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N. Cabibbo, R. Gatto: FURTHER REMARKS ON THE PROPOSED *u* - e
SELECTION RULE.

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N. Cabibbo and R. Gatto: FURTHER REMARKS ON THE PROPOSED μ -e SELECTION RULE (Submitted for publication to Nuovo Cimento).

In a recent paper published in the Physical Review Letters ⁽¹⁾ we proposed a selection rule that forbids transformations of μ into e. We were led to postulating such a selection rule on the basis of the following formal argument.

We assume that μ and e have identical interactions and they differ only in their rest mass. It can then be seen that, if electromagnetic interactions satisfy the requirement of minimality ⁽²⁾ and if weak interactions are neglected, there exists a symmetry property of the theory, that we called μ -e symmetry, related to an exchange of the two lepton fields that are necessary to describe μ and e. We then found that in order to maintain the μ -e symmetry also in a complete theory including weak interactions one has to postulate two neutrinos, one coupled to the electron, ν_e , and one coupled to the muon, ν_μ . This possibility of having two neutrinos has been considered since a long time from different authors on various grounds ⁽³⁾. Experimentally it seems to be suggested from the absence of $\mu \rightarrow e + \gamma$ ⁽⁴⁾, which should almost certainly be present if in $\mu \rightarrow e + \nu + \bar{\nu}$ there is only one kind of neutrino. Now, it is known that to a sym

metry property of the theory there corresponds, in general, a physical conservation law. The conservation law corresponding to μ -e symmetry is a multiplicative conservation law that forbids transformations of μ into e.

More specifically, one assigns to each particle a multiplicative muonic quantum number K, according, for instance, to the assignment shown in Table I, and one obtains that the only reactions that are permitted are those for which the product of the initial K values is equal to the product of the final K values. To the mesons, barions and photon we have assigned $K = 1$, but we could have chosen instead $K = e^{i\frac{\pi}{2}N}$ where N is the nucleonic quantum number of the particle, without altering any physical consequence, but only with a slight alteration of the formal argument leading to the postulated law.

Besides the multiplicative conservation law that we have discussed one can consider a more stringent additive conservation law; to each particle there corresponds an additive muonic quantum number M, such that, for instance

$$K = e^{i\frac{\pi}{2}M}$$

and only those reactions are allowed for which the sum of the initial M values is equal to the sum of the final M values. Corresponding to our previous assignments of K one can assign values of M as in table I.

It is obvious that if the additive muonic conservation law is verified also the multiplicative law is verified..

However if the multiplicative muonic conservation law is verified, it does not follow that also the additive conservation law is verified. It only follows that the difference ΔM between the sum of the initial M values and the sum of the final M values can only take values 0, ± 4 , ± 8 , (I. e., $\Delta M = 0 \text{ mod. } 4$).

Odd ΔM are always forbidden because of lepton conservation. $\Delta M = \pm 2, \pm 6$, etc are forbidden for both types of conservation laws, and this is sufficient to prevent reactions such as

$$\begin{aligned} \mu &\rightarrow e + \gamma \\ \mu &\rightarrow e + e + e \\ \mu + (\text{nucleus}) &\rightarrow e + (\text{nucleus}) \\ \mu &\rightarrow e + \nu_e + \bar{\nu}_e, \quad \pi \rightarrow \mu + \nu_e, \quad \text{etc.} \end{aligned}$$

Reactions with $\Delta M = \pm 4, \pm 8$, etc are forbidden from the additive law, but they are allowed from multiplicative law. Thus evidence for reactions like, for instance,

$$e^+ + \mu^- \rightarrow e^- + \mu^+ \quad (1)$$

$$e^+ + e^+ \rightarrow \mu^+ + \mu^+ \quad (2)$$

$$\nu_\mu + (\text{nucleus}) \rightarrow \nu_e + e^- + \mu^+ + (\text{nucleus}) \quad (3)$$

would exclude the additive conservation law and be consistent with the multiplicative conservation law. None of these reactions can occur by electromagnetic interaction alone and thus any evidence for them would be an evidence for some new mechanism, for instance of the kind we are considering. The reaction (1) is very interesting: it can occur as a charge exchange reaction in muonium

$$(e^+ + \mu^-)_{\text{bound}} \rightarrow (e^- + \mu^+)_{\text{bound}}$$

It has been studied theoretically by Pontecorvo (5), and by Feinberg and Weinberg (6). We refer to the work of these authors for further discussion.

The reaction (2) might in principle be studied by colliding beam experiments of the type considered at Stanford (7), but carried out at a much higher energy or with higher intensity. A possible interaction of the type

$$\mathcal{L}' = f(\bar{\psi}^{(\mu)} \gamma_\lambda a \psi^{(e)}) (\bar{\psi}^{(e)} \gamma_\lambda a \psi^{(\mu)})$$

(where $a = \frac{1}{2}(1+\gamma_5)$ and $\bar{a} = \frac{1}{2}(1-\gamma_5)$) through Fierz reordering can be written in the form (we use the Majorana representation).

$$\mathcal{L}' = 2f^{\dagger} (\psi^{(e)} \gamma_4 \bar{a} \psi^{(e)}) (\psi^{(\mu)*} \gamma_4 \bar{a} \psi^{(\mu)*})$$

The cross section is then clearly isotropic in the center of mass system. The total cross section is given by

$$\sigma_{total} = \frac{f^2 E^2}{\pi} \beta (1 + \beta^2)$$

where E is the center of mass energy of each colliding electron and β is the muon velocity in the center of mass system. If, tentatively, we identify f with $\sqrt{\beta} G$ where G is the weak coupling constant the cross section turns out to be $\sigma_{total} = 1.6 \times 10^{-37} (E/M)_{cm}^2$ where M is the nucleon mass.

The final muons are polarized longitudinally. The value of the polarization is

$$P^{\pm} = \pm \frac{2/\beta}{1+\beta^2}$$

where the upper sign holds for μ^+ , the lower sign for μ^- .

The reaction (3) can occur through an interaction of the type

$$\mathcal{L}' = f (\bar{\psi}^{(\mu)} \gamma_{\lambda} a \psi^{(\nu_e)}) (\bar{\psi}^{(\nu_e)} \gamma_{\lambda} a \psi^{(e)})$$

together with the absorption of a virtual nuclear gamma by one of the final charged leptons. An estimate of its cross section on lead gives $0.5 \times 10^{-39} (E_{\nu}$ in BeV) 2 cm 2 where E_{ν} is the incident neutrino energy in the laboratory system. The rapid increase of the cross section with energy will perhaps make this process one of the more convenient to decide between the two possibilities of an additive or of a multiplicative muon electron selection rule.

TABLE I

particles	quantum number K	quantum number M
$\mu^-, \nu_\mu, e^+, \bar{\nu}_e$	$-i$	-1
$\mu^+, \bar{\nu}_\mu, e^-, \nu_e$	$+i$	$+1$
mesons, baryons, γ	1	0

Bibliography

- 1) - N. Cabibbo and R. Gatto, Physical Review Letters, 5, 114 (1960); see also R. Gatto, Proceedings of the Annual International Conference on High Energy Physics in Rochester, pag. 609.
- 2) - The formal meaning of this requirement is that one can obtain the electromagnetic interactions by the substitution $\partial/\partial x_\mu \rightarrow \partial/\partial x_\mu - ieA_\mu$ in the original Lagrangian without electromagnetic interactions. The minimality requirement seems to be essential for the present theory of elementary particles. In particular it guarantees the conservation of strangeness in electromagnetic interactions. Theoretically it seems to be related to the condition of renormalizability. In the axiomatic approach to field theory a limitation equivalent to minimality might be directly related to causality but to our knowledge no systematic investigation of this question has been carried out so far.
- 3) - A. Salam, Proceedings of the seventh Annual Conference on High Energy Nuclear Physics, 1951, (Interscience publishers, New York 1957) pag. IX, 44; B. Pontecorvo, J. Exptl. Theoret. Phys. (USSR) 37, (1751); T.D. Lee and C.N. Yang, Phys. Rev. Lett. 4, 307 (1960).
- 4) - Berley et al., Phys. Rev. Lett. 2, 351 (1959). The experimental situation on the process $\mu^- + (\text{nucleus}) \rightarrow (\text{nucleus}) + e^-$, whose presence or absence is also directly related to the problem, seems rather confused at present (Conversi et al. to be published, Sard et al., to be published).
- 5) - B. Pontecorvo, J. Exptl. Theoret. Phys. 33, 549 (1957).
- 6) - G. Feinberg and S. Weinberg (private communication). See the discussion remark by Feinberg after the paper by R. Gatto in Proceedings of the 1960 Annual International Conference on High Energy Physics in Rochester, pag. 609.
- 7) - W. Panofsky, Proceedings of the 1960 Annual International Conference on High Energy Physics in Rochester, pag. 769.