

Laboratori Nazionali di Frascati

LNF-59/22 (Luglio 1959)

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Estratto dal: Report CNF-2 del CNEN

COMITATO NAZIONALE PER LE RICERCHE NUCLEARI

Searching for ϱ^0 Mesons

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Roma, luglio 1959

SEARCHING FOR ρ^0 MESONS

1. - Theoretical background

It is well known that there are some successful classification schemes of the elementary particles in terms of a few quantum numbers and that these schemes are not completely filled by observed particles. Among the others, the Gell Mann - Nishijima (1) and the Schwinger's (2) one are the most famous as providing a convincing set of intercorrelations between similar particles and selection rules for production and decays. These two schemes are essentially the same: the first is based on the assignment of three quantum numbers, the isospin T the strangeness S and the baryonic number B; the second on the assignment of T, B and the hypercharge Y. The charge of the particle is given by

$$Q = T_3 + \frac{1}{2} B + \frac{1}{2} S = T_3 + \frac{1}{2} Y$$

so that the obvious relation between Y and S is

$$Y = S + B ;$$

the Schwinger's scheme has the merit that the quantum numbers can be interpreted in terms of 'geometrical' symmetries of a 4 - dimensional euclidean formulation of field theories.

We must stress that no explicit relation is given between mass, spin, parity etc. of the elementary particles and these quantum numbers. Nevertheless, everyone knows that mesons, having zero baryonic number, have at the same time integer spin; and so on.

People usually explore the following range of values of T, S, B (3)

$$T = 0, \frac{1}{2}, 1$$

$$S = 0, \pm 1, \pm 2 \quad (1)$$

$$B = 0, \pm 1$$

but the schemes allow for a numerable infinity of particles: there are however some guiding principles (by no means firmly established till now) limiting the number of possibilities. One of these principles says that there are no doubly (or more) charged particles so that

$$\left| T_3 + \frac{1}{2} B + \frac{1}{2} S \right| = \left| T_3 + \frac{1}{2} Y \right| \leq 1$$

The mass seems to increase with the strangeness.

The baryonic number distinguishes particles from antiparticles and there is no meaning at present for $|B| > 1$ with the just mentioned limitations, it seems that the most promising holes in the schemes are those in the ranges (1). We will fix our attention on the missing $T = 0, S = 0, B = 0$ particle and call it in the following ρ^0 .

No prescription follows from the aforesaid scheme about the mass of the ρ^0 , the spin and the intrinsic parity.

The mass value determines essentially, through the threshold energy, the possibility of production with a given accelerator. For instance, with the Frascati 1 BeV synchrotron (4) and according to the reaction



the upper limit of the mass value one can observe is about 720 MeV.

Moreover, the mass value, together with the spin and parity, selects between a large variety of possible decay modes those which are compatible with fundamental conservation laws.

Now, to restrict the possibilities for all these dynamical parameters not included in the classification schemes, one must collect a number of physical arguments emerging from known (or, better, badly-known) facts; the goal is to allow for saying: well, we need the same ρ^0 for all these reasons, let us try to observe it.

In what follows we will be mainly concerned with the role of ρ^0 as common remedy for a lot of failures of *current* theoretical methods in explaining experimental facts. One must mention that there are some models of elementary particles calling for such a meson: for instance in the Fermi Yang model ⁽⁵⁾ there is room for two kind of mesons, the isotopic triplet

$$\pi^+ = p\bar{n} \ , \quad \pi^0 = \frac{1}{\sqrt{2}} (p\bar{p} + n\bar{n}) \ , \quad \pi^- = \bar{p}n$$

and the isotopic singlet

$$\rho^0 = \frac{1}{\sqrt{2}} (p\bar{p} - n\bar{n})$$

Whether this model or another one is the right way to approach theoretically the elementary particles problem is just, at present, a matter of opinion; one can just say that it sounds very attractive.

In this paper we will be mainly concerned with the efforts people has done to improve the matching of current theoretical methods with experimental data, by introducing a new, but non-freakish, meson.

2. - Connection with the experimental situation.

We should have mentioned in the preceding paragraphs that in most cases a valid substitute for ρ^0 mesons would be an adequate two or more pions bound state, that is a π - π interaction. An unstable bound state with a fast decay can simulate almost all the main features of processes involving ρ^0 mesons. But, assuming that the classification scheme for elementary particles applies to such particles in strict sense (as distinguished from compound particles), we will speak of ρ^0 mesons having in mind that there is a pictorially different possibility.

The ρ^0 has a long story, on the Physical Review time scale. In 1955, Teller and Johnson (6) made some calculations on the properties of nuclear matter by assuming that nucleons are linked together by a suitable boson field. It is an entirely classical approach, in the style of Thomas Fermi theory. We review it here for completeness, being aware that the strength of such an argument is quite doubtful. Briefly said, the assumptions of this picture are:

- 1) that the shell model is a reasonable one
- 2) that a boson sea can produce a smooth self-consistent potential for the nucleons.

On this ground, if one takes for the interaction term an expression like

$$g \psi O_i \psi \Phi_i \quad (\text{non relativistic})$$

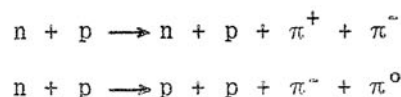
(where g is an interaction constant, ψ the many-nucleons wave function, O_i an operator built from nucleon variables and Φ_i the boson field (or a functional of the boson field)) by requiring that the interaction be invariant and inserting for ψ shell model wave functions, one obtains that

the field Φ_i (like O_i) must be a scalar, isotopic singlet. Φ_i could obviously be a complicated functional of well known boson fields, thus representing some sort of collective behaviour of pions due to $\pi\pi$ interaction (like phonons in a crystal lattice). Were it a particle or a quasiparticle, this scalar, isotopic singlet meson should have an effective mass 500 MeV; moreover the interaction constants involved in nuclear data fitting seems to be of the order of the pionic one.

Sometime after this proposal by Teller and Johnson, a scalar neutral meson has been tentatively suggested by Gupta (7). He had in mind the possibility offered by the Gell Mann scheme and in order to take advantage of it, he looked for nuclear phenomena resembling a mortice with one missing piece in current theory. Anomalous magnetic moments of the nucleons belong to this class and Gupta shows that adding to π and K particles contributions the contribution of a ρ^0 meson there results a trend toward the right answer. A few graphs are included in the calculation and the main assumption is that the ρ^0 's coupling constant is the same as for π -N, g^2 . Then g^2 and a coupling constant for K particles, f^2 , are determined from the experimental values of the magnetic moments and, as a check of the suitability of the improvement, they must agree with the expected value (at least for g^2). In fact, they do, quite insensitively to the ρ^0 's mass in the range of a few π masses.

Next, Gupta considers (8) multiple pion production in high energy events as NN collisions or $N\bar{N}$ annihilations. There are three main points:

- 1) the multiplicity of π is usually greater than foreseen by Fermi theory
- 2) Fermi theory gives a ratio 3.3 : 1 for the processes



whereas the observed ratio seems higher

3) there seems to be some angular correlation between charged π in multiple production.

Gupta's scalar ρ^0 , were it heavy enough, should have a fast decay according to

$$\rho^0 \rightarrow \pi^+ + \pi^-$$

The intermediate step of the process

$$N + N \rightarrow N + N + \rho^0 \rightarrow N + N + \pi^+ + \pi^-$$

goes immediately in favour of points 1) 2) and 3). In particular, for point 2) the intermediate step is effective for the first of the two reactions only. As for point 3), the correlation comes out from the fact that in the ρ 's c.m. system the two pions fly 180° apart. (As we said at the beginning of this section, suitable $\pi\pi$ interactions could accomplish the same task as ρ^0 's in multiple production).

Further comments can be found in Gupta's paper (8). Quite recently (9), Gupta remarks that a scalar ρ^0 provides an adequate spin-orbit coupling between two nucleons; a second order calculation of this coupling (based on a coupling constant like pions and a mass somewhat more than two pions) gives practically the same potential as the one phenomenologically introduced by Marshak and Signell to account for experimental data.

Before going to examine Nambu's proposal (10) (which the next paragraph is devoted to) we must mention two neutral mesons, whose existence has been recently suggested for special purposes. They are Baldin's π_0^0 (11) and Schwinger's σ^0 (12).

π_0^0 : Baldin has made an attempt to mend the fault of coherence in matching the data from low energy π photoproduction - scattering and the Panoisky ratio by introducing a $T = 0$ meson, the isotopic singlet partner of the usual triplet $\pi^{\pm 0}$. This neutral meson should have a mass somewhat li-

ghter than π^- and should be the one emitted in π^- absorption by Hydrogen. There seems to be no direct experimental evidence about its existence⁽²¹⁾;

σ^0 : in order to provide a theoretical basis for the large μ meson mass, Schwinger hypothesizes the existence of a neutral scalar field; this σ - field is coupled with the lepton field through a moderately large coupling constant and, to avoid undesired non-electromagnetic properties of the μ meson, is supposed to have quite a large mass. Further calculations⁽¹²⁾ show, for instance, that the hypothesis is compatible with the essentially electromagnetic behaviour of μ mesons if the σ -field has medium strong interactions with mesons and baryons and a mass of some (~ 3) nuclear masses; besides⁽¹³⁾, there is the possibility to arrange at the same time for self mass and anomalous magnetic moment of the muon by means of a heavy scalar σ -field. This neutral meson, because of its mass, cannot obviously be observed directly with the Frascati machine.

3. - The vector ρ^0 meson.

We have deserved a whole paragraph to Nambu's⁽¹⁰⁾ meson because of a lot of vague reasons. As doctor Nambu points out⁽¹⁴⁾ it "should enjoy more individuality (like π^0 or Σ^0) because the decay can occur only with the help of the electromagnetic interaction". In fact, if this ρ^0 had a mass $M_0 < (m_\pi, 3m_\pi)$ (m_π = pion mass) it should decay according to

$$\rho^0 \longrightarrow \pi^0 + \gamma \quad (\text{or } 2\pi + \gamma)$$

A 3γ coincidence is indeed the event we hope to detect and to classify as a ρ^0 meson. We are strongly tempted to assume $M_0 < 3 m_\pi$ so dismissing the troublesome ghost of an unstable 3π bound state.

Such a particle fails to provide a basis for the explanation of mul-

multiple pion production anomalies (Gupta's argument ⁽⁸⁾); this is not a trouble, however, since one can appeal to suitable $\pi\pi$ or πN bound states ⁽¹⁴⁾ (quite independently of Nambu's ρ^0) to fit the data.

The vector ρ^0 meson is called for essentially the following purposes ⁽¹⁰⁾:

- 1) to allow for the interpretation of the form factors of the nucleons
- 2) more successful evaluation of the anomalous magnetic moment of the nucleons
- 3) interpretation of the second maximum of πN scattering (around 1 BeV) as resonant ρ^0 production according to



- 4) better agreement of the ratio of charged to neutral components in high energy reactions
- 5) field-theoretical basis for the phenomenological hard core in nuclear forces. Besides, a vector meson provides, in a natural way, a spin-orbit coupling between the nucleons.

This is Nambu's list of possibilities offered by his ρ^0 . One more specific request for a *neutral vector field* comes from a theoretical argument based on the law of conservation of baryons ⁽¹⁵⁾: assuming invariance of all interactions under baryon gauge transformations B (which multiplies one-baryon states by $e^{i\alpha}$ and one-antibaryon states by $e^{-i\alpha}$), in order to get invariance of the free baryons lagrangian one must introduce a neutral vector field ρ_μ^0 transforming like

$$\rho_\mu^0 \longrightarrow \rho_\mu^0 + \delta_\mu \alpha$$

thus matching the baryon's transformation. This neutral field must interact in the same way with all the baryons (N, Λ , Σ , Ξ). Unfortunately (from our point of view) such a neutral vector field should have zero mass, at least if it satisfies a Klein Gordon equation.

Let us examine more closely the situation with a vector meson. Rough perturbation calculations of relevant effects have been done by Huff (16) K particles contributions are not included in these calculations. ρ^0 's are assumed to have vector coupling with nucleons; moreover, a direct vector coupling with electrons is hypothesized.

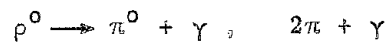
Consider first the situation with the decay modes; we will distinguish between $M_0 > 3m_\pi$ and $M_0 < 3m_\pi$.

a) $M_0 > 3m_\pi$. As we said, this 'unfortunate' case would lead to the decay



It would be very difficult to identify an elementary particle in such a prolific mother of pions unless its life is unexpectedly long.

b) $M_0 < 3m_\pi$. The main decay modes should be



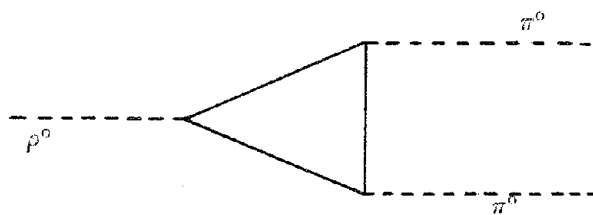
There are other modes of small branching ratio:



The reason why the branching ratio is small is obvious for all the mentioned processes but perhaps for the first: we want to add some comment on it. Consider first the decay



This process should not occur at all by Furry's theorem (17): the closed nucleon line (fig. 1)



has three corners, namely two neutral meson external lines and one ρ^0 - nucleons vector coupling vertex. So, according to one of the rules (ref.17) the process is strictly forbidden.

Next consider the decay

$$\rho^0 \longrightarrow \pi^+ + \pi^-$$

If the ρ^0 meson coincides with its charge conjugate, the process is forbidden at all orders by charge conjugation. (18) Nevertheless, charge independence assures forbiddenness at lowest order without any particular assumption on charge conjugation properties of the parent ρ^0 .

According to Nambu's paper (10) the lifetime of the main decay $\rho^0 \rightarrow \pi^0 + \gamma$ should be in the $10^{-19} - 10^{-20}$ sec range. It obviously depends on mass and coupling constant; moreover the process

$$\rho^0 \longrightarrow 2\pi + \gamma$$

should have a comparable rate, if energetically possible. We do not discuss the consequences of the ρ^0 hypothesis on nucleon properties because of the arbitrariness of the theoretical methods involved. To end this section we just say that the ρ^0 seems a convenient tool but it must be still invented how to handle this expediency: everyone will agree it is better to observe ρ^0 first.

4. - Kinematics of ρ^0 photoproduction and decay.

The processes we are interested in are



The feature distinguishing the kinematical calculations referring to P from the usual π case is the unknown meson mass. The mass M_0 will indeed be the main parameter of all the tables or graphs.

First of all, we give threshold energies ω_t of the γ rays for process P (M is the proton mass, $\hbar = c = 1$)

$$\omega_t = M_0 \left(1 + \frac{1}{2} \frac{M_0}{M} \right)$$

that is

M_0 (MeV)	ω_t (MeV)
200	220
300	350
400	485
500	630
600	790
700	960

Drawing the graphs, we assume we are able to measure the following quantities in the P process:

θ_p) proton angle θ_p with the γ ray direction

T_p) proton kinetic energy T_p (or proton momentum p)

θ_0) the angle θ_0 of the ρ^0 line of flight with the γ ray beam axis.

The knowledge of θ_p and T_p fixes the value of the γ ray energy (in the bremsstrahlung spectrum) as a function of M_o only (fig. 2, lab system)

$$\omega = \frac{1}{2} \frac{M_o^2 + 2T_p}{p \cos\theta_p - T_p}$$

(Graph n. 1 at the end of the paper, for the particular case $\theta_p = 30^\circ$)

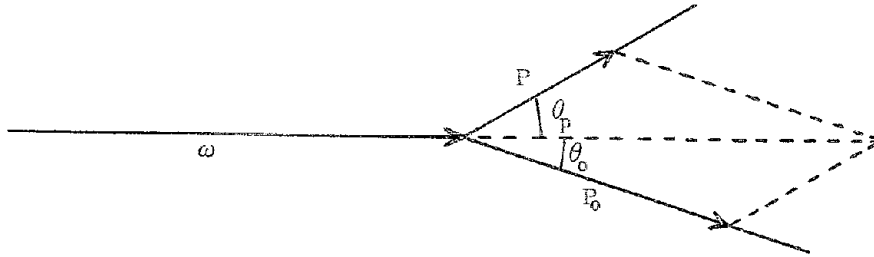


Fig. 2

At the same time θ_o is fixed as a function only of the parameter M_o . We made a lot of polar diagrams (graph n. 2 to 5): the radial coordinate measures energies or masses of the particles, while angles in the North-West quadrant refer to the ρ^o line of flight. In these graphs one finds simultaneous values of θ_o , M_o and T_o (ρ^o 's kinetic energy) for a fixed proton angle (30°) and four typical T_p values (small arrows in the North-East quadrant).

Now, if we suppose for a moment we are able to detect a ρ^o , the direction of its line of flight holds all the information we need; that is essentially the mass M_o .

Obviously, because of the short life and no electrical charge, detect ρ^o direction is a foolish task. So, we must apply to the decay products and dispose of them in order to squeeze from their kinematics a memory of the line of flight of the parent. At each one of these subsequent steps

the available phase space fraction narrows fearfully, and one must be careful to avoid unnecessary constraints.

Let us consider D process



This actually means observation of 3 γ rays, whose energies and angles are correlated in a very involved way. This correlation becomes very simple in the particular case the first occurring γ ray (γ_0) brings out all the momentum of the parent in the lab system whilst the π^0 is produced at rest. In this case:

$$p_0 = k = \gamma_0 \text{'s momentum}$$

$$(p_0^2 + M_0^2)^{1/2} = m_\pi + k$$

The γ rays from the π^0 fly 180° apart but their line of flight points completely at random over the whole 4π sphere; moreover, their energy is fixed and equals $\frac{1}{2} m_\pi$.

The main points of this simple case are:

a) γ_0 flies in the same direction as the ρ^0 , thus bringing the desired information about θ_0

b) the momentum of the ρ^0 is fixed by its mass

$$p_0 = \frac{M_0^2 - m_\pi^2}{2m_\pi}$$

Because of the last sentence b) we must reconsider the situation with the data concerning the P process: take again θ_p fixed. Now, from conservation of energy in the P process, T_p is no longer a free parameter since P_0 is not. Thus, the measurement of the proton energy is *not* necessary in

this case: the only care one must take is to do not operate at such a proton angle θ_p exceeding the maximum proton angle given by

$$\cos \theta_{\max} = \frac{M_o}{\omega} \left(1 + \frac{\omega}{M} - \frac{M_o^2}{4M^2} \right)^{1/2}$$

(this is an implicit relation since the primary x-ray energy ω depends on $\cos \theta_{\max}$). Eventually the condition $\theta_p < \theta_{\max}$ must be checked against the observed value of the mass M_o .

To summarize the situation, we write down now the equations linking measured to unknown kinematical parameters:

$$\omega^2 + p_o^2 - 2\omega p_o \cos \theta_o = p^2 \quad \text{cons. of mom. I)}$$

$$\omega^2 + p^2 - 2\omega p \cos \theta_p = p_o^2 \quad \text{cons. of mom. II)}$$

$$\omega = T_p + (p_o^2 + M_o^2)^{1/2} \quad \text{cons. of energy III)}$$

$$p_o = \frac{M_o^2 - m_\pi^2}{2m_\pi} \quad \text{condition for the special decay with a } \pi^0 \text{ at rest (lab syst.) IV)}$$

From II) and III) one eliminates ω and T_p (or p); as a consequence I) gives θ_o as a function of M_o only, if one studies the particular case in which P_o is given by IV).

We remark at this point, though perhaps unnecessary, that we will deal in the following only with the special case distinguished by condition IV).

To end this section we add a few words about the D process. A vector meson could be polarized in the production process by some interaction we do not know; this in turn could influence the angular distribution of the decay products. We forget this possibility in the following by assuming

that the distribution of the decay products in their center of mass system (ρ^0 decaying at rest) is isotropic (that is we average over the spin of the parent). The π^0 momentum in the c.m. system is thus

$$p^* = \frac{M_o^2 - m_\pi^2}{2M_o}$$

One must choose the ρ^0 's momentum in the lab system according to IV) in order to compensate p^* , so to speak, for a π^0 flying in the backward direction (in the c.m.).

Let us call θ_Y the angle γ_o (one of the direct D products) makes in the lab system with the ρ^0 's line of flight. The assumption of isotropic c.m. distribution now gives for the lab. angular distribution

$$P(\theta_Y) = \frac{1}{4\pi} \frac{1 - \beta_o^2}{(1 - \beta_o \cos \theta_Y)^2}$$

where β_o is the velocity of the parent ρ^0 in the lab system. That is

$$\beta_o = \frac{p_o}{(M_o^2 + p_o^2)^{1/2}} = \frac{M_o^2 - m_\pi^2}{[4m_\pi^4 + (M_o^2 - m_\pi^2)^2]^{1/2}}$$

having inserted for p_o the value given by condition IV). Since we are interested in forward produced γ_o 's, the relevant quantity is now

$$P(\theta_Y = 0) = \frac{1}{4\pi} \frac{1 + \beta_o}{1 - \beta_o}$$

The smallest $1 - \beta_o$, the highest the $P(0)$ value: for instance, in the case $M_o = 2 m_\pi$, $\beta_o = 0.83$ and $P(0) = 10.8/4\pi$.

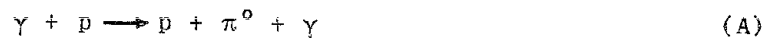
The angular width of the γ_o counter will eventually fix the probability of the special decay; needless to say that $P(\theta_Y)$ is quite insen-

sitive to θ_γ values around $\theta_\gamma = 0$ until

$$\theta_\gamma^2 < 1 - \beta^2$$

5. - Competitive background.

There is a process whose outcomes are exactly the same as in ρ^0 's detection, namely



A process contributing as unavoidable background to the counting rate is



Finally we mention the process (see par. 3)



which, though involving a ρ^0 mesons, does not contribute to its detectability. This process could even be more frequent than the main decay process we are interested in



C) and D) do not exclude one the other, so we relegate C) in the background. What one must examine is: 1) the possibility of discrimination of process D) against A); 2) the possibility of discrimination of D) against B).

1 - Process A) - We mainly refer to the case in which the final state contains a hard γ -ray. This event should be quite unfrequent as compared

with non-radiative single π^0 production (19). On the other side we do think that a ρ^0 experiment based on the abovesaid line can be successful if the production rate of the new meson is not too low as compared with single pions production.

Let us consider the kinematics of process A) in the lab system (for the case in which the π^0 is at rest). In correspondence of the first three equations I) II) III) par. 4 we have

$$\omega^2 + k^2 - 2\omega k \cos \theta_0 = p^2 \quad (\text{I}_A)$$

$$\omega^2 + p^2 - 2\omega p \cos \theta_p = k^2 \quad (\text{II}_A)$$

$$\omega = T_p + m_\pi + k \quad (\text{III}_A)$$

(k is the secondary photon energy).

Condition IV) is missing and this makes a discrimination possible. In fact we have γ rays at every value of the angle θ_0 since in this case ω , T_p , k are *unknown* quantities (whereas in the case of ρ^0 's production, p_0 , playing the same role as k, was fixed by the condition ' π^0 at rest', IV⁰). Thus, unless the angular distribution of the γ rays coming from A) is anomalously peaked in some direction, its contribution should be a flat background. Whether this background is really harmless or not could be further checked by suitable T_p or k measurements (a rough biasing window in a γ_0 counter, for instance). This seems unnecessary however, at least in the first trials.

2 - Process B) - We believe this will be the main background.

Discrimination against this background can be performed as for process A) because there is one more γ ray destroying nearly at all the memory of the flying π^0 direction (once again, one of the π^0 's is at rest in

lab system). Moreover the condition IV is missing in this case too and we do not measure the proton energy. Also in this case the γ rays angular distribution as a function of θ_0 should be flat thus allowing for discrimination of a possible ρ^0 's peak.

A trouble could arise with a γ_0 counter so large as to accept both the γ rays coming from the pion: but at the energies we deal with, this trouble is certainly not sensible.

In spite of the different angular behaviour, process B) could give such a rich background as to render a peak indistinguishable from fluctuations. We have no means to predict what will happen, unless we are content with phase space comparisons. We want to report such a calculation since it seems a favourable circumstance. The ratio of available phase space for the two cases

$$\xi = \frac{\gamma + p \rightarrow p + \rho^0}{\gamma + p \rightarrow p + 2\pi}$$

is approximately given by⁽²⁰⁾

$$\xi = 60 \pi^2 \frac{m_\pi^3 (W - M - M_0)^2}{(W - M - 2m_\pi)^5} \quad (M_0 > 2m_\pi)$$

where M is the nucleon mass and

$$W^2 = M^2 + 2\omega M$$

ω being the energy of the primary quantum in the lab system (as before).

ξ has a maximum as function of ω in correspondence of the value:

$$\left\{ \begin{array}{l} \omega_{\max} = \frac{W_{\max}^2 - M^2}{2M} \\ W_{\max} = M + \frac{5}{3} M_0 - \frac{4}{3} m_\pi \end{array} \right.$$

The maximum is quite high over the interesting range:

M_0	ω_{\max}	ξ_{\max}
330 MeV	450 MeV	200
400	600	24
460	780	6.7
530	970	2.8

A formula for ξ_{\max} as a function of M_0 is

$$\xi_{\max} \approx 20 \frac{m_{\pi}^3}{(M - 2m_{\pi})^3}$$

We agree one must not give much credit to this result; nevertheless one can accept it as an agreeable aperitif.

6. - Sketch of the experimental arrangement.

In this section we want just describe a more realistic experimental situation. Namely, we want to examine the influence of energy and geometry spreads, introduced by the counters, around the central values used in the preceding sections, in order to decide whether it is possible to resolve a ρ^0 peak in the θ^0 -distribution without lowering too much the counting rate, or not.

To do this, we refer to fig. 3. The only approximation we make is to have a point target t

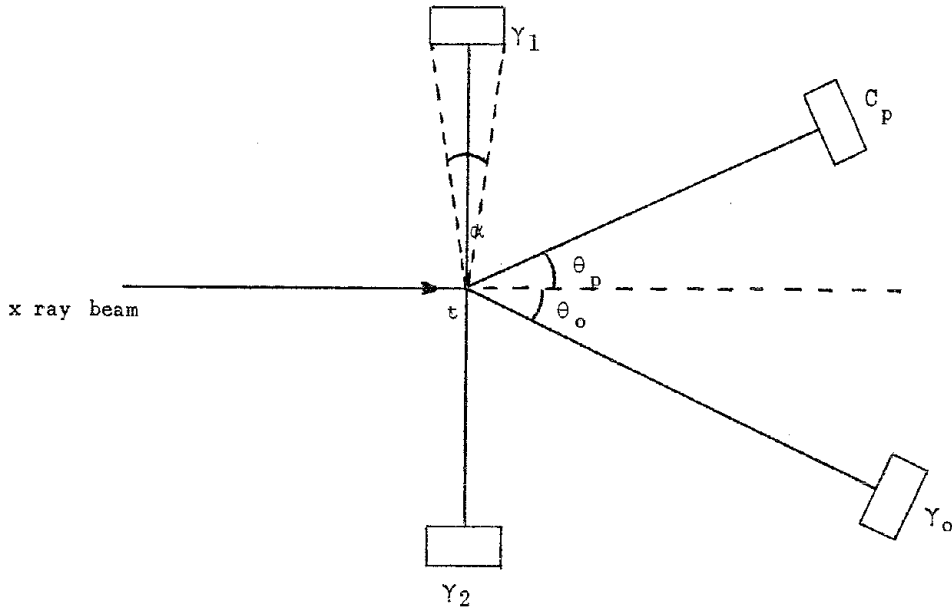


Fig. 3

γ_1 , γ_2 are two counters in coincidence, giving a pulse everytime they are traversed by γ rays whose energy is $m_\pi/2 \pm \Delta k$. Moreover, the linear dimensions of γ_1 (γ_2) as seen from the target correspond to an angular aperture α . The proton counter C_p centered at θ_p collects protons from an angular interval $\pm \Delta\theta_p$ wide; the counter γ_0 collects γ rays from an angular interval $\pm \Delta\theta_o$ around θ_o .

Obviously, we can arrange any number of γ_1 , γ_2 pairs provided that we select coincidences from proper pairs (the line joining each pair must pass through the target and every pair must converge to an independent coincidence unit).

Consider first the definition of the ' π^0 at rest': the π^0 can have a non-zero momentum along the line $\gamma_1 t \gamma_2$ because of the uncertainty Δk . It can have a non-zero momentum orthogonal to the line $\gamma_1 t \gamma_2$ because of the non-point geometry of the counters. By properly adding the two uncertainties one has

$$\Delta p_{\pi} = (4 \Delta k^2 + \alpha^2 m_{\pi}^2)^{1/2} \quad (1.6)$$

as a measure of the momentum uncertainty for the ' π^0 at rest'. The angle θ_{γ} (see back, end of par. 4) makes a deviation from the true θ_0 (referring to the line of flight of the ρ^0) by an amount

$$\Delta\theta_{\gamma} = \frac{\Delta p_{\pi}}{P_0}$$

where P_0 is given by IV par. 4. As a consequence $\Delta\theta_0$ can be chosen to be *at least* as large as $\Delta\theta_{\gamma}$ (see below).

The angular spread $\Delta\theta_p$ in the geometry of the proton detector determines the width of the energy interval of the primary bremsstrahlung spectrum of the synchrotron that contributes to the production in this arrangement.

In fact, from II and III par. 4, one can deduce (see Appendix A)

$$\Delta\omega = \left[\left(\frac{\delta\omega}{\delta\theta_p} \right)^2 \Delta\theta_p^2 + \left(\frac{\delta\omega}{\delta P_0} \right)^2 \Delta P_0^2 \right]^{1/2} \quad (3.6)$$

where $\Delta P_0 = \Delta p_{\pi} = P_0 \Delta\theta_{\gamma}$. Actually, $\Delta\theta_p$ is strictly related to $\Delta\theta_0$ (not to be confused with $\Delta\theta_{\gamma}$): in fact, looking back to the equations I to IV, par. 4 we see that having fixed p_0 , determines the value of the other variables, in particular θ_0 . Thus we can say that to every value of θ_p corresponds a center θ_0 of the peak in the angular distribution of ρ^0 's. The width of the peak is given by (see Appendix A):

$$\Delta\theta_0 = \left(\Delta\theta_{\gamma}^2 + \left(\frac{\delta\theta_0}{\delta\theta_p} \right)^2 \Delta\theta_p^2 \right)^{1/2} \quad (4.6)$$

where $\Delta\theta_{\gamma}$ and $\Delta\theta_p$ are independent quantities: $\Delta\theta_{\gamma}$ comes from that part of

the device operating on the decay kinematics (D process), whilst $\Delta\theta_p$ comes from that part operating on the production kinematics (P process).

Eventually, a few words on the coplanarity of the primary X ray beam, γ_0 and the proton. For a perfectly collimated X ray beam the only deviation from coplanarity comes from the definition of the π^0 at rest; thus an angle $\Delta\theta_\gamma$ is allowed for the γ_0 direction out of the proton - X ray plane. This essentially fixes the lowest angular dimension of the C_p and γ_0 counters along the azimuthal coordinate measuring rotations around the X ray beam axis. These azimuthal dimensions could be quite large; but one must defer the decision about them to the consideration of the actual background counting rate (process B involves essentially non-coplanar γ rays). It is obvious that a coplanarity test can be used to check a successful result. This test, together with the fact that, changing θ_p , θ_0 (center of the peak) must displace according to a perfectly known law (I to IV par. 4), provides the best control of the data, being based on purely kinematical grounds.

7. - Considerations about the counting rate.

Let us call $\frac{\delta^2\sigma}{\delta\Omega\delta T_0}$ (θ_0 , T_0) the cross section per unit solid angle and energy interval for production of a ρ^0 by a photon (on a proton) at an angle θ_0 and kinetic energy T_0 . N is the number of hydrogen atoms per unit volume in the target; t is the thickness of the target.

The X - ray spectrum will be approximately given by

$$N(\omega) d\omega = Q \frac{d\omega}{\omega} \quad (1.7)$$

where Q is the number of equivalent quanta per minute from the synchrotron ($Q\omega_{\max} = \int_0^{\omega_{\max}} \omega N(\omega) d\omega$ in general).

The ρ^0 direction points into a solid angle $\Delta\Omega_0$ as given by 4.6 combined with the considerations on coplanarity: that is

$$\Delta\Omega_0 \cong \Delta\theta_Y \Delta\theta_0$$

where the \cong sign refers to the possibility of increasing the factor $\Delta\theta_Y$ (end of par. 6).

Thus the number of ρ^{0v} s per photon in this range will be

$$Nt \frac{d^2\sigma}{d\Omega dT_0} (\Delta\Omega_0)_{cm}$$

per unit energy. Now, we select a narrow energy interval by condition IV par. 4 and 1.6, so that the number of ρ^{0v} s per photon will be

$$Nt \frac{d^2\sigma}{d\Omega dT_0} (\Delta\Omega_0)_{cm} \frac{dT_0}{dP_0} \Delta P_\pi$$

To have the counting rate we must account for:

1) The number of photons of right energy. This is simply

$$Q \frac{\Delta\omega}{\omega} \quad \text{per minute}$$

according to (3.6) and (1.7).

2) the geometrical efficiency of the counter pair $\gamma_1\gamma_2$. If there are v such pairs, the efficiency is

$$v \frac{\alpha^2}{2\pi} \quad (\text{fig. 3})$$

3) The fact that only a few ρ^{0v} s of the right momentum p_0 give a π^0 at rest; this is expressed by a probability (see and of par. 4)

$$P(0) \Delta\theta_Y^2$$

Combining all these factors, one obtains the counting rate per minute

$$n_{\rho^0} = N t \frac{d^2\sigma}{d\Omega dT_0} \left(\frac{\Delta\Omega}{\omega} \right) \frac{dT_0}{dP_0} \Delta P_{\pi} Q \frac{\Delta\omega}{\omega} v \frac{\alpha^2}{2\pi} P(0) \Delta\theta_{\gamma}^2 \quad (2.7)$$

This number must be further lowered by a factor measuring the detection efficiency of the various counters.

Formula (2.7) can be used as a starting point to decide the minimum $d^2\sigma/d\Omega dT_0$ one can measure by this method.

One can add the following practical remarks:

- a) from (1.6) it can be seen that α can be quite large if M_0 (and p_0 in consequence) is large enough. In any case, one can make $\alpha \simeq \Delta k/m_{\pi}$, that is perhaps $\alpha \simeq 0.15$
- b) for the same reason t can be as large as the $\gamma_1(\gamma_2)$ counter linear dimensions.
- c) $\Delta\theta_p$ must not be too large, so that the width of the peak is mainly committed to $\Delta\theta_{\gamma}$
- d) the other factors in (2.7) are all practically frozen by the necessity of taming the three body kinematics.

For comparison, we shall derive a formula equivalent to (2.7) for the case of double π^0 production (that is for the background counting rate $n_{2\pi^0}$). Once again one of the π^0 's is at rest in the lab system.

Let us denote by index 1 the π^0 at rest, index 2 the π^0 flying with its momentum at an angle θ_2 with respect to the primary X ray beam. This $(\pi^0)_2$ lies in the same plane as the proton and the primary photon since

the process is essentially equivalent to a two-body one. If we call

$$\frac{d^3\sigma_{\pi^0\pi^0}}{d\Omega_2 dT_1 dT_2}$$

the cross section for production of a pair of π^0 mesons (having integrated over the possible directions of $(\pi^0)_1$), putting $P_0(\theta_\gamma) \Delta\Omega_\gamma$ for the probability $(\pi^0)_2$ produces a γ ray at an angle θ_γ with its line of flight into the solid angle $\Delta\Omega_\gamma$, we obtain

$$Nt \int_0^{\Delta p_\pi} dp_1 \frac{dT_1}{dp_1} \int_{\Omega_2} \frac{d\Omega_\gamma}{d\Omega_2} P_0(\theta_0 - \theta_2) \frac{d^3\sigma_{\pi^0\pi^0}}{d\Omega_2 dT_2 dT_1}$$

for the number of γ rays arriving at the γ_0 counter per photon and per unit energy. Actually one must integrate over the X-ray spectrum from the machine; finally inserting the factor $v \frac{\alpha^2}{2\pi}$ for the efficiency of the $\gamma_1\gamma_2$ pair of counters, one has

$$n_{2\pi^0} = Nt Q v \frac{\alpha^2}{2\pi} \Delta\theta_p \int_0^{\Delta p_\pi} dp_1 \frac{dT_1}{dp_1} \int_0^{T_2^m} dT_2 \int_{\Omega_2} d\Omega_2 \times$$

$$\times \frac{d^3\sigma_{\pi^0\pi^0}}{d\Omega_2 dT_1 dT_2} \cdot \frac{d\Omega_\gamma}{d\Omega_2} P_0(\theta_0 - \theta_2) \frac{dT_2}{d\theta_p} \cdot \frac{d\omega}{dT_2} \cdot \frac{1}{\omega} \quad (3.7)$$

The integration over Ω_2 is practically an integration over θ_2 ; that is, one can put $d\Omega_2 \cong \Delta\theta_\gamma d\theta_2$. Besides, we shall put

$$\frac{d^2\sigma_{\pi^0\pi^0}}{d\Omega_2 dT_2} = \int_0^{\Delta p_\pi} dp_1 \frac{dT_1}{dp_1} \frac{d^3\sigma_{\pi^0\pi^0}}{d\Omega_2 dT_2 dT_1} \quad (5.7)$$

The ratio $n_{\rho^0}/n_{2\pi^0}$ is (from 2.7, 3.7):

$$\frac{n_{\rho^0}}{n_{2\pi^0}} = \frac{\beta_0 \Delta\theta_0 \Delta p_\pi \frac{d^2\sigma}{d\Omega_0 dT_0} \frac{\Delta\omega}{\omega} P(0) \Delta\theta_\gamma^2}{\Delta\theta_p \int_0^{T_2^m} dT_2 \int_0^{\theta_2^m} d\theta_2 P_0(\theta_0 = \theta_2) \frac{d^2\sigma_{00}}{d\Omega_2 dT_2} \frac{1}{\omega} \frac{d\omega}{d\theta_p} \frac{d\Omega_\gamma}{d\Omega_2}}$$

T_2^m is the maximum kinetic energy of $(\pi^0)_2$ (mainly depending on the ω_{\max} of the X ray spectrum and θ_p); θ_2^m is the maximum angle of $(\pi^0)_2$; it is obviously a function of T_2 .

The averaging (5.7) represents a substantial decrease of the background events. Nothing more precise can be said because the cross section for $2\pi^0$ production is not known: one can make more or less arbitrary guess on the denominator of the ratio (6.7).

What one obviously learns from (6.7) is that a further reduction of the background can be achieved by requiring the energy of the proton to be in a narrow interval. In fact, if this interval is properly chosen it should not influence the characteristic angular distribution of γ rays coming from ρ^0 mesons while restricting the integration range over T_2 in (3.7). The proper proton energy must however be found by repeated trials if no definite enough M_0 value comes out of a first experiment performed without this expedient.

To end this survey, one can tentatively choose some of the parameters appearing in (2.7). Let us assume

$$t = 6 \text{ cm}$$

$$\Delta\theta_\gamma = 0.2 \text{ rad}; (\Delta\Omega)_{\text{cm}} \simeq 3(\Delta\Omega)_{\text{lab}}$$

$$\Delta\theta_p \ll \Delta\theta_\gamma \quad (\text{see appendix A})$$

$$\Delta k = 20 \text{ MeV}$$

$$Q = 5 \times 10^{11} \text{ equiv. quanta/minute}$$

$$\alpha = 0.2 \text{ rad}$$

$$\frac{\Delta\omega}{\omega} = 0.2 \quad (\text{a reasonable order of magnitude for } M_0 \simeq 2m_\pi)$$

Then it follows

$$n_{\rho^0} \simeq 5 \nu \left(\frac{d^2\sigma}{d\Omega_0 dT_0} \right) \quad \begin{array}{l} \text{counts/minute} \\ \mu\text{barn/sterad MeV} \end{array}$$

If $\Delta\theta_Y$ does practically coincide with $\Delta\theta_0$ as here assumed ($\Delta\theta_p \ll \Delta\theta_Y$), the peak width should be $\approx 10^\circ$ and the determination of the mass M_0 of the ρ^0 should be in error of

$$\frac{\Delta M_0}{M_0} = \frac{1}{M_0} \frac{dM_0}{d\theta_0} \Delta\theta_0 \simeq \frac{(M_0^2 - m_\pi^2)^2}{4\beta_0 M_0^2 (M_0^2 + m_\pi^2)} \Delta\theta_0$$

(having assumed θ_p small enough; see appendix B). That is for $M_0 \simeq 2m_\pi$, $\Delta\theta_0 = 0.2$

$$\frac{\Delta M_0}{M_0} \simeq \text{a few percent}$$

Appendix A.

We give here formulae for the quantities

$$\frac{\delta\omega}{\delta\theta_p}, \quad \frac{\delta\omega}{\delta p_0}, \quad \frac{\delta\theta_0}{\delta\theta_p}$$

appearing in (3.6) and (4.6).

Instead of using I to III par. 4 it is simpler to start from the following equivalent formulae:

$$p \sin \theta_p = p_o \sin \theta_o$$

$$p \cos \theta_p + p_o \cos \theta_o = \omega$$

$$\omega = T_p + E_o \quad (E_o^2 = M_o^2 + p_o^2)$$

Calling β_p the proton velocity, one has

$$\frac{\delta \omega}{\delta \theta_p} = \frac{\beta_p p (\sin \theta_p + \cos \theta_p \operatorname{tg} \theta_o)}{\cos \theta_p - \beta_p - \sin \theta_p \operatorname{tg} \theta_o} \quad (1.A)$$

$$\frac{\delta \omega}{\delta p_o} = - \frac{\beta_p}{\cos \theta_o (\cos \theta_p - \beta_p - \sin \theta_p \operatorname{tg} \theta_o)} \quad (2.A)$$

$$\frac{\delta \theta_o}{\delta \theta_p} = \frac{p (1 - \beta_p \cos \theta_p)}{\cos \theta_o (\cos \theta_p - \beta_p - \sin \theta_p \operatorname{tg} \theta_o)}$$

The common feature of these formulae is the denominator which determines the order of magnitude of the three derivatives. Note that the first two contains a factor β_p which can make them quite small. Actually we want (1.A) and (2.A) to be large, while (3.A) must be as small as possible, since 1.A and 2.A determine the number of photons intervening in the detected reaction while 3.A determines the width of the peak in the θ_o distribution.

Now, vanishing of the denominator in (1, 2, 3.A) is just the condition for the angle θ_p to be a maximum for a given p_o . In fact

$$\frac{d\theta_p}{dp} = \frac{1}{p} \frac{\cos \theta_p - \beta_p - \sin \theta_p \operatorname{tg} \theta_o}{\sin \theta_p + \cos \theta_p \operatorname{tg} \theta_o}$$

Clearly the prescription is to stay far from the maximum proton angle as we already remarked in par. 4. By putting

$$\frac{1}{H} = \cos\theta_o (\cos\theta_p - \beta_p - \sin\theta_p \operatorname{tg} \theta_o)$$

we can further write

$$\frac{\delta\omega}{\delta\theta_p} = \omega\beta_p \sin\theta_o H \quad (5.A)$$

$$\frac{\delta\omega}{\delta p_o} = -\beta_p H \quad (6.A)$$

$$\frac{\delta\theta_o}{\delta\theta_p} = \frac{P}{p_o} (1 - \beta_p \cos\theta_p) H \quad (7.A)$$

and from these formulae one only learns that the best thing to do is to work at a proton angle *as small and as sharply defined as possible* since (6.A) assures a non-vanishing $\Delta\omega$ while (7.A) shows that the peak width can be reasonably narrow.

Appendix B.

Defining H as in appendix A, one has

$$\frac{1}{H} \frac{d\theta_o}{dM_o} = 2\beta_o \frac{M_o (M_o^2 + m_\pi^2)}{(M_o^2 - m_\pi^2)^2} - 2 \frac{M_o \sin\theta_p}{p_o (M_o^2 - m_\pi^2)} (\omega - p\beta_p)$$

In the case $\sin\theta_p \rightarrow 0$ we obtain the result at the end of par. 7. by just

neglecting the second term on the right of this formula. The approximation is valid whenever

$$\sin\theta_p \ll \frac{p_o}{\omega - p\beta_p} \beta_o \frac{M_o^2 + m_\pi^2}{M_o^2 - m_\pi^2}$$

or

$$\sin\theta_p \ll \beta_o^2 \frac{M_o^2 + m_\pi^2}{M_o^2 - m_\pi^2}$$

noting that

$$\frac{p_o}{\omega - p\beta_p} > \frac{p_o}{\omega} \approx \frac{p_o}{E_o} = \beta_o$$

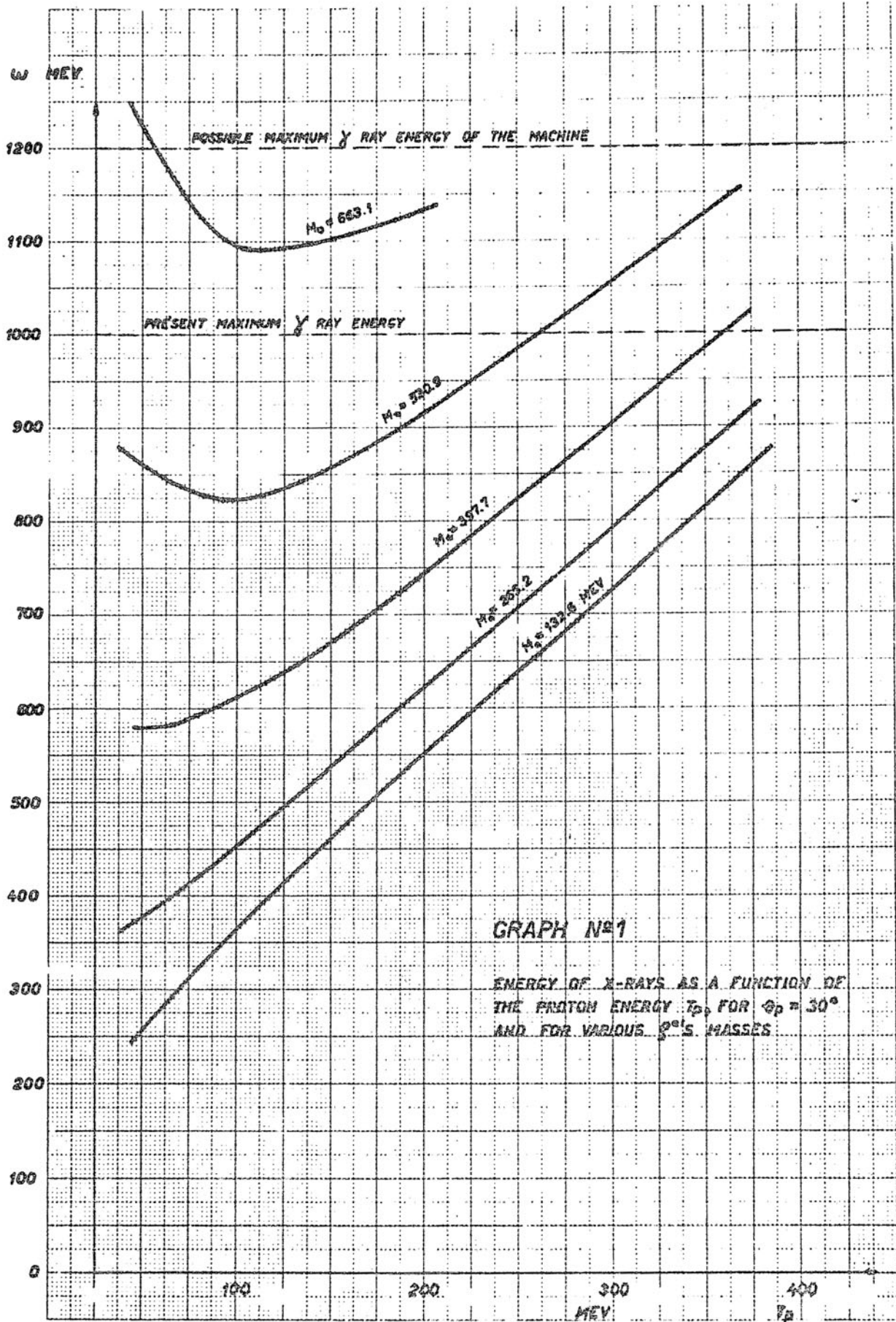
Besides, in the same approximation $H \approx 2$.

3. - Acknowledgements

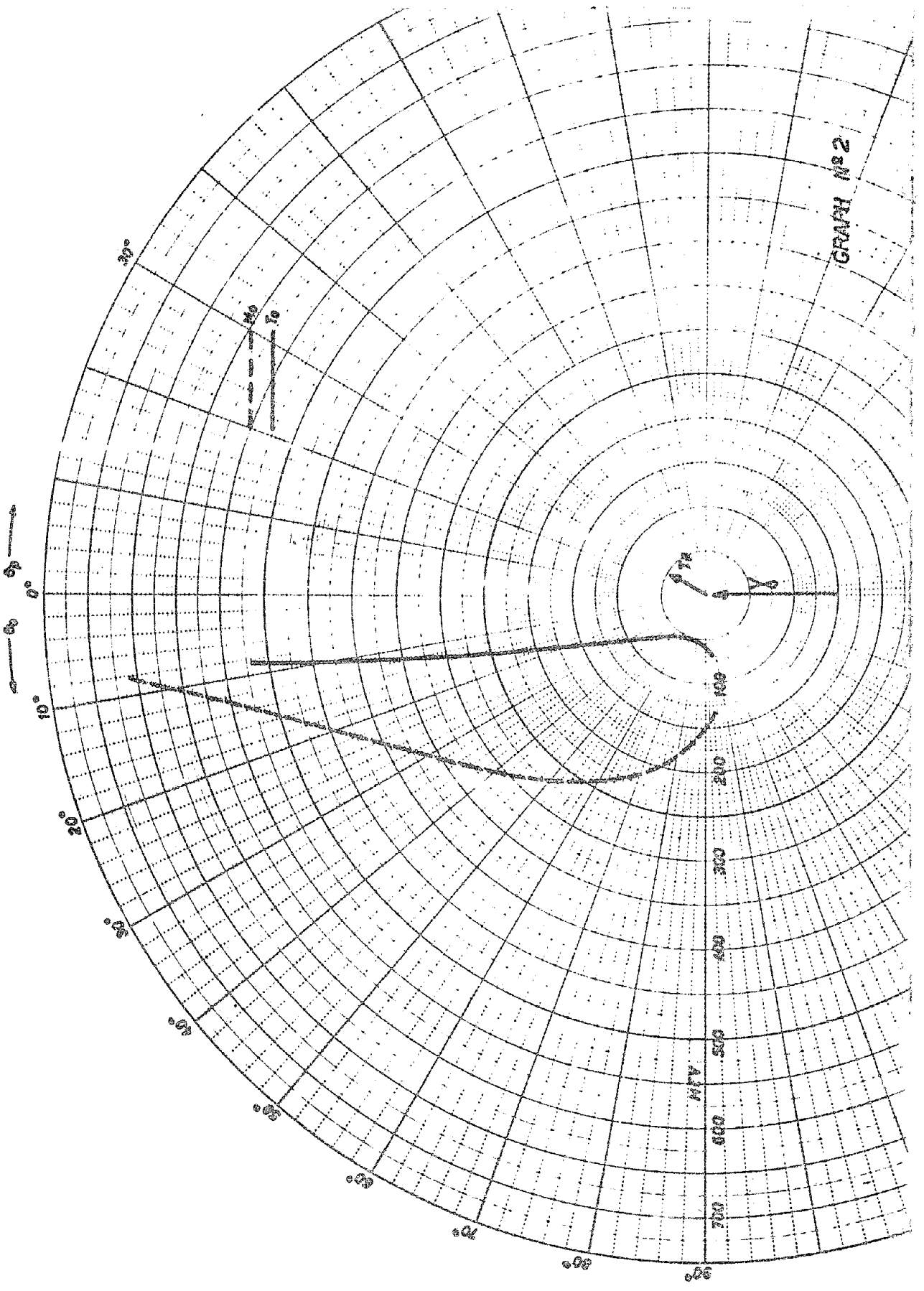
We want to thank professor Y. Nambu and professor S.N. Gupta for having kindly send to us their opinion. We heard some very useful comments and informations from prof. M. Cini, prof. R. Gatto, prof. G. Morpurgo, prof. G. Salvini and prof. E. Segre', to whom we are very grateful.

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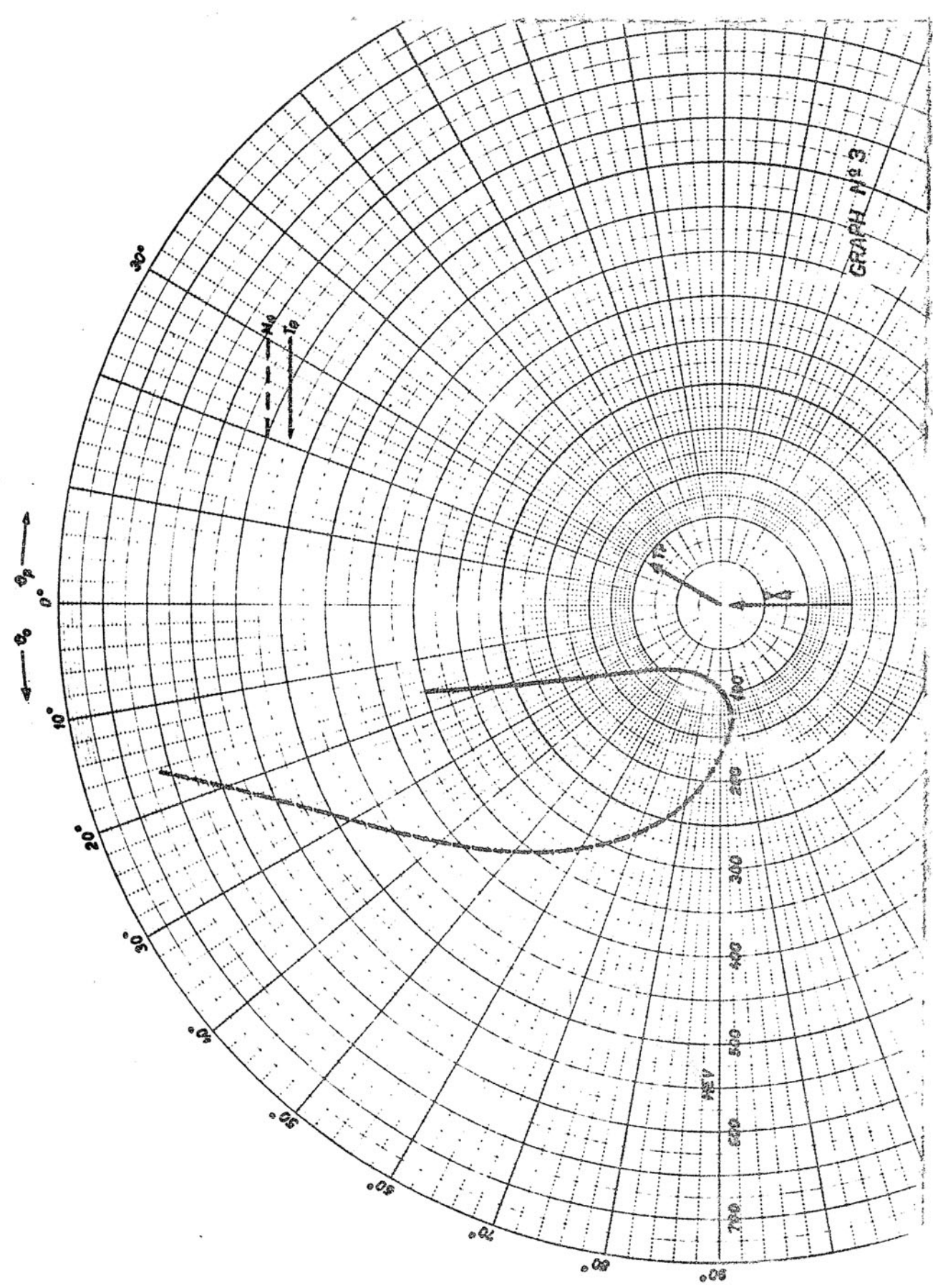
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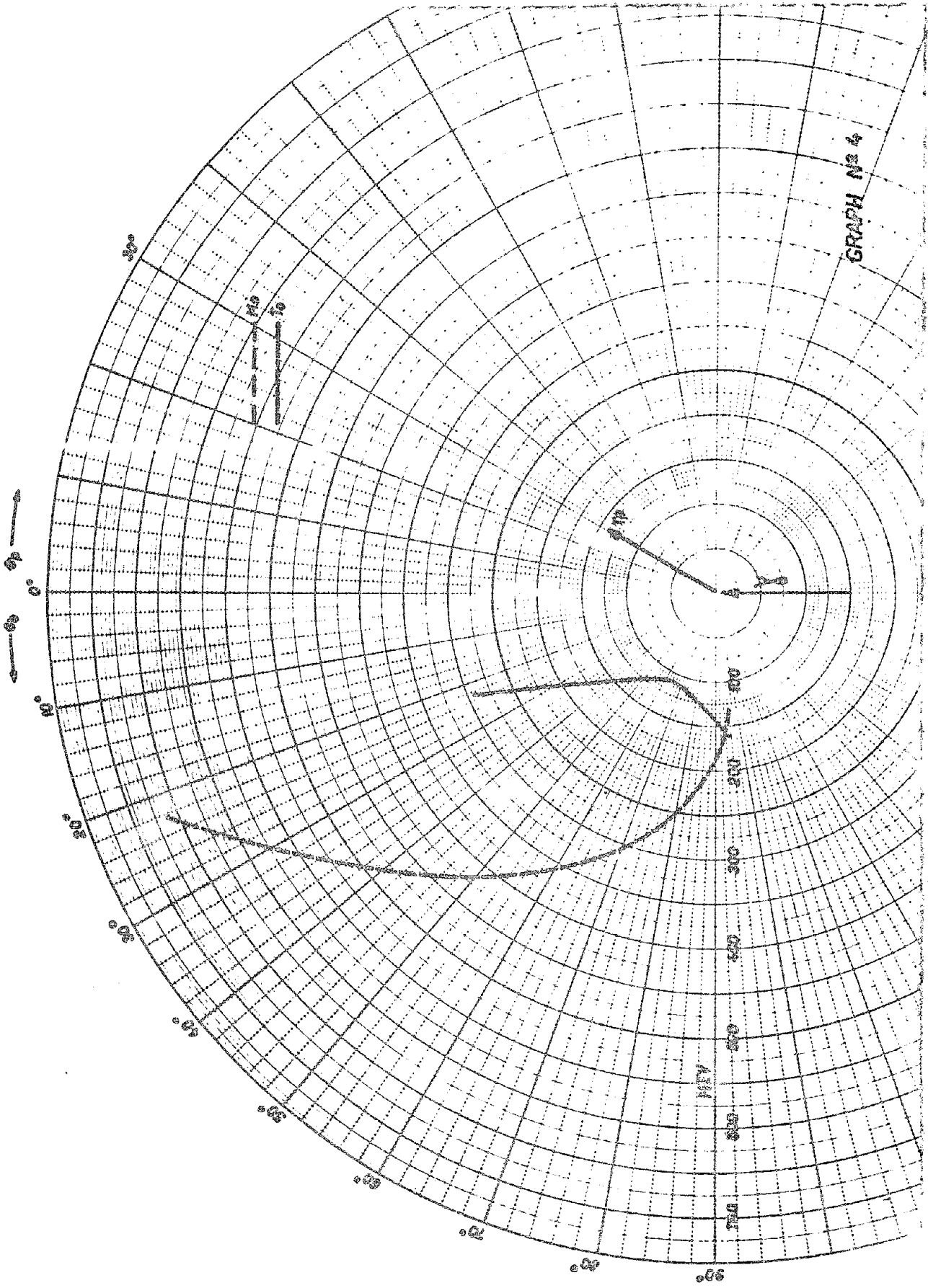
GRAPH N° 2



GRAPH No 3



GRAPH No 4



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