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Touschek B.: A STUDY OF THE MECHANISM OF INJECTION INTO A STORAGE RING.

In the following I want to study the mechanism of injection for a machine of the type of the Frascati storage ring (AdA). This machine is distinguished from similar designs by the fact that no pulsed magnetic fields are employed the only agent contributing to the capture of the particles being the damping of the betatron oscillations caused by the replacement of the radiation losses by means of the radio frequency.

The proposed experimental arrangement is the following:

The γ-ray beam of the synchrotron (or for that matter of any other apparatus capable of producing a strong γ-beam) is converted into pairs on a target placed inside the acceleration chamber of the storage ring. The combination of the magnetic field and the radiofrequency picks electrons of the right energy and phase out of the continuous spectrum of the particles produced. The recapture of part of the 'accepted electrons' by the target is avoided by the damping of betatron oscillations.

It is assumed that the original γ-ray beam has a maximum energy K and that its spectral distribution can be approximated by a thin target distribution, so that one can assume that there are dk/k (for 0 < k < K) quanta with energy in dk. Γ is the yield of 'equivalent quanta' of the machine - for the Frascati synchrotron about 10^{10} $secs^{-1}$. Assuming that the converter target is thin it is then seen that the incident beam will produce

$$(1) \qquad d\dot{n} = \frac{\Gamma t}{Q} \cdot \frac{K-E}{K} \frac{dE}{E}$$

electrons of energy E in dE in one second and per cm^2 of target area. Q is the cross section of the incident beam and t (in radiation lengths) the thickness of the converter target.

The number $\dot{N}$ of electrons which are accepted in the storage ring in one second can then be written

$$(2) \qquad \dot{N} = \Gamma t \frac{\Delta E}{E} \frac{K-E}{K} P$$

Here ΔE - the effective width of the synchrotron oscillations - is defined by $2 \pi E = \bar{J}$, where $\bar{J}$ is the maximum of the phase integral of the synchrotron oscillations evaluated in the E, φ - plane (= phase angle of the synchrotron oscillations). For $\varphi_s \approx 0$ (which is always approximately true in a storage ring) one has

$$\Delta E = \frac{8}{\pi} \sqrt{\frac{E_s V}{2 \pi k \alpha}}$$

where E is the energy of the storage ring V the RF voltage, k the harmonic index of the RF and α the momentum compaction factor. For the Frascati ring $\Delta E = 0.72$ MeV. The quantity P in (2) represents the probability that an electron with the right energy can actually be stored in the ring. In the following section P will be determined for a completely linear machine.

Determination of P

A newborn electron leaving the target is characterised by the following 6 parameters:

(I) the radial coordinate at which the electron leaves the target. It is denoted by $x > a$ and a is the distance of the edge of the target from the equilibrium orbit.

(II) the ordinate $-b < y < b$ at which the electron leaves the target; the height of the target is assumed to be $2b$.

(III) $0 < \theta < \theta_0$ the polar angle of emission of the electron. The polar axis being parallel to the tangent of the equilibrium orbit in the target plane.

(IV) the azimuth of emission $0 < \varphi < 2\pi$, $\varphi = 0$ corresponding to emission in the equatorial plane.

(V) $0 < r < \bar{r}$ the radial amplitude of the synchrotron oscillations and

(VI) $0 < \psi < 2\pi$ the phase angle of the synchrotron oscillations. For $\psi = 0$ synchrotron oscillations carry the electron away from the target.

The probability of survival of an electron is given as a function of these 6 parameters $w = w(x, y, \theta, \varphi, r, \psi)$. For the probability P defined in the preceding section we can then write

$$(3)\quad P = \frac{1}{Q\,\theta_0^2\,\bar{r}^2\,\pi^2}\int_a dx \int_{-b}^{+b} dy \int_0^{\theta_0} \theta\, d\theta \int_0^{2\pi} d\varphi \int_0^{\bar{r}} r\, dr \int_0^{2\pi} d\psi\; w(x, y, \theta, \varphi, r, \psi)$$

Here we have assumed that the electrons are distributed with equal probability inside a cone of solid angle $\theta_0^2 \pi$, θ_0 being determined as a function of the multiple scattering of the electrons in the target. The synchrotron oscillations have been idealized by a linear approximation. Since w is a rapidly decreasing function of $x - a$ (only the electrons born near the edge of the target have a chance of survival) the upper limit of the x integration can be replaced by ∞. It is further assumed that the target covers the entire height of the acceleration chamber. Neglecting end effects we can then assume that w will only depend on the radial amplitude of the electrons. This amplitude is given by

$$(4) \qquad s = \left(x^2 + \frac{R^2\Theta^2\cos^2\psi}{1-n}\right)^{1/2} + r(1-\cos\psi)$$

It is obvious that only amplitudes which differ little from a will contribute to the survival probability. We therefore put $s-a = a\varepsilon$ and treat ε as an infinitesimal quantity. It is then easily seen that the probability that the electron hits the target after one revolution is given by $\pi(\varepsilon) = \frac{\sqrt{2}}{\pi}\varepsilon^{1/2}$. This quantity, however, has to be averaged over the synchrotron oscillations, which will move the beam periodically relative to the position of the target. Now it is worth noting that the amplitude of the synchrotron oscillations will be of the order of some millimeters, whereas the useful width of the edge of the target will be (owing to the minute spiralization) of the order of 10^{-3} mm. If ψ is not too close to zero the synchrotron oscillations will therefore lead to the loss of the particle - making its orbit dip for millimeters into the target. On the other hand synchrotron oscillations with $\psi \approx 0$ will periodically remove the electron from the target. This is favourable effect and will be responsible for the major contribution to the survival probability. If r/a is big compared with ε - but of course still small compared to one - we get for the probability of rehitting the target, averaged over the synchrotron period

$$(5) \qquad \langle\pi\rangle = \frac{1}{2\pi}\sqrt{\frac{a}{r}}\,\varepsilon$$

Now owing to the damping of the betatron oscillations the amplitude ε considered as a function of the number ν of revolutions can be written as

(6) $$\mathcal{E}(\nu) = \mathcal{E} - \delta\nu$$

where δ is the logarithmic decrement of the betatron oscillation viz:

(7) $$\delta = \frac{n}{1-n}\,\frac{V_s}{E_s}$$

and V_s is the average radiation loss per revolution. (In the Frascati ring one has $\delta \doteq 3.1\ 10^{-6}$). An electron started with an amplitude $\mathcal{E}$ will therefore be out of danger after an average of $\mathcal{E}/\delta$ revolutions. The probability of escape is then given by

(8) $$\vartheta = e^{-\int_0^{\mathcal{E}/\delta} \langle\pi\rangle d\nu} = e^{-\frac{1}{4\pi}\sqrt{\frac{a}{r}}\,\frac{\mathcal{E}^2}{\delta}}$$

The integral (3) can now be easily evaluated on the basis of the following simplifications and substitutions. Instead of θ and φ we introduce the coordinate $\lambda = \theta\cos\varphi$ and $\mu = \theta\sin\varphi$. Since only small angles θ_0 will contribute we can then replace approximately

$$\iint \theta\, d\theta\, d\phi \simeq \frac{\pi}{4}\int_{-\theta_0}^{+\theta_0} d\lambda \int_{-\theta_0}^{+\theta_0} d\mu$$

Since μ does not explicitely appear in w, we get from (3) and (8):

(9) $$P = \frac{ab}{Q\theta_0 \bar{r}^2\pi}\int_0^{\bar{r}} r\,dr \int_{-\pi}^{+\pi} d\psi \int_0^{\infty} d\xi \int_{-\theta_0}^{+\theta_0} d\lambda\; e^{-\frac{1}{4\pi}\sqrt{\frac{a}{r}}\,\frac{1}{\delta}\left(\xi + \frac{R^2\lambda^2}{2(1-n)a^2} + \frac{r}{2a}\psi^2\right)^2}$$

This integral we can now evaluate by introducing the following dimensionless parameters

$$\rho = \frac{r}{a}$$

$$(10)\qquad \xi_1 = (4\pi\delta)^{-1/2}\rho^{-1/4}\xi$$
$$\xi_2 = (4\pi\delta)^{-1/4}\rho^{-1/8}\frac{R}{a\sqrt{2(1-n)}}\lambda$$
$$\xi_3 = \frac{1}{\sqrt{2}}(4\pi\delta)^{+1/4}\rho^{3/8}\psi$$

We then get

$$(11)\qquad P = \frac{4\pi a^2 b\sqrt{1-n}\,\delta}{RQ\theta_0}\,k$$

where

$$(12)\qquad k \approx \int_0^{\infty} d\xi_1 \int_{-\infty}^{+\infty} d\xi_2 \int_{-\infty}^{+\infty} d\xi_3 \; e^{-(\xi_1+\xi_2^2+\xi_3^2)^2}$$

This integral can be evaluated in closed form. It has the value $\pi/2$, so that finally one gets

$$(13)\qquad P = \frac{2\pi^2 a^2 b\sqrt{1-n}\,\delta}{RQ\theta_0}$$

For the Frascati ring this gives (a=2cms, b=1.5cms, R=67cms, $Q=100cm^2$, $\theta=1/50$)

$$P = 1.8 \times 10^{-6}$$

Application to AdA.

At present AdA is situated at a distance of about 12 metres from the Synchrotron. The total yield of the injection process can be determined from (2) and one finds that about 5 electrons per second can be injected into AdA with the present arrangement. Assuming that it would indeed be possible to reach a vacuum of 10^{-10}mm we should then expect to accumulate about 4.8 million electrons in a run of 250 hrs (the lifetime of the beam corresponding to 10^{-10} mm). This is still a factor 100 below what is wanted to make the machine a success. The hopes of improvement are:

(1) Putting AdA nearer to the synchrotron: this may give a factor 5.

(2) Improving the intensity of the synchrotron; another factor 4, say.

(3) Non linear effects. These seem particularly promising in connection with the rôle played by the synchrotron oscillations. Putting the target to the limit of the stable region might enhance the breathing effect of these oscillations increasing the difference between the capture probability and its average. Admitting our ignorance this might well be worth a factor 10.

(4) Improvement of the target. I have estimated this previously as being worth a factor 50. But I do not think that (3) and (4) are independent of one another. It migh then only be reasonable to expect that (3) and (4) together give a factor 100.

On the debit side we have to note that it is quite unrealistic to expect the Frascati synchrotron to work for 10 days without a pause. 2 days might be more realistic. This gives the factor:

(1-) Stability of machinery (not only the synchrotron but more likely something may go wrong with AdA herself). A factor 1/10.

(2-) part of the time (or of the intensity) has to be used to make positrons: factor 1/2.

All these factors together give one an improvement of 100 on the previous estimate, which would make the machine precisely 'successful' in the sense of the definition that a successful machine should yield 1 pulse per second for the reaction $e^+ + e^- = 2\gamma$.

There is, however, one outstandingly positive result coming from all these calculations: if the statistical theory is right AdA is bound to work with a linear accelerator as a source of γ- rays.