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Some Rare Decay Modes of the K-Meson.

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Summary. — In view of future experiments with high intensity K-meson beams the branching ratios for $K^{\pm}\to\pi^{\pm}+e^{+}+e^{-}$ and $K_1^0\to 2\gamma$ are evaluated.

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1. – The expected availability of high intensity K-meson beams makes it necessary to evaluate the branching ratio to be expected for presumably very rare decay modes of the K-mesons. In the first part of this note we consider the possible decay mode

$$K^{\pm}\to\pi^{\pm}+e^{+}+e^{-} \tag{1}$$

due to the virtual step $K^{\pm}\to\pi^{\pm}+\gamma$ and $\gamma\to e^{+}+e^{-}$. The process is of the second order in the e.m. coupling constant, but it is expected to have a fairly low branching ratio in view of the fact that for a spin zero K-meson the first of the two virtual steps is forbidden when the photon is transverse and on the energy shell. In the second part of the paper we consider the decay modes $K^0\to 2\gamma$. In discussing such reactions one has to analyze first the consequences of the application of the *CP* selection rules to the K^0-decays [1].

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[1] L. Landau: *Nucl. Phys.*, **3**, 127 (1957); R. Gatto: *Phys. Rev.*, **106**, 168 (1957). T. D. Lee and C. N. Yang: *Elementary particles and weak interactions*, B.N.L. 443 (1957).

From CP invariance the matrix elements for the 2γ decay of the shortlived K^0, K_S^0, is of the form $(\boldsymbol{\epsilon}_1, \boldsymbol{\epsilon}_2)$ where $\boldsymbol{\epsilon}_1$, $\boldsymbol{\epsilon}_2$ are the polarization vectors of the photons, while the matrix element for the longlived K^0, K_L^0, is of the form $\boldsymbol{q}(\boldsymbol{\epsilon}_1 \wedge \boldsymbol{\epsilon}_2)$, where $\boldsymbol{q}$ is the relative final momentum. The lowest mass intermediate states for K_S^0 is the 2π state, while for K_L^0 it is the 3π state. Our calculation concerns K_S^0 via the lowest mass intermediate state, but we expect a similar branching ratio for K_L^0 considering that its effective coupling is smaller and also its lifetime is longer.

2. – The simplest diagram which represents reaction (1) (let us choose the case of a K^+) is the following one, involving a vertex $(K^+\pi^+\gamma)$.

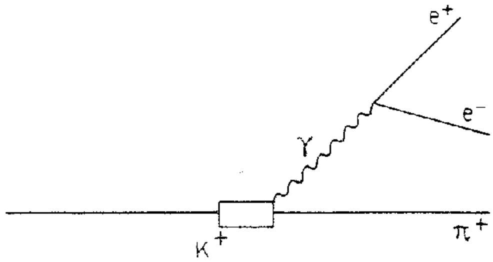

Fig. 1.

The vertex $(K^+\pi^+\gamma)$ must vanish for real photons because of gauge invariance.

The matrix element for process (2) will then be ($q = k - p$, where k and p are the 4-momenta of the K^+ and the pion, and p^+, p^- the momenta of the two electrons)

$$\langle pp^+p^- \,|\, k\rangle = \frac{(2\pi)^{-3}}{q^2} \langle p \,|J_\mu|\, k\rangle \,\delta^4(k - p - p^+ - p^-)\,[u(p^-)\gamma_\mu v(p^+)]\,. \tag{2}$$

We make the position $\langle p\,|J_\mu|\,k\rangle = (2\pi)^{-3}(4E_pE_K)^{-\frac{1}{2}}C_\mu$, where $(p-k)_\mu C_\mu = 0$ due to gauge invariance. This condition fixes C_μ to be proportional to $\{k_\mu[\mu^2 + (pk)] + p_\mu[m_K^2 + (pk)]\}$. We evaluate the decay rate by performing first a summation over the spins and momenta of the electrons and we get

$$\text{Rate}\,(K^+ \to \pi^+ + e^+ + e^-) = \frac{e^2}{12m_K(2\pi)^{12}} \int' \frac{d^3p\,(C_\mu C_\mu)}{E_p[-(p-k)^2]}\,.$$

We shall now try to evaluate C_μ.

We can describe the $(K^+\pi^+\gamma)$ vertex in the simplest way by means of an intermediate $\bar{\Lambda}$-p loop, obtaining the following diagrams:

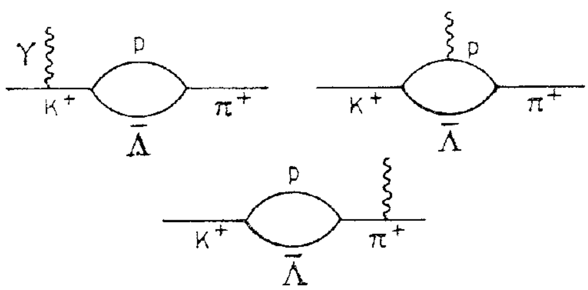

Fig. 2.

The left hand side vertex in the diagrams gives substantially the $(K^+p\Lambda)$ coupling constant, while the right hand side vertex is related to the lifetime of the Λ^0-hyperon. Of course in a better calculation the effect of intermediate Σ's should also be included.

One finds that the dominant

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part of the contribution of the above graphs (*i.e.* the contribution obtained by letting the masses of the intermediate fermions go to infinity) vanishes identically, as a consequence of a general theorem stated by CABIBBO and GATTO which can be easily extended to cover this case ([2]).

The consequence is that the contribution of the above graphs is of the order $(m/M)^2$, where m is of the order of magnitude of the masses of the external particles, and M of the particles of the loop. The integrals are now convergent. A detailed calculation to the order $(m/M)^2$ gives the gauge invariant result ([3])

$$C_\mu \simeq i(2\pi)^2 \frac{egG}{3M} [k_\mu(\mu^2 + pk) + p_\mu(m_K^2 + pk)] , \tag{3}$$

G is a constant related to the rate R_Λ ([4]) of the process $\Lambda^0 \to p + \pi$ through

$$G^2 = \frac{4\pi}{Q} \frac{m_\Lambda^2}{[(m_\Lambda^2 - M^2) - \mu^2(m_\Lambda^2 + m^2)} R_\Lambda ,$$

(Q is the 3-momentum of the π^- in the Λ^0 rest system).

Finally one gets ($\alpha \quad e^2/4\pi$)

$$R_{e^+e^-} = \alpha^2 \left(\frac{g_{\Lambda K_D}}{4\pi}\right)^2 \frac{1}{108\pi^3} \left(\frac{\mu}{M}\right)^4 \frac{m_K}{Q} F_1 F_2 ,$$

where

$$F_1 = \frac{m_\Lambda^2 M^2}{[(m_\Lambda^2 - M^2) + \mu^2(m_\Lambda^2 + M^2)]} ,$$

$$F_2 = \frac{1}{4}\left(\frac{m_K}{\mu}\right)^2 \left[\left(\frac{m_K^2 - \mu^2}{2m_K\mu}\right)^2 - \frac{3}{2}\right] + \frac{3}{2} \ln \frac{m_K}{\mu} .$$

The branching ratio is ([5])

$$\frac{K^+ \to e^+ + e^- + \pi^+}{K^+ \to \text{all other modes}} = 1.0 \cdot 10^{-7} .$$

This ratio is indeed very small, smaller than it would be thought on the

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([2]) N. CABIBBO and R. GATTO: *Phys. Rev.*, **116**, 1334 (1959); see also S. OKUBO. *Nuovo Cimento*, **16**, 963 (1960).

([3]) The terms of higher order introduce an error $\sim 4\%$. The error induced by neglecting in the integrals the Λ-p mass difference is much smaller. M=nucleon mass, μ=pion mass.

([4]) *High Energy Particle Data*, UCRL, vol. **2**, p. 85.

([5]) The order of magnitude of $g_{\Lambda K_D}$ can be inferred from dispersion relations: P. T. MATTHEWS and A. SALAM: *Phys. Rev.*, **110**, 769 (1958).

basis of an order of magnitude calculation ([6]). This is due to the « selection rule » provided by the mentioned theorem for the ($K^+\pi^+\gamma$) vertex.

3. – The rate of the reaction

$$K_s^0 \rightarrow 2\gamma \tag{4}$$

can be deduced by associating the interaction responsible for the decay of the K_s^0 into two charged pions with the interaction of these pions with the electromagnetic field. The process will be described by a diagram of this type.

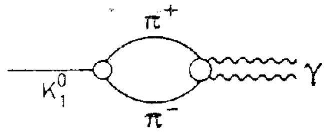

Fig. 3.

The left-hand side vertex will be associated to the rate of the normal $K_s^0 \rightarrow \pi^+ + \pi^-$ decay; the right-hand side vertex can be safely evaluated by means of a standard perturbation calculation. The S-matrix element of the process is given by the following expression:

$$\langle k_1 k_2 | p \rangle = \frac{1}{\sqrt{2}}\, e^2 G\, \delta^4(p - k_1 - k_2)(2\pi)^{-9/2}(2EE_1E_2)^{-\frac{1}{2}}(\varepsilon_1\varepsilon_2)I\,,$$

G is a constant which is connected to the rate R_K of the $K_1^0 \rightarrow \pi^+ + \pi^-$ decay by

$$G^2 = 16\pi\, m_K^2 (m_K^2 - \mu^2)^{-\frac{1}{2}} R_K\,,$$

p, k_1,k_2 are the 4-momenta of the K_s^0 and of the γ's; E, E_1, E_2 are the corresponding energies. In the K_s^0 rest-system we have $E = M_K$, $E_1 = E_2 = \frac{1}{2}M_K$ ε_1, ε_2 are the polarization vectors of the γ's. The factor $1/\sqrt{2}$ comes from the symmetrization of the final state. The expression $(\varepsilon_1 \cdot \varepsilon_2)I$ is the result of the evaluation of the following integral:

$$(\varepsilon_1\varepsilon_2)I = \int \mathrm{d}^4 q\, \frac{4(\varepsilon_1 q)(\varepsilon_2 q)}{(q^2 + \mu^2)[(q - k_1)^2 + \mu^2][(q + k_2)^2 + \mu^2]} +$$
$$+ \int \mathrm{d}^4 q\, \frac{4(\varepsilon_1 q)(\varepsilon_2 q)}{(q^2 + \mu^2)[(q - k_2)^2 + \mu^2][(q + k_1)^2 + \mu^2]} - \int \mathrm{d}^4 q\, \frac{(\varepsilon_1\varepsilon_2)}{(q^2 + \mu^2)[(q - k_1 - k_2)^2 + \mu^2]}\,.$$

Since the integrals are logarithmically divergent, a cut-off parameter must be introduced. Due to the fact that the mass of the intermediate state is less than the mass of the initial state, the integrals will not be pure imaginary

([6]) An order of magnitude estimate was made by R. H. Dalitz: *Phys. Rev.*, **99**. 915 (1955): the branching ratio expected was $5 \cdot 10^{-5}\,(C/A)^2$, where C and A are two unknown constants. Comparison with the present result shows that $C/A = 1/20$.

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(as in the usual evaluation of Feynman diagrams) but will have also a real part, which in the present case, however, turns out to be small.

The final expression of the rate $R_{2\gamma}$ for process (4) is ($\alpha = e^2/4\pi$)

$$R_{2\gamma} = \frac{1}{8}\,\frac{\alpha^2}{(2\pi)^3}\,G^2\,\frac{|I|^2}{m_K} = \alpha^2 R_K N\,. \tag{5}$$

This result is of the expected order of magnitude, since the coefficient N is of the order unity with a cut-off of reasonable order of magnitude. For a cut-off equal to the nucleon mass M, we get $N = 0.37$: the variation of this number with the cut-off is very slow. The branching ratio of this process is

$$\frac{K_S^0 \to 2\gamma}{K_S^0 \to \text{all other modes}} \simeq 2.3\cdot 10^{-5}\,.$$

As it has been said in the Introduction, the corresponding ratio for the K_L^0 is expected to be of the same order of magnitude. A calculation of the preceding type cannot be easily performed because of the presence of the unknown 3π - 2γ vertex.

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We thank Prof. R. GATTO for helpful discussion.

RIASSUNTO

In vista di futuri esperimenti con fasci di K di alta intensità vengono valutati i rapporti di decadimento per $K^\pm \to \pi^\pm + e^+ + e^-$ e $K_1^0 \to 2\gamma$.

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