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N. Cabibbo, E. Ferrari: SOME RARE DECAY MODES OF THE K MESON.

N. Cabibbo and E. Ferrari[+]: SOME RARE DECAY MODES OF THE
K MESON (Submitted to Nuovo Cimento for Pubblication).

1) The expected availability of high intensity K meson beams
makes it necessary to evaluate the branching ratio to be ex
pected for presumably very rare decay modes of the K-mesons.
In the first part of this note we consider the possible de-
cay mode

$$K^{\pm} \longrightarrow \pi^{\pm} + e^{+} + e^{-} \qquad (1)$$

due to the virtual step $K^{+} \longrightarrow \pi^{+} + \gamma$ and $\gamma \longrightarrow e^{+} + e^{-}$.
The process is of the second order in the e.m. coupling con
stant, but it is expected to have a fairly low branching
ratio in view of the fact that for a spin zero K meson the
first of the two virtual steps is forbidden when the pho-
ton is transverse and on the energy shell. In the second
part of the paper we consider the decay modes $K^{o} \longrightarrow 2\gamma$.
In discussing such reactions one has to analize first the
consequences of the application of the CP selection rules
to the K^{o} decays[1].

From CP invariance the matrix elements for the 2
decay of the shortlived K^{o}, K_{S}^{o} is of the form $(\vec{\mathcal{E}}_{1} \cdot \vec{\mathcal{E}}_{2})$

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(1) - R. Gatto, Phys. Rev. 106, 168 (1957).

where $\vec{\varepsilon}_1$, $\vec{\varepsilon}_2$ are the polarization vectors of the photons, while the matrix element for the longlived K^o, K_L^o, is of the form $\vec{q}(\vec{\varepsilon}_1 \wedge \vec{\varepsilon}_2)$ where $\vec{q}$ is the relative final momentum. The lowest mass intermediate states for K_s^o is the 2π state, while for K_L^o it is the 3π state. Our calculation concerns K_s^o via the lowest mass intermediate state, but we expect a similar branching ratio for K_L^o considering that its effective coupling is smaller and also its lifetime is longer.

2) The simplest diagram which represents reaction (1) (let us choose the case of a K^+) is the following one, involving a vertex $(K^+\pi^+\gamma)$
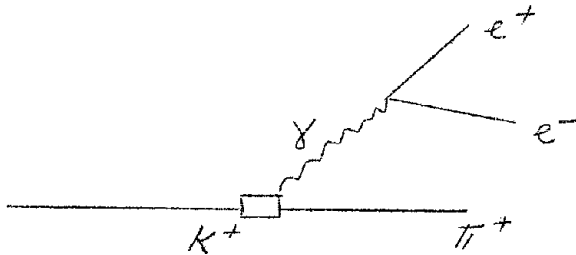
The vertex $(K^+\pi^+\gamma)$ must vanish for real photons because of gauge invariance.

The matrix element for process (2) will then be ($q = k - p$, where k and p are the 4 moments of the K^+ and the pion, and p^+, p^- the momenta of the two electrons):

$$\langle p\, p^+ p^- | k \rangle = \frac{(2\pi)^{-3}}{q^2} \langle p | J_\mu | k \rangle \, \delta^4(k-p-p^+-p^-) \times \quad (2)$$

$$\times \left[\bar{u}(p^-)\, \gamma_\mu\, v(p^+) \right]$$

We make the position $\langle p | J_\mu | k \rangle = (2\pi)^{-3}(4E_p E_k)^{-1/2} C_\mu$ where $(p-r)_\mu\, C_\mu = 0$ due to gauge invariance. This condition fixes C_μ to be proportional to $\{ k_\mu \left(\mu^2 + (pk) \right) + p_\mu \left[m_k^2 + (pk) \right] \}$. We evaluate the decay rate by performing first a summation

over the spins and momenta of the electrons and we get:

$$Rate\ (K^+ \to \pi^+ + e^+ + e^-) = \frac{e^2}{12\,m_K\,(2\pi)^{12}} \int \frac{d^3p\,(C_\mu\,C_\mu)}{E_p\,[-(p-k)^2]}$$

We shall now try to evaluate C_μ.

We can describe the $(K^+\pi^+\gamma)$ vertex in the simplest way by means of an intermediate $\bar{\Lambda}-p$ loop, obtaining the following diagrams:
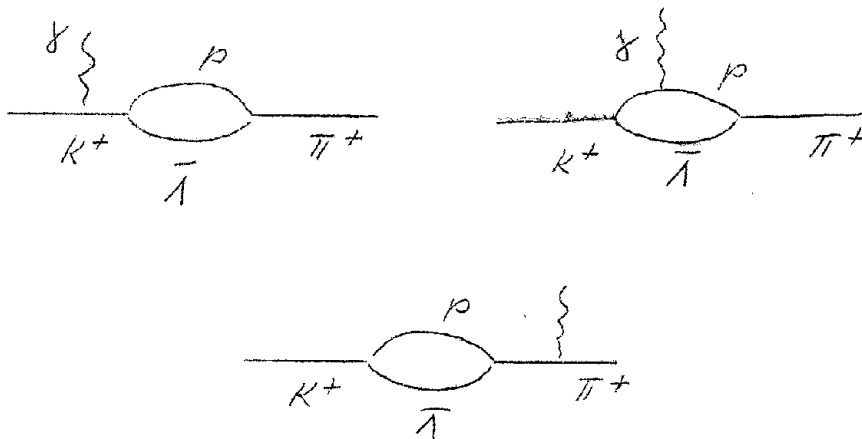
The left hand side vertex in the diagrams gives substantially the $(K^+p\Lambda)$ coupling constant, while the right hand side vertex is related to the lifetime of the Λ^o-hyperon. Of course in a better calculation the effect of intermediate Σ's should also be included.

One finds that the dominant part of the contribution of the above graphs (i.e. the contribution obtained by letting the masses of the intermediate fermions go to infinity) vanishes identically, as a consequence of a general theorem stated by Cabibbo and Gatto which can be easily extended to cover this case[2].

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(2) - N. Cabibbo and R. Gatto, Phys. Rev. 116, 1334 (1959).

See also S. Okubo, Nuovo Cimento, 16, 963, (1960).

The consequence is that the contribution of the above graphs is of the order (m/M), where m is of the order of magnitude of the masses of the external particles, and M of the particles of the loop. The integrals are now convergent. A detailed calculation to the order $(m/M)^2$ gives the gauge invariant result[3]

$$C_\mu \simeq i(2\pi)^2 \frac{eg G}{3M} \left[K_\mu (\mu^2 + pK) + p_\mu (m_k^2 + pK) \right] \qquad (3)$$

G is a constant related to the rate R_Λ of the process $\Lambda^0 \longrightarrow p + \pi^-$ through

$$G^2 = \frac{4\pi}{\wp} \frac{m_\Lambda^2}{\left[(m_\Lambda^2 - M^2) - \mu^2 (m_\Lambda^2 + m^2) \right]} R_\Lambda$$

($\wp$ is the 3-momentum of the π^- in the Λ^0 rest system).

Finally one gets

$$R_{e^+ e^-} = \alpha^2 \left(\frac{g_{\Lambda K p}}{4\pi} \right)^2 \frac{1}{108 \pi^3} \left(\frac{\mu}{M} \right)^4 \frac{m_K}{\wp} F_1 F_2$$

where

$$F_1 = \frac{m_\Lambda^2 M^2}{\left[(m_\Lambda^2 - m^2) + \mu^2 (m_\Lambda^2 + M^2) \right]}$$

$$F_2 = \frac{1}{4} \left(\frac{m_K}{\mu} \right)^2 \left[\left(\frac{m_K^2 - \mu^2}{2 m_K \mu} \right)^2 - \frac{3}{2} \right] + \frac{3}{2} \ln \frac{m_K}{\mu}$$

The branching ratio is

$$\frac{K^+ \longrightarrow e^+ + e^- + \pi^+}{K^+ \longrightarrow all\ other\ modes} = 1.0 \cdot 10^{-7}$$

(3) - The terms of higher order introduce an error $\simeq 4\%$.
The error induced by neglecting in the integrals the Λ - p mass difference is much smaller. M = nucleon mass, μ = pion mass.

This ratio is indeed very small, smaller that it would be thought on the basis of an order of magnitude calculation[4]. This is due to the 'selection rule' provided by the mentioned theorem for the $(K^+\pi^+\gamma)$ vertex.

3) The rate of the reaction

$$K_s^o \longrightarrow 2\gamma \tag{4}$$

Can be deduced by associating the interaction responsible for the decay of the K_s^o into two charged pions with the interaction of these pions with the electromagnetic field. The process will be described by a diagram of this type:
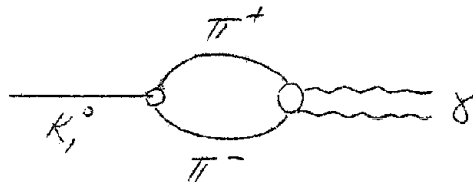
The left-hand side verter will be associated to the rate of the normal $K_s^o \longrightarrow \pi^+ + \pi^-$ decay; the right-hand side vertex can be safely evaluated by means of a standard perturbation calculation. The S-matrix element of the process is given by the following expression:

$$\langle K_1 K_2 | p \rangle = \frac{1}{\sqrt{2}} e^2 G \delta^4 (p - K_1 - K_2)(2\pi)^{-9/2} (2\varepsilon \epsilon_1 \epsilon_2)^{-1/2} (\vec{\epsilon_1} \vec{\epsilon_2}) I$$

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(4) — An order of magnitude estimate was made by R.H. Dalitz, Phys. Rev. 99, 915 (1955): the branching ratis expected was $5.10^{-5} (C/A)^2$, where C and A are two unknown constants. Comparison with the present result shows that C = 1/20 A.

G is a constant which is connected to the rate R_K of the $K_1^o \longrightarrow \pi^+ + \pi^-$ decay by

$$G^2 = 16 \pi \, m_K^2 \, (m_K^2 - \mu^2)^{-1/2} \, R_K$$

p, k_1, k_2 are the 4 momenta of the K_S^o and of the $\gamma's$; E, E_1, E_2 are the corresponding energies. In the K_S^o rest-system we have $E = M_K$, $E_1 = E_2 = 1/2 \, M_K$. ε_1, ε_2 are the polarization vectors of the $\gamma's$. The factor $1/\sqrt{2}$ comes from the symmetrization of the final state. The expression $(\varepsilon_1 . \varepsilon_2) I$ is the result of the evaluation of the following integral:

$$(\varepsilon_1 \varepsilon_2) I = \int d^4 q \, \frac{4 (\varepsilon_1 q)(\varepsilon_2 q)}{(q^2 + \mu^2)[(q - k_1)^2 + \mu^2][(q + k_2)^2 + \mu^2]} \; +$$

$$+ \int d^4 q \, \frac{4 (\varepsilon_1 q)(\varepsilon_2 q)}{(q^2 + \mu^2)[(q - k_2)^2 + \mu^2][(q + k_1)^2 + \mu^2]} - \int d^4 q \, \frac{(\varepsilon_1 \varepsilon_2)}{(q^2 + \mu^2)[(q - k_1 - k_2)^2 + \mu^2]}$$

Since the integrals are logarithmically divergent, a cut-off parameter must be introduced. Due to the fact that the mass of the intermediate state is less than the mass of the initial state, the integrals will not be pure imaginary (as in the usual evaluation of Feynman diagrams) but will have also a real part, which in the present case, however, turns out to be small.

The final expression of the rate $R_{2\gamma}$ for process (4) is ($\alpha = e^2/4\pi$)

$$R_{2\gamma} = \frac{1}{8} \frac{1}{(2\pi)^3} G^2 \frac{|I|^2}{m_K} = \alpha^2 R_K N \qquad (5)$$

This result is of the expected order of magnitude, since the coefficient N is of the order unity with a cutoff of reasonable order of magnitude. For a cutoff equal to the nucleon mass M, we get N = 0,37: the variation of this number

with the cutoff is very slow. The branching ratio of this process is

$$\frac{K_s^o \rightarrow 2\gamma}{K_s^o \rightarrow \text{all other modes}} \simeq 2,3 \ 10^{-5}$$

As it has been said in the Introduction, the corresponding ratio for the K_L^o is expected to be of the same order of magnitude. A calculation of the preceding type cannot be easily performed because of the presence of the unknown $3\pi - 2\gamma$ vertex.

We thank Prof. R. Gatto for helpful discussion.