Laboratori Nazionali di Frascati

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Erdas F.[x] and von Gehlen G.: SPIN CORRELATION IN MUON PAIR PRODUCTION.

1) - Introduction.

The correlation of the electron spins in electron pair production by unpolarized photons has been studied by H. Olsen and L.C. Maximon(1). In their calculation sommerfield. Mane wave functions are used, and account has been taken of screening and Coulomb effects. In the corrisponding case of the spin correlation of muon pairs obviosly one is not allowed to make on extreme relativistic approximation for the muons, but the momentum transfers are much larger, so that screening becomes unimportant. Disregarding the problem of coulomb corrections we calculate the differential cross section simply by inserting the covariant spin projection operators into the Feynman-Dyson matrix elements and using the trace technique. Introducting a cut-off into the muon propagators, we investigate the influence of an eventual breakdown of quantum Electrodinamics.

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2) <u>The matrix elements in Born approximation with spin projection operators.</u>

Using hermitian γ-matrices $\gamma_\mu = \gamma_\mu^+$ (μ = 1, 2, 3, 4) and $p^2 = \vec{p}^2 + p_4^2$ $p_4 = ip_0$ the differential cross section for pair production is

$$(1) \qquad d\sigma = \frac{e^4}{8(2\pi)^5} \frac{|\vec{q}||\vec{Q}|dQ_0}{K_0^3} d\Omega_q d\Omega_Q \, T_{\mu\nu} A_\mu(\bar{K}-\bar{q}-\bar{Q}) A_\nu(\bar{q}+\bar{Q}-\bar{K})$$

with

$$(2) \qquad T_{\mu\nu} = \frac{K_0^2}{2} Sp \Big\{ \Big[\gamma_\mu \frac{i\gamma(Q-K)+m}{2QK}\gamma_\lambda + \gamma_\lambda \frac{i\gamma(q-K)-m}{-2qK}\gamma_\mu \Big] \cdot$$

$$\cdot \frac{1}{2}(1-i\gamma_5 \gamma S)(i\gamma Q+m) \cdot$$

$$\cdot \Big[\gamma_\lambda \frac{i\gamma(Q-K)+m}{2QK}\gamma_\nu + \gamma_\nu \frac{i\gamma(q-K)-m}{-2qK}\gamma_\lambda \Big] \cdot$$

$$\cdot \frac{1}{2}(1+i\gamma_5 \gamma T)(i\gamma q-m) \Big\}$$

where K, Q, q are the momenta of the incident γ and of the two muons, respectively, m the muon mass. S is the spin operator of the muon with momentum Q as intruduced by H.A. Tolhook[2]

$$(3) \qquad S = \left(\vec{J} + \frac{(\vec{J}\vec{Q})\vec{Q}}{m(Q_0+m)} \,,\, i\frac{(\vec{J}\vec{Q})}{m} \right)$$

with $\vec{J}$ the spatial polarization vector of the muon, similary is the covariant spin operator of the muon with momentum q.

Carring out the trace operations in (2), we get

$$T_{\mu\nu} = \frac{K_i^2}{2}\left[-\frac{A_{\mu\nu}}{4(QK)^2} + \frac{B_{\mu\nu}}{4(qk)^2} - \frac{C_{\mu\nu}}{4(QK)(qK)}\right] \tag{4}$$

with

$$\begin{aligned} A_{\mu\nu} = 4m^2\{&\delta_{\mu\nu}[(1-ST)(Qq-QK-qk-m^2) + \\ &+(TQ-TK)(Sq-SK)] - (1-ST)(Q_\mu q_\nu + Q_\nu q_\mu) + \\ &+(Sq-SK)(T_\mu K_\nu + T_\nu K_\mu - T_\mu Q_\nu - T_\nu Q_\mu) - \\ &-(TQ-TK)(q_\mu S_\nu + q_\nu S_\mu) + \\ &+(Qq-QK-qk-m^2)(S_\mu T_\nu + S_\nu T_\mu) + \\ &+\frac{1}{m^2}QK\,qK\,\delta_{\mu\nu} + (1-ST-\frac{QK}{m^2})(K_\mu q_\nu + K_\nu q_\mu)\}\end{aligned} \tag{5a}$$

$$\begin{aligned} B_{\mu\nu} = 4m^2\{&\delta_{\mu\nu}[(1-ST)(Qq-QK-qK-m^2) + \\ &+(TQ-TK)(Sq-SK)] - (1-ST)(Q_\mu q_\nu + Q_\nu q_\mu) + \\ &+(TQ-TK)(S_\mu K_\nu + S_\nu K_\mu - S_\mu q_\nu - S_\nu q_\mu) - \\ &-(Sq-SK)(Q_\mu T_\nu + Q_\nu T_\mu) + \\ &+(Qq-QK-qK-m^2)(S_\mu T_\nu + S_\nu T_\mu) + \\ &+\frac{1}{m^2}QK\,qK\,\delta_{\mu\nu} + (1-ST-\frac{qK}{m^2})(K_\mu Q_\nu + K_\nu Q_\mu)\}\end{aligned} \tag{5b}$$

$$
\begin{aligned}
G_{\mu\nu} = 4\{ & 2Q_\mu Q_\nu[(1-ST)qK - Sq\,TK] + \\
& +(Q_\mu q_\nu + Q_\nu q_\mu)[(2Qq - QK - qK)(1-ST) + TQSK + SqTK] + \\
& +(Q_\mu K_\nu + Q_\nu K_\mu)[-Qq(1-ST) + qKST - Sq(TQ+TK)] + \\
& +(Q_\mu S_\nu + Q_\nu S_\mu)[-TK(Qq - m^2) + TQ\,qK] + \\
& +(Q_\mu T_\nu + Q_\nu T_\mu)[-SK(Qq + m^2) + Sq(2Qq - 2qK - QK)] + \\
& +2q_\mu q_\nu[(1-ST)QK - TQSK] + \\
& +(q_\mu K_\nu + q_\nu K_\mu)[-Qq(1-ST) + QKST - TQ(Sq + SK)] + \\
& +(q_\mu T_\nu + q_\mu T_\nu)[-SK(Qq - m^2) + Sq\,QK] + \\
& +(q_\mu S_\nu + q_\nu S_\mu)[-TK(Qq + m^2) + TQ(2Qq - 2QK - qK)] + \\
& +2K_\mu K_\nu[m^2 - STQq + TQSq] + (K_\mu S_\nu + K_\nu S_\mu)\cdot \\
& \cdot[-TQ(qK + m^2) + TK\,Qq] + (K_\mu T_\nu + K_\nu T_\mu)\cdot \\
& \cdot[-Sq(QK + m^2) + SK\,Qq] + (S_\mu T_\nu + S_\nu T_\mu)\cdot \\
& \cdot[-2Qq(Qq - QK - qK - m^2)] + 2\delta_{\mu\nu}[-Qq(Qq - QK - qK - m^2)\cdot \\
& \cdot(1-ST) - ST\,QK\,qK + SKTQ(m^2 + qK) + TKSq(m^2 + QK) - \\
& - Qq(SKTK + SqTQ) + SqTQ(QK + qK)]\}
\end{aligned}
\tag{5c}
$$

Inserting for $A_\mu(\vec{K}-\vec{Q}-\vec{q})$ the Coulomb potential. We have for (1):

$$(6) \qquad d\sigma = \left(\frac{Z}{2\pi}\right)^2 \alpha^3 \, \frac{|\vec{q}||\vec{Q}|\, dQ_0}{K_0^3} \, d\Omega_q \, d\Omega_Q \left[- \frac{T_{44}^{(1)}}{(\vec{K}-\vec{Q}-\vec{q})^4} \right]$$

3) <u>The spin correlation of the muon pair.</u>

The spin correlation of the muons is defined by

$$(7) \qquad C = \frac{d\sigma^{(p)} - d\sigma^{(a)}}{d\sigma^{(p)} - d\sigma^{(a)}}$$

where (p) stands for 'parallel spin' ($\vec{J}_1 = \vec{J}_2$), and (a) for antiparallel spins ($\vec{J}_1 = -\vec{J}_1$).
(7) becomes in our case

$$(8) \qquad C = \frac{\left(A_{44}^{(p)} - A_{44}^{(a)}\right)\frac{N}{M} + \left(B_{44}^{(p)} - B_{44}^{(a)}\right)\frac{M}{N} - \left(C_{44}^{(p)} - C_{44}^{(a)}\right)}{\left(A_{44}^{(p)} + A_{44}^{(a)}\right)\frac{N}{M} + \left(B_{44}^{(p)} + B_{44}^{(a)}\right)\frac{M}{N} - \left(C_{44}^{(p)} + C_{44}^{(a)}\right)}$$

with

$$(9) \qquad M = \frac{1}{K_0}(QK) \qquad\qquad N = \frac{1}{K_0}(qK)$$

In the electron case, the pairs are produced almost completely in the forward direction. Olsen and Maximon(1) integrated over the electron angles so that oll spin directions orthogonal to $\vec{K}$ are equivalent in their formula. In the muon case the cross section is not so, much peaked forward, so that we distinguish the plane formed by the muon pair.

Taking $\vec{K}$, $\vec{Q}$ and $\vec{q}$ coplanar and the spins orthogonal to this plane T_{44} takes an expression like to

$$T_{44} = \gamma(\beta - \alpha\, \vec{\sigma}_1 \vec{\sigma}_2)$$

so that the spin correlation becomes:

$$C = -\frac{\alpha}{\beta} \tag{10}$$

We choosed the symmetrical situation, $\theta = \varphi = 10°$ (θ and φ are the angles between $\vec{K}$ and $\vec{Q}$, and $\vec{K}$ and $\vec{q}$, respectively). The results are given in fig. 1, in which we have fixed $Q_0 = 3mc^2 \simeq$ 317 MeV and varied K_0 with q_0.

4) - Influence of a cut-off in the muon propagator.

We assume that the muon and photon propagators are modified by some cut-off parameter and we calculate the influence of such a cut-off on the spin correlation.

It is easily seen from (2) and (6) that only modifications on the muon propagator enter into our expression for the spin correlation, while the photon propagator divides out in our Born approximation.

Following Drell[3] we sobstitute in (2)

$$\left[i(\not{Q}-\not{K})+m\right]\left[\frac{1}{2(QK)} - \frac{1}{2(QK)-\Lambda^2}\right]$$

for

$$\frac{i(\not{Q}-\not{K})+m}{2(QK)}$$

(and analogously with q instead of Q).

We get again (8), but M and N are now given by

$$\frac{1}{M} = K_0\left[\frac{1}{(QK)} - \frac{1}{(QK)-\frac{\Lambda^2}{2}}\right] \tag{11}$$

$$\frac{1}{N} = K_0\left[\frac{1}{(qK)} - \frac{1}{(qK)-\frac{\Lambda^2}{2}}\right]$$

instead of (9).

We evaluated (8) with (10) again for the case of coplanar $\vec{K}$, $\vec{Q}$, and $\vec{q}$ and transversal spins ortogonal to the plane of $\vec{Q}$ and $\vec{q}$, with $\vartheta = \varphi = 10°$ and $Q_0 = 3mc^2 \simeq 317$ MeV. The results for different values of the cut-off Λ are given in fig. 2.

One sees that about $Q_0 = 475$ MeV the spin correlation depends strongly on the cut-off. Taking into account that only relative measurement are needed, it should not be impossible with this method to probe the electrodinamics of the muon down to about 0,4 .

References:

(1) H. Olsen, L.C. Maximon: Phys. Rev. 114, 887 (1959)
(2) H.A. Tolhoek: Rev. Mod. Phys. 28, 277 (1956)
(3) S.D. Drell: Annals of Physics 4, 75 (1958).

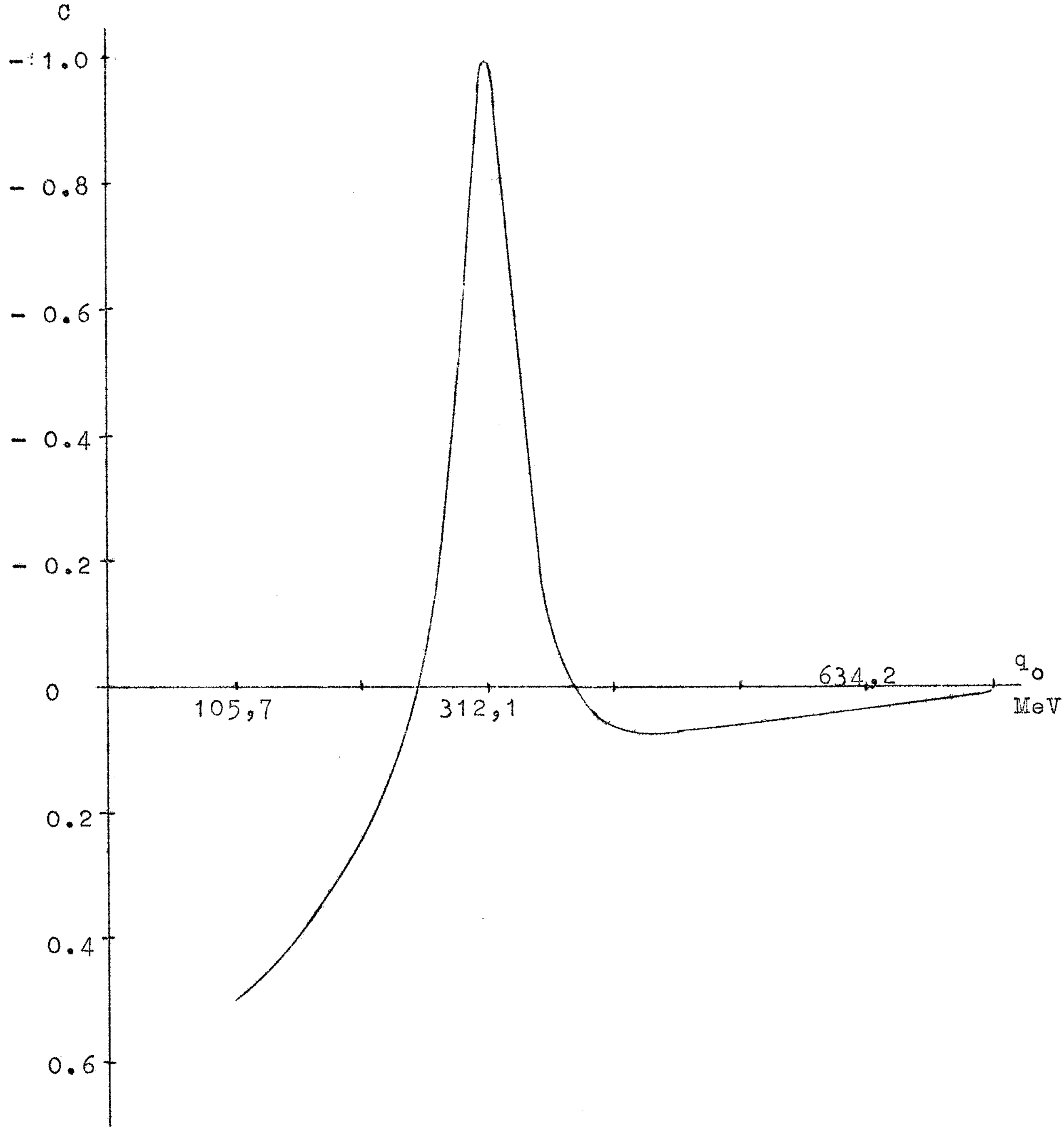

FIG. 1 - Spin correlation for muon pairs produced by unpolarized photons in the case $\vec{K}$, $\vec{Q}$ and $\vec{q}$ coplanar, the spins orthogonal to this plane and $\theta = \varphi = 10°$.

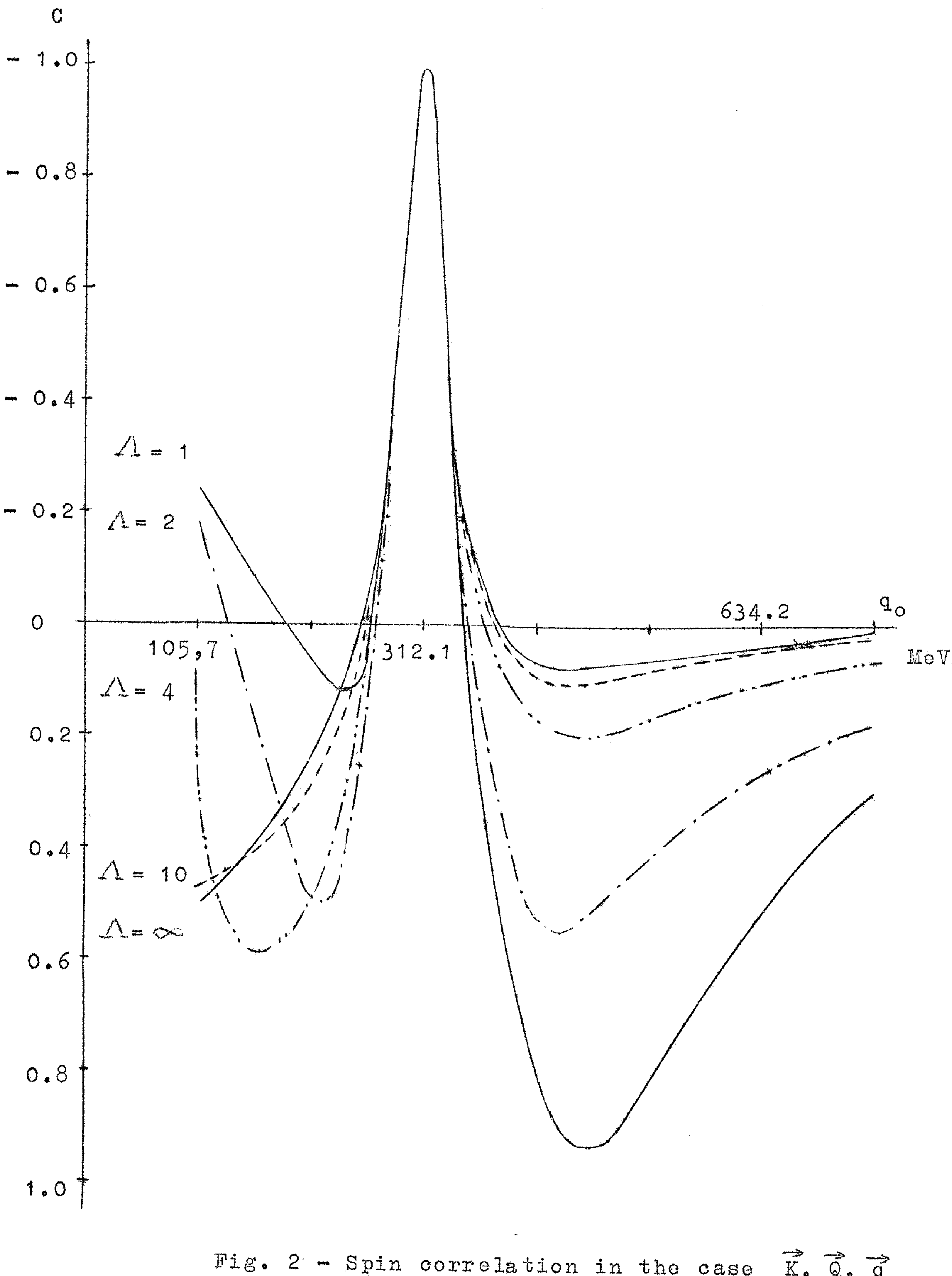

Fig. 2 - Spin correlation in the case $\vec{K}$, $\vec{Q}$, $\vec{q}$ coplanar, the spin orthogonal to this plane and θ = φ = 10° for

$\Lambda = \lambda_\mu^{-1}, 2\lambda_\mu^{-1}, 4\lambda_\mu^{-1}, 10\lambda_\mu^{-1}, \infty$