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On the Connection between Scattering and Photoproduction of Pions at High Energies.

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It is known that between the elements of the S matrix for scattering and photoproduction of pions on nucleons

$$\left\{\begin{array}{l} \pi+\mathcal{N}\rightarrow\mathcal{N}+\pi\,, \\ \gamma+\mathcal{N}\rightarrow\mathcal{N}+\pi\,, \end{array}\right. \tag{1}$$

a relation exists which follows from the unitarity and time reversal invariance of S (1). One can indeed show that, if the amplitudes of (1) with well defined total angular momentum J, isotopic spin T and parity P are written as $\langle\pi|S|\pi\rangle = A\exp[2i\alpha]$, $\langle\pi|S|\gamma\rangle = C\exp[i\gamma]$, with A, α, C, γ real numbers, then — at energies which do not allow two or more pions in the final states — one obtains $A=1$, and also

$$\cos 2(\alpha-\gamma) = -1\,. \tag{2}$$

Recently the measurements of cross sections for reactions (1) have been extended to energies where two pion processes contribute appreciably, and we wish to present in this note a formula of the same type as (2), which takes these processes into approximate account (2).

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(1) K. M. WATSON: *Phys. Rev.*, **95**, 228 (1954); see also M. GELL-MANN and K. M. WATSON: *Ann. Rev. Nucl. Sci.*, **4**, 219 (1954).

(2) An analysis of the same type as the one given here has been made by R. H. CAPPS: *Phys. Rev. Lett.*, **2**, 475 (1959). Due to his exact treatment, Capps deduces only an inequality for the phase differences.

Let us introduce elements

$$\langle 2\pi, \lambda | S | \pi \rangle = B_\lambda \exp [i\beta_\lambda] \qquad \text{and} \qquad \langle 2\pi, \lambda | S | \gamma \rangle = D_\lambda \exp [i\delta_\lambda] \,,$$

which are responsible for two pion reactions. The index λ which figures here explicitly serves to indicate all the quantum numbers which, together with JTP, are necessary to specify the state of a two-pion, one-nucleon system. Invariance of S for time reversal means that S is symmetrical, $\langle b | S | a \rangle = \langle a | S | b \rangle$, and the unitarity condition $SS^+ = 1$ is now written

$$(3) \qquad 0 = \langle \pi | SS^+ | \gamma \rangle = \sum_n \langle \pi | S | n \rangle \langle n | S^+ | \gamma \rangle = \\ = \langle \pi | S | \pi \rangle \langle \pi | S^+ | \gamma \rangle + \langle \pi | S | \gamma \rangle + \sum_\lambda \langle \pi | S | 2\pi, \lambda \rangle \langle 2\pi, \lambda | S^+ | \gamma \rangle \,.$$

We have of course made $\langle \gamma | S | \gamma \rangle = 1$. We have also restricted ourselves to an energy region in which final states with three or more pions can be neglected. In this region S and P waves only should be adequate in describing two pion processes.

In order to proceed further and be able to connect (3) with the cross sections, we now assume the isobaric model ([3]). This describes the two pion processes as reactions which — due to the strong resonant interaction in the $P\,\frac{3}{2}\,\frac{3}{2}$ state of the pion nucleon system — proceed essentially through an intermediate state in which an excited nucleon $\mathcal{N}^*$ with spin $\frac{3}{2}$, isotopic spin $\frac{3}{2}$, parity $+$ and sharp mass is present. Thus we write

$$(4) \qquad \begin{cases} \pi + \mathcal{N} \to \pi + \mathcal{N}^* \to \pi + \pi + \mathcal{N} \,, \\ \gamma + \mathcal{N} \to \pi + \mathcal{N}^* \to \pi + \pi + \mathcal{N} \,. \end{cases}$$

Experimental evidence supports this model ([4]).

Reactions (4) proceed then, in this approximation, as though they were two body processes, and the sum over λ in (3) reduces now to only one term. This allows us to write

$$AC \exp [i(2\alpha - \gamma)] + C \exp [i\gamma] = -BD \exp [i(\beta - \delta)] \,.$$

Finally, squaring this equation, and using the other condition, $\langle \pi | SS^+ | \pi \rangle = 1$, which furnishes $A^2 + B^2 = 1$, we obtain the result

$$(5) \qquad \cos 2(\alpha - \gamma) = \frac{1}{\sqrt{1 - B^2}} \left[\frac{1}{2} B^2 \left(1 + \frac{D^2}{C^2} \right) - 1 \right] .$$

Let us indicate now by X^2 any one of the squared matrix elements B^2, C^2, D^2. The connection with the corresponding partial cross section of the same JTP is then

([3]) S. J. LINDENBAUM and R. M. STERNHEIMER: *Phys. Rev.*, **109**, 1723 (1958).

([4]) V. ALLES-BORELLI, S. BERGIA, E. PEREZ-FERREIRA and P. WALOSCHEK: *Nuovo Cimento*, **14**, 211 (1959).

given by the well known formula

$$\sigma = \frac{\pi \bar{\lambda}^2 (2J+1)}{2n} X^2 .$$

Here $\bar{\lambda}$ is the wave lenght of the incident particle, and n is 1 if this particle is a pion, 2 if it is a photon.

Unfortunately, the experimental results for reactions (4) are too scanty to allow a numerical evaluation of (5). There is one point to be noted, however, and this is that the existence of only S and P waves in (4), together with the isobar hypotesis, prevents the $J=\frac{1}{2}$, $P=-1$ state from going into a two pion channel. This follows from the usual rules for the addition of angular momenta, and makes relation (2) still valid at the energies here considered for the S wave of reactions (1).

A question could be raised, which concerns the inadequacy of the isobar model, due to the coexistent pion-pion interaction. Actually, experimental evidence seems to indicate that this interaction becomes important only at energies where three pion processes should also be important, that is at energies where the present analysis becomes inadequate anyway.

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In conclusion, we should like to thank Prof. G. Stoppini for posing the problem and for his encouragement, and Prof. M. Cini for a clarification regarding the pion-pion interaction.