

Laboratori Nazionali di Frascati

LNF-60/37 (3. 8. 60)

N. Cabibbo, R. Gatto: CONSISTENCY OF THE $K^+ \rightarrow \bar{\pi}^+ + \bar{\pi}^0 + \gamma$ RATE
WITH THE $\Delta T = 1/2$ RULE.

Nota interna n° 47
3 Agosto 1960

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CONSISTENCY OF THE $K^+ \rightarrow \pi^+ + \pi^0 + \gamma$ RATE WITH THE $\Delta T = \frac{1}{q}$
RULE

(Submitted to Physical Review Letters for publication)

It was proposed by Gell-Mann and Pais⁽¹⁾ that in the decays of hyperons and K-mesons into strong interacting particles the isotopic spin change is given by $\Delta T = 1/q$ when electromagnetic interactions are neglected. The subsequent detailed investigation of the implications of the rule⁽²⁾ led to predictions which seem now to be satisfied by the most recent data⁽³⁾.

For $K^+ \rightarrow \pi^+ + \pi^0$ the $\Delta T = 1/q$ rule predicts no rate at all in the absence of electromagnetic corrections. Experimentally the $K^+ \rightarrow \pi^+ + \pi^0$ rate is found to be almost two hundred times smaller than the rate for $K^0 \rightarrow 2\pi$. This result is in the direction of the prediction from the $\Delta T = \frac{1}{q}$ rule. The question then arises of discovering the mechanism which makes the amplitude for $K^+ \rightarrow \pi^+ + \pi^0$, arising from electromagnetic corrections of order e^2 , large enough to account for the observed rate. Independently of this question it has been pointed out that the amplitude for $K^+ \rightarrow \pi^+ + \pi^0 + \gamma$ would be of the order e and might thus compete favorably with the amplitude for $K^+ \rightarrow \pi^+ + \pi^0$ ⁽⁴⁾. Such a possibility would strongly contradict present experimental evidence based on a few $K^+ \rightarrow \pi^+ + \pi^0 + \gamma$ events which can be accounted for in terms of the internal bremsstrahlung processes accompanying the $K^+ \rightarrow \pi^+ + \pi^0$ decay.

In this note we show that there is no contradiction between the observed low frequency of $K^+ \rightarrow \pi^+ + \pi^0 + \gamma$ events

and the $\Delta T = 1/q$ rule. Specifically we show that the total $K^+ \rightarrow \pi^+ + \pi^0 + \gamma$ amplitude is not expected to be much larger than the internal bremsstrahlung contribution. This contribution would be of the order e^3 , according to the exact $\Delta T = 1/q$ rule, but it may thus compete favorably with the direct amplitude of order e . Such a rather unique situation would make very interesting a measurement of the π^+ spectrum of $K^+ \rightarrow \pi^+ + \pi^0 + \gamma$.

We shall construct an amplitude for $K^+ \rightarrow \pi^+ + \pi^0 + \gamma$ satisfying the requirements of gauge invariance. The total amplitude will consist of an internal bremsstrahlung amplitude (of order e^3 if $\Delta T = 1/q$ holds exactly) and of a direct amplitude (of order e). The (necessarily off-shell) amplitude for $K^+ \rightarrow \pi^+ + \pi^0$ satisfying $\Delta T = 1/q$ can be approximated by retaining only those terms with the lowest dependence on momentum transfer. Such an approximation should be valid if high mass intermediate states contribute to the decay, as it is expected. The form of the amplitude is then uniquely fixed by $\Delta T = 1/q$ and corresponds to an effective Lagrangian

$$\mathcal{L}' = \sqrt{2} fM \frac{\partial \bar{\Phi}}{\partial x_\nu} \left(\psi^* \frac{\overleftrightarrow{\partial}}{\partial x_\nu} \psi \right) + \text{H.c.} \quad (1)$$

where f is a constant, M is the K mass, $\bar{\Phi}$ is the K^+ field, ψ is the charged π field, and $\frac{\overleftrightarrow{\partial}}{\partial x_\nu} = \frac{\partial}{\partial x_\nu} - \frac{\partial}{\partial x_\nu} (6)$. The amplitude for $K^+ \rightarrow \pi^+ + \pi^0$ on the mass shell (which necessarily violates $\Delta T = 1/q$) can be derived from the simplest Lagrangian

$$\mathcal{L}'' = gM^3 \bar{\Phi} \psi^* \psi + \text{H.c.} \quad (2)$$

on the basis that any structure effect from this vertex would certainly be negligible considering the smallness of the K^+ decay constant g . The current producing the radiative decay $K^+ \rightarrow \pi^+ + \pi^0 + \gamma$ includes contributions from the π and K currents and from the transition current derived from (1) through the substitution $\frac{\partial}{\partial x_\nu} \rightarrow \frac{\partial}{\partial x_\nu} - ieA_\nu$. The total amplitude is given by

$$\frac{e}{(4\pi)^2} \frac{1}{(ME_0 E_+ E_\gamma)^{1/2}} \mathcal{E}_\mu \dot{j}_\mu \quad (3)$$

where

$$\dot{j}_\mu = \left(\frac{p_\mu^{(+)}}{p_\nu^{(+)} \cdot q_\nu} - \frac{K_\mu}{K_\nu \cdot q_\nu} \right) \left[gM^2 - \sqrt{2} f q_\lambda \cdot (p_\lambda^{(+)} - p_\lambda^{(o)}) \right] \quad (4)$$

In (3) and (4) E_0, E_+, E_γ are the energies of the final particles, \mathcal{E}_μ is the polarization vector, $p_\mu^{(o)}, p_\mu^{(+)}, K_\mu$ and q_μ are the momenta of the final pions, of the K-meson and of the photon respectively. It may be noted that the current (4) does not lead to any magnetic dipole emission: this is a consequence of our approximation which neglects higher structure terms. The pion spectrum from (3) and (4) is given by

$$\frac{1}{w_0} \frac{dw}{d\omega} = \left(\frac{e^2}{4\pi} \right) \frac{2qp}{\pi E \beta} \left[\mathcal{B} + x \mathcal{Y} + x^2 \mathcal{D} \right] \quad (5)$$

with

$$\mathcal{B} = \frac{2}{q^2} \left[\frac{\omega}{p} \ln \frac{\omega+p}{\mu} - 1 \right]$$

$$\mathcal{Y} = \frac{\sqrt{2} E}{M^2 q} \left[6 - \frac{\mu^2 + 2M\omega}{Mp} \ln \frac{M(\omega+p) - \mu^2}{M(\omega-p) - \mu^2} - \frac{M-4\omega}{p} \ln \frac{M-\omega+p}{M-\omega-p} \right]$$

$$\mathcal{D} = \frac{E^2}{M^4} \left[2 \frac{\mu^2 + M\omega}{Mp} \ln \frac{M(\omega+p) - \mu^2}{M(\omega-p) - \mu^2} + 4 \frac{M-2\omega}{p} \ln \frac{M-\omega+p}{M-\omega-p} - 12 \right]$$

in the following notations: q is the photon momentum in the (γ, π^0) center of mass system, ω and p are the energy and momentum of π^+ in the K rest system, E is the total energy of (γ, π^0) in their center-of-mass system, β is the velocity of the non-radiative decay, $x = f/g$, μ is the π mass, and w_0 is the rate for non-radiative decay. The expressions for E , q and β are

$$E^2 = M^2 + \mu^2 - 2M\omega$$

$$2Eq = M^2 - 2\omega M$$

$$M^2 \beta^2 = M^2 - 4\mu^2$$

In (5) \mathcal{B} is the bremsstrahlung term arising from the small amplitude (2) for $K^+ \rightarrow \pi^+ + \pi^0$, \mathcal{D} arises entirely from the off-mass-shell amplitude (1), and \mathcal{V} is an interference term. The bremsstrahlung term coincides with that given by Good (5), but the form of the other terms differs from estimates contained in reference (5) on the basis of dimensional and invariance considerations. In fig.1 we report $A\mathcal{B}$, $Ax\mathcal{V}$, and $Ax^2\mathcal{D}$ with $A = qp/E\beta$, as given by (5), and for $x = 15$. In the ratio x/g can be derived from the $K^+ \rightarrow \pi^+ + \pi^0$ rate. A simplest model that gives a unique value for f is the following. The $K \rightarrow 2\pi$ decays are supposed to proceed through virtual dissociation of the K-meson into a nucleon and an antilambda (for both K^+ and K^0); the Λ decay interaction (subject to $\Delta T = 1/q$) contains the derivative of the pion field. It then follows that $K^0 \rightarrow 2\pi$ comes from an effective Lagrangian

$$fM \frac{\partial \Phi_0}{\partial x_\nu} \left(\vec{\psi}, \frac{\partial \vec{\psi}}{\partial x_\nu} \right) \quad (6)$$

with the same f as in (1). The value of f can then be obtained by comparing the expression for the total $K^0 \rightarrow 2\pi$ rate derived from (6) with its experimental value. In this way are obtained $x = \pm 15$. According to the choice of the sign one finds that the number of $K^+ \rightarrow \pi^+ + \pi^0 + \gamma$ events in best explored region of 55 - 75 MeV is bigger by a factor 1.6 (for negative interference term) or 5 (for positive interference) than the number predicted on the basis of internal bremsstrahlung alone (i.e 1 event for the 8653 decays examined, among which two anomalous decays were reported). We consider our conclusion to be in agreement with experiment also in view of our ambiguous determination of f . For instance if our model to determine f is extended to include Σ -particles one can derive in perturbation theory, assuming special forms for the hyperon decay Lagrangians (7), cut-off independent relations between f and the K^0 decay constant of (6). Such relations do not give however unique results because of the possibility of varying some signs in the hyperon decay Lagrangians (8).

References and footnotes

- (1) M.Gell-Maun and A.Pais, Proceedings of the Glasgow Conference (1954);
- (2) R.Gatto, Nuovo Cimento, 3, 318 (1956);
G.Wentzel, Phys. Rev., 101, 1214 (1956);
- (3) see F.S.Crawford et al., Phys. Rev. Letters, 2, 266 (1959);
- (4) M.Gell-Maun, Nuovo Cimento, 5, 758 (1957);
- (5) J.D.Good, Phys. Rev., 113, 352 (1959);
- (6) We report briefly the argument. The rule $\Delta T = 1/q$ implies that the final pions appear in the combinations

$$\varepsilon_{ikl} \varphi^{(k)} \varphi^{(l)}, \quad \varepsilon_{ikl} \frac{\partial \varphi^{(k)}}{\partial x_j} \frac{\partial \varphi^{(l)}}{\partial x_j},$$

$$\varepsilon_{ikl} \frac{\partial \varphi^{(k)}}{\partial x_j} \varphi^{(l)}, \quad \text{or} \quad \varepsilon_{ikl} \varphi^{(k)} \frac{\partial \varphi^{(l)}}{\partial x_j}.$$

Of such expressions only the last two differ from zero and they are equal apart from sign;

- (7) O.Hori, Nuclear Physics, 17, 227 (1960)
- (8) Particular models including Σ -particles are being investigated by dr. Bassetti; whom we wish to thank for useful discussions.-

Caption to figure 1

The different contributions to the π^+ spectrum in $K^+ \rightarrow \pi^+ + \pi^0 + \gamma$. A β is the bremsstrahlung term, $Ax^2\mathcal{D}$ is the direct term, and $Ax\mathcal{I}$ is the interference between the two. The value of the parameter x is here chosen to be 15.-

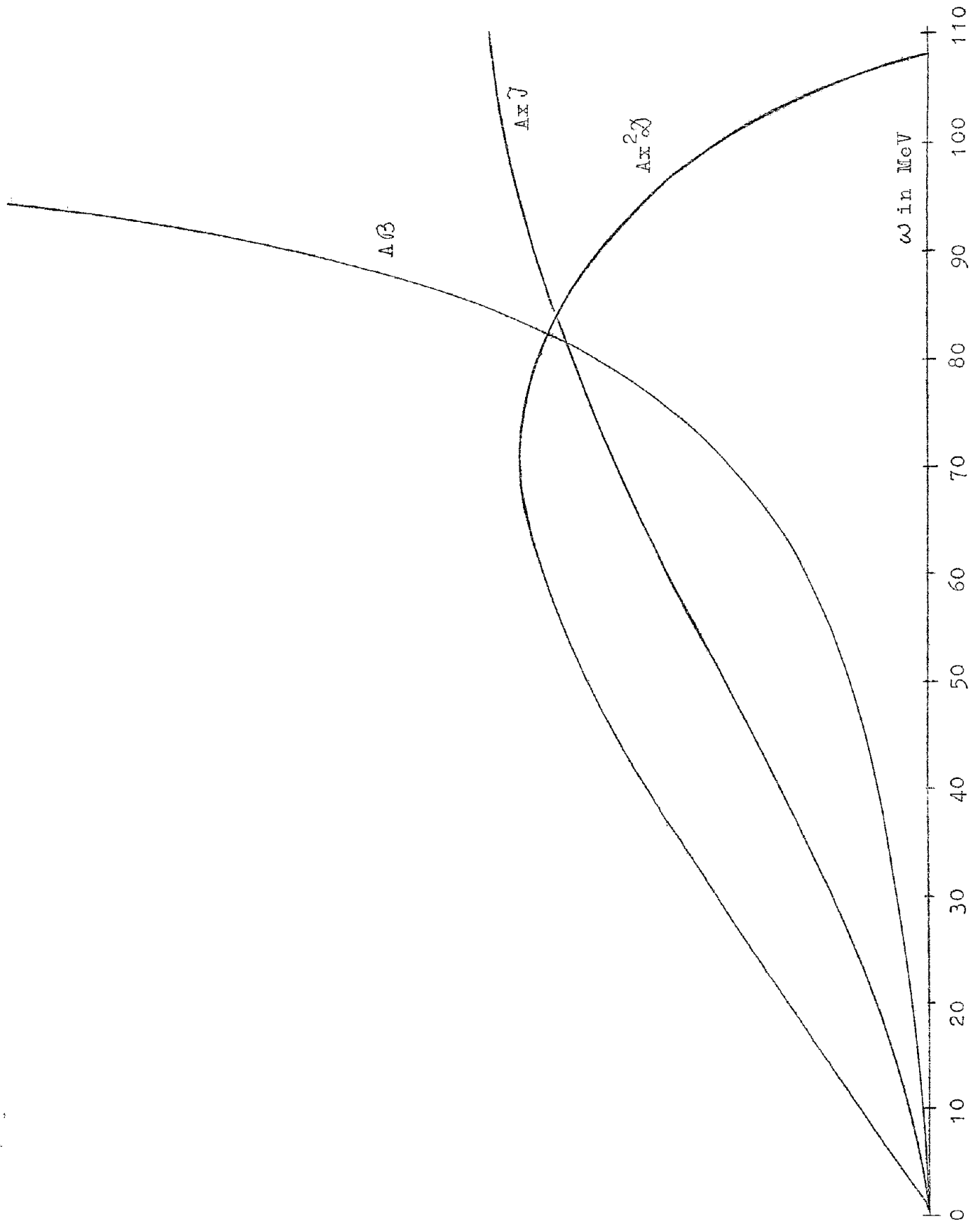


FIG. 1
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