The limit is then set by the instabilities. This problem has not yet been considered in detail, but we can evaluate the order of magnitude of the quantities involved.

If the interaction of the electron beam on one positron in the crossing region is considered as a focussing elastic force, by matrix multiplication one can find the value of N_-/S which makes the $\cos\mu$ of one period of the machine close to ± 1; this value will depend on the focussing type and structure, on the length of the bunches, and on their number. This dependence is nevertheless not very strong, and, in this very rough approximation, one can see that for $N_-/S = 5 \times 10^{13} \div 10^{14}$ el/cm^2 the troubles arise for every type of machine, at 1.5 GeV.

The gradient errors should give an instability band when the betatron frequency is integral or half-integral (which means when the $\cos\mu$ of the whole machine is approaching ± 1); this limit gives a figure for N_-/S somewhat lower, by a factor of about 5. However this instability, if the machine is accurately built, has a very slow build-up time, so that probably , taking into account the damping, the instability bands can be crossed by the positron beam.

With $N_+ = N_- = 1.2 \times 10^{10}$, $S = 2.4 \times 10^{-4}$ cm^2 and $\sigma = 10^{-32}$ cm^2, the (2) gives $\dot{n} = 1.5 \times 10^{-2}$ events per second, for the strong focussing solution; for weak focussing, with $N_+ = N_- = 2.5 \times 10^{11}$, $S = 5 \times 10^{-3}$ cm^2 and $\sigma = 10^{-32}$ cm^2, we get $\dot{n} = 0.3$ events per second. The total currents are 10 mA in strong focussing and 200 mA in weak focussing; if we want to compare the currents with the same value of $\dot{n} = 1.5 \times 10^{-2}$ events per second, we get 10 mA for strong focussing and 45 mA for weak focussing. These are the space charge limited values for N_+ and N_- when the cross section of the beams has the value calculated for E = 1.5 GeV with the (26); for lower energies, in the same accelerator, S goes like $E^{-3/2}$, and the space charge limit like E. This means that, increasing both currents like $E^{-1/2}$, up to the space charge limit, $\dot{n}$ decreases like $E^{1/2}$.

A deeper study of the coupled space charge is of course necessary; its results can be conclusive on the feasibility of a colliding beam accelerator, or on its most convenient maximum energy.

7) - The technical problems.

A colliding beam accelerator presents new technical problems as compared to an usual particle accelerator.

The most difficult of them is probably the vacuum system. As we said, a machine like the one here considered requires an average pressure of 10^{-9} mmHg in a donut of elliptical cross section of about 100 ÷ 200 cm², 60 m long.

The ultra-high vacuum technique requires the use of materials with low outgassing rate, like glass or stainless steel; in our case the most convenient will be probably the stainless steel. It must be heated at about 400°C to be outgassed; this requires a magnet structure allowing the heating of the donut without damage to the magnet poles.

At CERN it has been studied an electropolishing treatment of the surfaces, which seems to allow the outgassing at moderate temperature, at about 200°C. This would of course make much simpler the heating of the donut.

The pumping system could be made by oil diffusion pumps with special liquid air traps; we are thinking to use a mixed system, with oil diffusion pumps down to pressures of 10^{-7} ÷ 10^{-8} mm Hg, and getter pumps from there on. This method should avoid the troubles with the argon in the getter pumps; at the same time with these pumps there is no danger of air reentry because of errors or main faults.

It is interesting to mention that an ultra-high vacuum system (with pressure down to 2×10^{-10} mm of Hg) has been built by RCA for the stellarator; the vacuum envelope is a tube about 20 cm in diameter and 30 meters long and the pumps are oil diffusion pumps, with special liquid air traps[5].

The connection between the linac and the donut vacuum systems must be carefully thought of; a solution can be a very thin Al diaphragm, a few microns thick to keep the scattering angle within acceptable limits, with a differential pumping.

The radiofrequency system, by itself, is easier than others already built for accelerators; the total voltage required per turn, at 1.5 GeV, is about 120 KV in strong focussing and 220-250 KV in weak focussing, at a frequency of 10 Mc/sec; it can be obtained with more than one cavity, lowering the total losses. The troubles come in when one think to put these cavities, which are necessarily very big (their diameter is of the order of one fourth of a wave length), in an ultrahigh vacuum.

At the moment the best solution seems to be use of cavities mostly in air, with a dielectric separating the vacuum. The feasibility of such a cavity depends strongly on its maximum voltage; this might possibly be a conclusive circumstance for the choice of the strong focussing. In addition to the already examined space charge effects, the radiofrequency sets a higher limit in the electron intensity; for a current of 0.1 A the power transferred from the cavities to the beam becomes of the order of 7 ÷ 10 KW, which can be thought as an acceptable value.

The choice of a higher harmonic for the radiofrequency acceleration would decrease the technological difficulties in building the cavities; however as can be seen from (2) and (16), the higher frequency means a lower counting rate, for the same intensity, and a higher RF voltage, for the same beam lifetime.

The second harmonic gives four collision sections: two of them can be employed for RF cavities, the other two for experiments; this seems a reasonable compromise between the different requirements.

The magnet and its power supply do not seem to present special problems, if we except what we already said about the heating of the donut for its outgassing.

Careful attention must be payed to the reliability of operation of the whole machine; we must care to avoid as much as possible the failures which cause the loss of the accumulated beam. This means that the controls of the differents parts need to be designed very accurately from the reliability standpoint, which is not, usually, a very important point in scientific apparatus.

8) - The choice of the accelerator parameters.

The design of a colliding beam accelerator involves more variables than the design of an usual accelerator; the choice of its parameters is then a multi-dimensional problem which we shall not attempt to present here.

Let us examine only the leading criteria. If we come back to the interaction rate $\dot{n}$, given by (2), we can express t_s and S as functions of the accelerator parameters, using the (26):

$$\dot{n} = 2.1\times10^{30}(I_+I_-)_{(A^2)}\,\sigma_{(cm^2)}\,\frac{1}{K}\left\{\frac{J_rJ_v}{P(1+\frac{J_r}{J_s})}\right\}^{1/2}\times \qquad (35)$$

$$\times\ \left(\frac{E_{(GeV)}}{R_{(m)}}\right)^{3/2} Q^3\ \ (ev/sec)$$

This formula holds if the cross section of the beams is determined only by the radiation effects and gas scattering, not by the space charge, and if there is no coupling between radial and vertical oscillations; moreover the trajectories of the two beams must coincide, at least in the collision se ction.

With this assumptions, we see from the (37) that $\dot{n}$ is a function of the ratio E/R, which is proportional to the magnetic field B: this means that in two accelerators with different E and R, but the same B, $\dot{n}$ is the same, if the other parameters remain constant. This consideration may be useful for the choice of the maximum energy.

Going to the choice of the magnet structure, we must first remark that a colliding beam accelerator, unlike an usual accelerator, does not give an external beam; the 'target' is in the donut and the experimental apparatus must be mounted close to the donut, in a straight section. Therefore the design of the accelerator must take into account the requirements for the experiments, the first of which is, of course, a straight section leaving at least 1.5 to 2 meters free for the detectors.

This condition sets a higher limit on the number of straight sections, that is to say, on the number of magnetic periods; with an average radius of 10 m, this limit can be taken around 10. Another condition on the number of straight sections is given by the fact that it must be a multiple of 2 K, to have all the collision regions in a straight section. With K = 2, we have then only two possibilities, namely 4 or 8 periods.

Without going into the details of the strong focussing, we recall here that a low number of periods means necessarily lower strength of the focussing forces.

The Q values attainable with 8 periods range between

1.75 and 2.25, as compared to 0.7 ÷ 0.8 for weak focussing.

Also with these low values of Q, the strong focussing presents many advantages over the weak focussing; let us try to summarize the most important of them:

a) higher energy acceptance, therefore higher peak current injected per pulse, by a factor Q^2 (roughly 5 to 8);
b) possibility of indipendent adjustment of the vertical and radial betatron frequencies with quadrupoles;
c) smaller dimensions of the beam due to quantum fluctuations, therefore a factor Q^3 in the interaction rate (10 to 20) and a factor of $Q^2/F^{1/2}$ (3.5 to 5) in the radial width of the donut, if it is governed, at the injection energy, by the induced oscillations (it is the case for the cycling in energy, with E_{max} bigger than about 400 ÷ 500 MeV);
d) lower RF voltage, by a factor of 2 to 2.5.

Against these advantages, the main disadvantage of the strong focussing is the antidamping of the radial betatron oscillations; if Robinson's theory proves to be adequate, the changes to be done in the structure to get damping are not difficult; the radial focussing sectors must have a lower field than the defocussing ones, holding the same focussing properties, that is to say holding constant n/ρ^2 (n is the field index and ρ the radius of curvature of the equilibrium orbit) and the length of the sectors.

A weak focussing accelerator could be built with 4 periods only and a smaller radius of curvature; this would in part compensate some of the disadvantages, but in the same time would require still higher RF voltage, as the radiation losses are inversely proportional to the radius of curvature.

These considerations hold if the coupled space charge is not taken into account. Probably its effect will be to cancel the advantage of the strong focussing of having a smaller beam cross section.

Our first aim is now to settle all the controversial questions; in this sense a program of theoretical calculations is being carried on, mainly by C. Bernardini, C. Pellegrini and M. Bassetti, with the very helpful cohoperation of D. Ritson, in leave of absence from MIT.

The conclusions of the theoretical work will allow to decide if a colliding beam accelerator is feasible, and which are its most convenient parameters.

We hope also to have, within a few months, some experimental information on the accumulation process from the Stanford storage ring and from AdA, the 250 MeV electron and positron storage ring built in Frascati(7).

References.

(1) W.C. Barber et al.: An experiment on the limits of quantum electrodynamics - Rapport HEPL - 170, (1959).

(2) C. Tzara: Rendements en positrons et en photons d'annihilation d'accélérateur lineaire d'electrons - Rapport SPNME n° 6, 13.1.1961.

(3) A.A. Kolomenski and A.N. Lebedev: The effects of radiation on the motion of relativistic electrons in a synchrotron Proc. of CERN Symposium on High Energy Accelerators, 1956.

(4) K.W. Robinson: Radiation effects in circular electron accelerators - Phys. Rev., 111, 373 (1958).

(5) Yu. F. Orlov, E.K. Tarasov and S.A. Khejfets: Damping of particle oscillations in an electron synchrotron with strong focussing - Pribory Tekh. Eksper. 1 17 (1959)

(6) K. Dreyer and J.T. Mark: The ultra-high-vacuum system for the C. Stellarator - RCA Review, 21, n° 4 (1960)

(7) C. Bernardini, G.F. Corazza, G. Ghigo and B. Touschek: The Frascati storage ring - N. Cim., 18, 1293 (1960).