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Summary

Radiative corrections to the processes $e^+ + e^- \longrightarrow \longrightarrow$ n pions are evaluated in relation to the planned colliding beam experiments. A theorem is proved which shows the possibility of separating experimentally the contribution of the γ - n pion vertex from the contribution of the 2 γ - n pion vertex.

In a recent work Cabibbo and Gatto (1) have discussed the possibility of direct measurement of the form factors of the photon-pion vertex through processes of the type:

$$e^+ + e^- \longrightarrow \text{n pions} \qquad (1)$$

They have also obtained in the first electromagnetic approximation the expression of the corresponding cross-section in the center - mass system. We will here discuss in wich way their results can be modified if the radiative corrections in the second electromagnetic approximations are taken into account. The general situation is then characterized by Feynmann's diagrams of fig. (1), where F and G

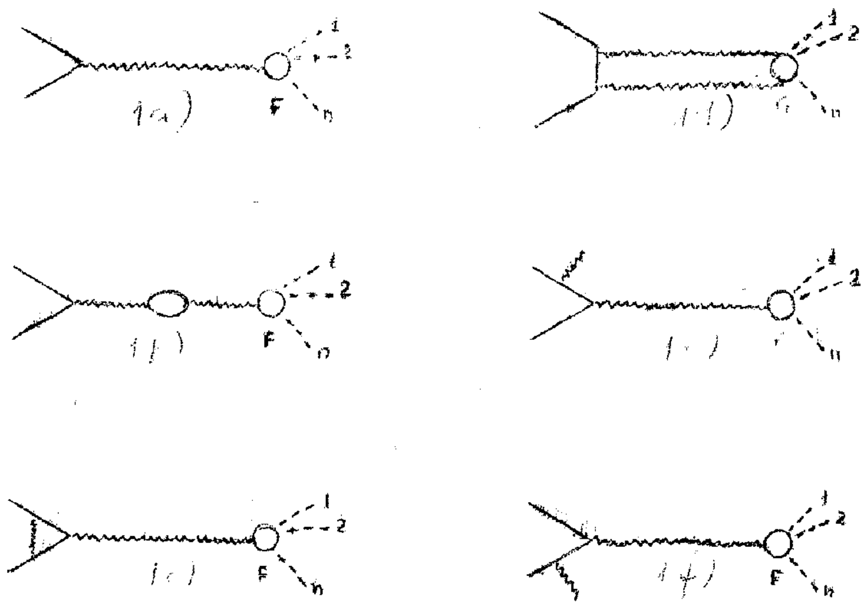

FIG. 1

represent the 'complete' vertices γ -nπ and 2 γ -nπ; that is they correspond to the sum of all the diagrams with external lines of this type.

Experiments on processes of type (1) should precisely measure F and G.

The corrections corresponding to diagrams 1b), 1c), 1e), 1f) do not involve new photon-pion verteces and therefore they will be latter calculated with the usual techniques.

The correction corresponding to the diagram with exchange of two virtual photons (fig. 1d) contains the new unknown vertex, 2 γ - pions.

On the basis of general considerations we will show it is easy to isolate experimentally the contributions to the cross - section hom the graphs with exchange of two photons; i.e. it is possible to distinguish the contributions of the two form factors. Our proof is based on three points:

a) In processes of type (1) the initial state, being eigenstate of the charge with eigenvalue 0, is a superposition of two eigonstates of the charge-parity operator C relative to eigenvalues + 1 and − 1.

$$|i) = |i^{+}) + |i^{-}) \text{ with } C|i^{\pm}) = \pm |i^{\pm}) \qquad (2)$$

b) calling S' and S'' the terms of the operator S corresponding to diagrams of fig. 1a) and 1d) respectively we have:

$$S'|i^{+}) = 0 \quad S''|i^{-}) = 0 \qquad (3)$$

In fact for instance:

$$S'|i^{+}) = (ie)^2 \int dx dx_2 \; i\,\Delta_f(x_1 x_2)\; j_\pi(x_1)\,|0)(0|j_e(x_2)|i^{+}) = 0$$

since $(0|j_e(x_2)|i^{+}) = (0\,C^{-1}|C\,j_e(x_2)C^{-1}|Ci^{+}) = -(0|j_e(x_2)(i^{+})$.

In the same way it is shown that $S''|i^{-}) = 0$.

c) We will now show that if the set of final states F distinguished from the measurement is an invariant subspace for the operator C, the contribution to the cross section of the interference term between the matrix elements corresponding to diagrams of fig. 1a) and 1d) is zero. In fact in this case we can assume as basis in F a set of vectors $|f_s)$ wich are eigenstates of C.

$$C\,|f_s) = \pm\,|f_s) \qquad (5)$$

The contribution to the transition probability in e^6 from the initial state $|i)$ to the set of final states F, due to the diagrams with exchange of two photons is:

$$\begin{aligned}
&2\,\mathrm{Re}\left\{\sum_s (i|S'^{+}|f_s)(f_s|S''|i)\right\} = \\
&2\,\mathrm{Re}\left\{\sum_s (i^{-}|S'^{+}|f_s)(f_s|S''|i^{+})\right\} = \\
&2\,\mathrm{Re}\left\{\sum_s (i^{-}C^{-1}|C S'^{+} C^{-1}|C f_s)(f_s C^{-1}|C S'' C^{-1}|C i^{+})\right\} = \qquad (6)\\
-&2\,\mathrm{Re}\left\{\sum_s (i|S'^{+}|f_s)(f_s|S''|i)\right. = 0
\end{aligned}$$

Let us now examine to wich experimental situation the condition that F be invariant subspace for C corresponds to. It is evidently sufficient that the apparatus revealing the final particles do not distinguish the π^+ from the π^-. This condition can certainly be verified in the experiments with intersecting beams in project at Stanford and in Frascati.

For the sinplest processes of type (1), that is $e^+ + e^- \longrightarrow \pi^+ + \pi^-$, it follows that, apart from particular experimental situations, the term of the cross section in e^6 due to the diagram d) is an odd function of cos θ (where θ is the angle between the momenta of the electron and of the π^-). Therefore it does not contribute to the differential cross-section for θ = 90 and to the integral cross-section. As instead the contributions of the other diagrams with exchange of one photon are even functions of cos θ, it is possible to separate the two types of contributions in an experience with variable θ; this means that it is possible to measure separately the form factors for the vertices $2\pi - \gamma$ and $2\pi - 2\gamma$. The above is valid also for similar processes like $e^+ + e^- \longrightarrow \mu^+ + \mu^-$, $e^+ + e^- \longrightarrow K^+ + K^-$, etc. Thus in an experiment $e^+ + e^- \longrightarrow \mu^+ + \mu^-$ it is possible to distinguish the contributions to the cross section in e^6 due to diagram with are unimportant for a check of renormalization theory, from the contributions of vertex and self-energy corrections.

Keeping in mind the above demonstration, we have calculated the expression of the cross-section to be used if one wants to measure the form factor of the 'complete' vertes $\gamma - n\pi$ through an experience of the (1) type.

The formula is valid for the revealing conditions stated above no contribution from the diagram with vertex $2\gamma - n\pi$; we have taken into account the fact that the electrons are certainly relativistic, and we have assumed that the maximum energy $\mathcal{E}$ of the bremsstrahlung photons (see fig. 1e and 1f) is small with reference to the energy of the emitting particles.

In such hypothesis the expressions of the correction is independent of the number and type of the final pions, and of the specific form of vertex γ-nπ.

Using the usual techniques to calculate radiative corrections we find:

$$d\sigma_n^{(1)} = d\sigma_n^{(0)}(1 + \delta_{SE} + \delta_V + \delta_B)$$

where $d\sigma_n^{(0)}$ and $d\sigma_n^{(1)}$ are the cross-sections in the first and in the second electromagnetic approximation of process(1), while δ_{SE} ; δ_V and δ_B are the percentage corrections due to diagrams of fig. 1b), 1c), 1e) and 1f). Expressions for $d\sigma_n^{(0)}$ are given in (1); for the corrections we have obtained:

$$\delta_{SE} = \frac{2\alpha}{\pi}\frac{2}{3}\left\{\ln\frac{2E}{m} - \frac{5}{6}\right\} \qquad (8)$$

$$\delta_V = -\frac{2\alpha}{\pi}\left\{\left(1-2\ln\frac{2E}{m}\right)\ln\frac{\lambda}{m} + \left(\ln\frac{2E}{m}\right)^2 - \frac{3}{2}\ln\frac{2E}{m} + 1 + \frac{\pi^2}{6}\right\}$$

$$\delta_B = -\frac{2\alpha}{\pi}\left\{\left(1-2\ln\frac{2E}{m}\right)\ln\frac{2\varepsilon}{\lambda} + \left(\ln\frac{2E}{m}\right)^2 - \ln\frac{2E}{m} + \frac{\pi^2}{6}\right\} \qquad (9)$$

where α is the constant of fine structure, E is the energy and m the mass of the electron (or positron), ε is the maximum energy of the bremsstrahlung photons, λ is the fictitious mass of the photon, that disappears in the global correction expression.

It is to be noted that (7) and (8) are also valid for $e^+ + e^- \longrightarrow \mu^+ + \mu^-$, if $d\sigma^{(0)}$ and $d\sigma^{(1)}$ are the corresponding cross sections in e^4 and e^6.

In this formula the effects due to creation of virtual particles heavier than electrons by the intermediate photon have been neglected; a most important contribution could come from the two pion intermediate states(2) (3).

In agreement with (3) they are negligible.

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References

(1) - N. Cabibbo and R. Gatto; Phys. Rev. Letters 4, 313(1960)
(2) - L.M. Brown and F. Calogero; Phys. Rev. Lett. 4, 315(1960)
(3) - Yung - Su - Tsai; Phys. Rev. 120, 269 (1960).

Riassunto.

Sono state calcolate le radiazioni radiative ai processi $e^+ + e^- \longrightarrow n$ pioni in relazione agli esperimenti a fasci incrociati in progetto. Viene dimostrato un teorema che indica la possibilità di separare sperimentalmente il contributo del vertice γ -n pioni dal contributo del vertice 2γ - n pioni.