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Space-Charge Effects in Electron-Synchrotrons.

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Summary. — Space charge effects are considered (taking into account positive ions) with the aim of deciding whether they can set up serious intensity limitations for the beams of weak focusing electron synchrotrons, or not. The possibility of linear resonances and non linear beam splitting is shown. An estimate of the rate of production of positive ions in the doughnut is given.

1. – Introduction.

Space charge effects in electron-synchrotron are usually settled on by the statement that, if the injection energy is high enough, magnetic forces compensate nearly at all electrostatic forces (the well known $1-\beta^2$ factor [1]). This statement is, of course, valid in view of the till now obtained intensities for such machines, *i.e.* having in mind a figure of 10^{10} for the order of magnitude of the number of circulating electrons per pulse.

It is still assumed, even if not explicitly said, that the electron beam moves in vacuum but for the problem of scattering losses. Consideration of scattering losses gives an intensity independent rule for a troubleless pressure of the residual gas; now, we want to show that this rule does not eliminate the possibility of ion production, in the beam-occupied region, in such a quantity as to give rise to troubles with the focusing properties of the external magnetic field plus space charge forces.

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(1) See for instance: W. Harman, *Fundamentals of Electronic Motion* (New York, 1953), pag. 291.

A complete formulation of the problem of positive ions production and of their interaction with the electron beam is complicated by the number of concomitant circumstances; nevertheless, orders of magnitude and semiqualitative results can be obtained by the use of simple models.

Two main results will be shown in the following paragraphs:

a) The possibility of reaching dangerous resonant values of the betatron frequencies.

b) The possibility of beam-splitting into two or more parallel beams.

It is hard to maintain that the strange facts observed in some electron-synchrotron laboratories (2) find an explanation in the results of this paper. However, we think that such results can give a guide for an easy experimental program with the aim of deciding if positive ion effects are to be accounted for (or rejected) in planning high intensity machines.

Eventually, we anticipate that typical appearance values of the effects in weak focusing machines (which we are interested in) are $5\cdot10^{-6}$ mm Hg for the pressure, $5\cdot10^{10}$ for the number of electrons per pulse. The main perturbation parameters depend on the product pressure $\times$ intensity.

2. – Resonance effects.

In this paragraph we want to study the effect of an uniformly distributed sea of positive ions, charge $+\zeta e$, on the betatron oscillation of the electrons in the beam. We think of the ion density as constant.

Let us call N_e the total number of circulating electrons, N_+ the number of ions in the beam occupied region and put

$$\varepsilon = \frac{\zeta N_+}{N_e}\,.$$

Suppose for semplicity the beam is made up of four bunches, uniformly filled, with electrons and cylindrical in shape, the length, along the axis (parallel to the direction of the bunch), being $R\Delta\Phi$, the circular section having a radius r_0. The total radial force on an electron of the beam, at a distance r from the beam axis will be

$$F_r = \frac{1}{2}\,N_e\,\frac{e^2}{R\Delta\Phi r_0^2}\,(1-\beta^2-\varepsilon)r\,.$$

(2) *CERN symposium*: **1**, 67, 301 (1956).

The $1-\beta^2$ dependent part is due to electron-electron forces, the ε part is due to electron-ion forces.

We neglect the $1-\beta^2$ part, thus assuming

$$1-\beta^2 \ll \varepsilon ;$$

the equations of betatron oscillations take the form

$$x'' + (1-n+\delta n)x = 0 \qquad \text{(radial motion)}$$

$$z'' + (n+\delta n)z = 0 \qquad \text{(vertical motion)},$$

where

$$\delta n = \frac{1}{2}\,\varepsilon N_e\,\frac{e^2 R}{mc^2\Delta\Phi r_0^2},$$

and the other symbols need no explanation.

The dependence on electron energy of this space charge parameter δn is not easy to guarantee: one could say that the quantity $m\Delta\Phi r_0^2$ behaves roughly as $B^{-\frac{1}{2}}$ (B being the main magnetic field in the machine) due to adiabatic dampings. But this energy dependence could be wrong because of a number of influencing circumstances (like scattering, radiation fluctuations or space charge effects themselves). We take a constant δn value, corresponding to injection values of the parameters; « injection values » means, of course, values reached soon after RF capture, when the electron beam density is nearly steady.

For typical values as

$$R = 360\ \text{cm}, \qquad (mc^2)_{\text{injec.}} = 2.5\ \text{MeV},$$

$$r_0 = 1\ \text{cm}, \qquad \Delta\Phi = 1\ \text{rad},$$

one obtains

$$\delta n = 0.1\,\varepsilon\left(\frac{N_e}{10^{10}}\right).$$

We can write down a resonance relation for the betatron frequencies, for a racetrack synchrotron with four straight sections of length L, in the form:

$$p + \left(1+\frac{L}{\pi R}\right)\left(q\sqrt{1-n+\delta n} + r\sqrt{n+\delta n}\right) = 0,$$

where p, q, r are integers ($\gtrless 0$) ([3]).

([3]) E. Persico: report T1 della Sez. Acc. dell'INFN (1953).

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In the case of $L/\pi R = 0.106$, $n = 0.61$ (as for the Frascati machine), there are three *linear* resonances near the unperturbed working point, namely

$$p = 1 \qquad q = 0 \qquad r = -1 \qquad \delta n \simeq 0.2$$
$$p = -2 \qquad q = 1 \qquad r = 1 \qquad \delta n \simeq 0.3$$
$$p = 1 \qquad q = -1 \qquad r = 0 \qquad \delta n \simeq 0.4$$

The first resonance ($\delta n = 0.2$) reveals itself by the build up of vertical oscillations due to first harmonic median plane irregularities. The second one ($\delta n = 0.3$) is a coupling resonance excited by field or median plane second harmonic irregularities. The last resonance ($\delta n = 0.4$) is a field first harmonic excited radial motion (*).

Non-linear resonances are not considered here because of the surely longer amplitude build-up time, which could allow for ion density rearrangements destroying the resonant value of δn.

3. – The ion density.

We want to estimate the quantity δn as a function of pressure, beam intensity, atomic number Z of the residual gas, etc. We neglect the secondary electron production since, as can be easily shown, secondary electron velocities are $\gg$ than ion velocities, so that they rapidly leave the beam region.

Let us call N the number of the atoms per cm³. At 300 °K of temperature:

$$N = 3.2 \cdot 10^{16} P \; (\text{cm}^3)^{-1} ,$$

where P is the pressure in mm Hg.

σ_i is the ionization cross-section: it is energy independent for relativistic electrons [4]. We shall take as in reference [4]

$$\sigma_i \simeq 0.2 Z \cdot 10^{-18} \text{ cm}^2 .$$

Put t_w for the time an ion needs to escape from the beam-occupied region.

(*) Resonant blow up of the beam, of the just considered kind, seems likely to be observable only in the vertical direction because of the need of having ions over the whole path of the electrons during the oscillation amplitude build up time. This has been kindly pointed out to me by doctor A. Tollestrup.

(4) M. J. Moravcsik: *Phys. Rev.*, **100**, 1009 (1955).

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It follows that

$$N_+ \simeq c\sigma_i N \int_0^t N_e(t') \exp[-t'/t_w]\,\mathrm{d}t' .$$

We suppose N_e is made up of two parts:

$$N_e = N_a + N_p \exp[-t/\tau] ,$$

where N_a and N_p are constant; τ is a characteristic time of slow losses.

When $t \gg t_w$, τ, ε reaches a steady value

$$\varepsilon = \zeta c t_w \sigma_i N \left(1 + \frac{\tau}{\tau + t_w} \frac{N_p}{N_a}\right);$$

the factor $1 + (\tau/(\tau + t_w))(N_p/N_a)$ is likely to be about two, so that

$$\varepsilon \simeq 2\zeta c t_w \sigma_i N .$$

The main problem is to calculate t_w: The ions, since the time of their production, move along the lines of force of the external magnetic field, that is along the z axis. Thus, for an ion moving with velocity v, and produced at a random point of the beam section,

$$t_w = \frac{8r_0}{3\pi |v|} .$$

What we need is the average $\langle 1/t_w \rangle$ over the possible $|v|$ values, $|v|$ being the modulus of the z component of the ion velocity (see Appendix A). One could take the average on thermal velocities (*), thus obtaining

$$\frac{1}{t_w} = \frac{3}{2r_0} \left(\frac{2\pi kT}{M}\right)^{\frac{1}{2}} ,$$

(*) Recoil energies of the ions do not change appreciably the termal energy distribution. The transferred momenta q are certainly less than 10 keV/c in a collision, so that the ion mean recoil energy is certainly less than

$$\frac{q^2}{2M} \sim \frac{10^8}{4Z10^9} = \frac{0{,}025}{Z}\ \text{eV} ,$$

which is less than thermal energies by a factor $1/Z$.

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where M is the mass of the ions; but ion-ion and electron-ion electrostatic effects are certainly not negligible from the point of view of the ion distribution. There is the possibility of ion trapping by the main beam, which we shall examine in Appendix B. For the moment being, assume that the thermal distribution of velocities is valid; it follows that

$$t_w \simeq 3 r_0 \sqrt{z} \quad \mu\text{s} \qquad (r_0 \text{ in cm})$$

assuming $M \simeq 2Z$ (mass of the H atom).

The formula giving ε is

$$\varepsilon \simeq 1200\, \zeta r_0 Z^{3/2} P\,,$$

where P is in mm Hg. The formula for δn is thus

$$\delta n \simeq 120 \frac{\zeta Z^{\frac{3}{2}} P}{r_0} \left(\frac{N_e}{10^{10}}\right) \qquad \begin{cases} r_0 \text{ in cm} \\ P \text{ in mm Hg}\,. \end{cases}$$

We see that for

$$\zeta = 1\,, \quad Z = 10\,, \quad P = 5 \cdot 10^{-6}\,, \quad r_0 = 1 \text{ cm}\,, \quad N_e = 5 \cdot 10^{10}$$

one obtains

$$\delta n \simeq 0.1\,.$$

4. – Beam splitting.

So far we were concerned with a simplified model having cylindrical symmetry. Beam splitting effects can be put in evidence by simply removing this assumption.

We will treat in Appendix C the general case of ion distribution; for the sake of clearness we now consider a simple model in wich the ion distribution has elliptic asymmetry:

$$n_+(r, \varphi) r \,\mathrm{d}r \,\mathrm{d}\varphi = \frac{N_+}{4\pi r_0^2 R \Delta\Phi} \frac{1 + 2e_0 \sin^2\varphi}{1 + e_0} r \,\mathrm{d}r \,\mathrm{d}\varphi \qquad r < r_0$$

$$= 0 \qquad r > r_0$$

where n_+ stays for the ion density as a function of polar co-ordinates around the electron beam axis, r and φ. e_0 is a constant measuring the ellipticity, satisfying $e_0 > -\frac{1}{2}$ because of $n_+ > 0$.

The electric field components due to such a charge distribution are, at points far from the head or tail of a bunch

$$E_x = C\left(1 - \frac{2e_0}{1+e_0}\left(\frac{3}{4} + \ln\frac{r}{r_0}\right)\frac{x^2 - z^2}{r^2}\right)x - 4C\,\frac{e_0}{1+e_0}\left(\frac{1}{4} + \ln\frac{r}{r_0}\right)\frac{xz}{r^2}\,z\,,$$

$$E_z = C\left(1 - \frac{2e_0}{1+e_0}\left(\frac{3}{4} + \ln\frac{r}{r_0}\right)\frac{x^2 - z^2}{r^2}\right)z + 4C\,\frac{e_0}{1+e_0}\left(\frac{1}{4} + \ln\frac{r}{r_0}\right)\frac{xz}{r^2}\,x\,,$$

where

$$x^2 + z^2 = r^2\,, \qquad z = x\,\mathrm{tg}\,\varphi\,,$$

$$C = \frac{1}{2}\,\frac{\zeta e N_+}{r_0^2 R\,\Delta\Phi}\,.$$

Limiting the analysis of possible motions to a simple case, *i.e.* to the motion on the median plane $z = 0$, we see that the equations of betatron oscillations for the electrons of the main beam are of the following kind:

$$\ddot{x} + \left(\omega_0^2 + e_0\omega_1^2 \ln\frac{|x|}{r_0}\right)x = 0\,. \tag{1}$$

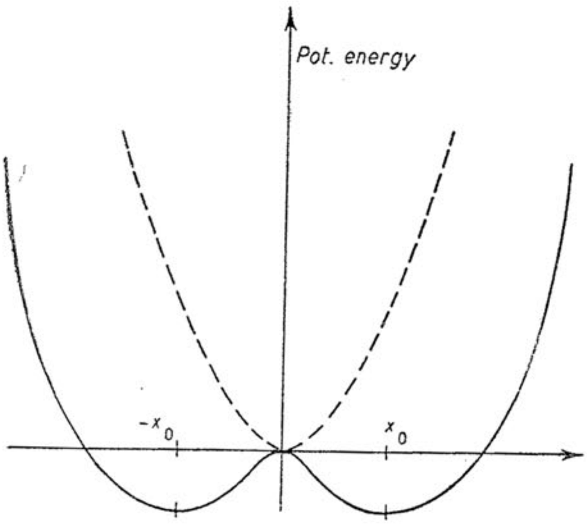


Fig. 2.

The log term is a new feature for betatron oscillations; ω_0, ω_1, are two constants readily obtainable in terms of the parameters defining the fields E_x, E_z and the external magnetic field.

The motion described by eq. (1) can be visualized by means of the graph (Fig. 1) of the potential energy of the force

$$\left(\omega_0^2 + e_0\omega_1^2 \ln\frac{|x|}{r_0}\right)x\,,$$

for the case $e_0 > 0$. (Were actually $e_0 < 0$, the same conclusions hold true for the motions out of the median plane, round $x = 0$).

Two equilibrium positions are found for the beam; the axis ($x = 0$) is no more a stable equilibrium position.

The parameters defining the well-depth and well centers are quite sensitive to the ellipticity e_0; in the case shown here, for instance,

$$|x_0| = r_0 \exp\left[-\frac{\omega_0^2}{e_0\omega_1^2}\right],$$

where $\pm x_0$ are the well-center co-ordinates (*).

5. – Conclusion.

This report summarizes a list of possibilities for space charge effects due to residual gas in the doughnut.

We do not maintain these effects will be so strongly evident in a well working machine as can be conjectured from some of the reported formulas: these are, in fact, rough calculations containing too many unknowns.

Nevertheless we are afraid that intensity limitations for weak focusing machines follow on this reasoning line; the point is whether they are so near the actual working points as here shown or not and this, we think, can be only decided by a suitable experimental program.

* * *

We would like to thank Professor G. SALVINI for stimulating help.

APPENDIX A

Simple Boltzmann equation for the ion density.

Let us call $n_+(x, z, v, t)$ the density of ions in a space of coordinates x, z, v (v being the z component of the velocity). Put $n_e(x, z, t)$ for the electron density in the beam and $f(v)$ for the maxwellian distribution function of the velocity v. Then

$$\frac{\partial n_+}{\partial t} = -v\,\frac{\partial n_+}{\partial t} + \sigma_i N c n_e(x, z, t) f(v)\,.$$

Let us consider the case in which

$$n_e(x, z, t) = \psi(x, z)\, N_e(t)\,,$$

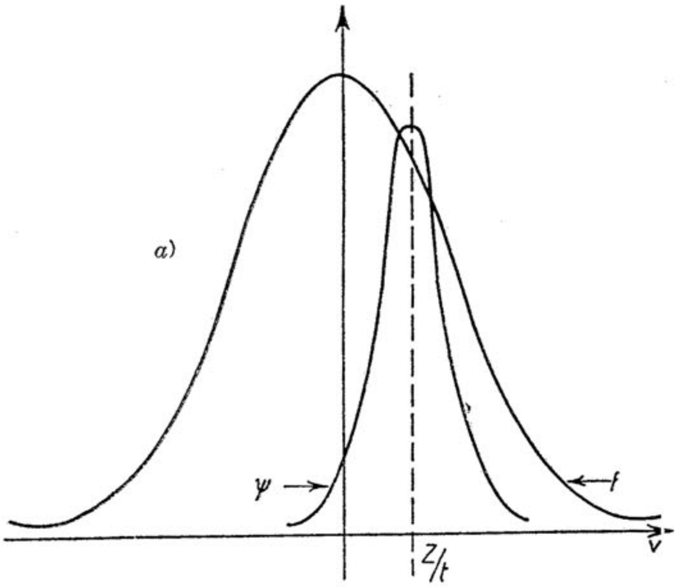
(*) Once two beams are produced further splitting could be envisaged for each one of the two beams on the same basic mechanism. Prof. R. WILSON pointed out to me this possibility.

where

$$\iint_{\text{whole space}} \psi(x, z)\,\mathrm{d}x\,\mathrm{d}z = 1 .$$

A solution is readily found with the only condition that there are no ions at the time $t = 0$.

$$n_+(x, z, v, t) = c\sigma_i N f(v) \int_0^t \psi[x, z - v(t' - t)]\, N_e(t')\,\mathrm{d}t' .$$



This formula allows us to perform the correct average over the velocities. Namely, what we need is

$$I(z, x, t) = \int_{-\infty}^{+\infty} f(v)\,\psi(x, z - vt)\,\mathrm{d}v .$$

Now, $f(v)$ is a gaussian function of width $\langle v^2 \rangle$; ψ, as a function of $z - vt$, is a strongly peaked function around the value $z - vt = 0$ of the argument. The width of this peak is of the order r_0. When (Fig. 2a)

$$\langle v^2 \rangle t^2 \gg r_0^2 ,$$

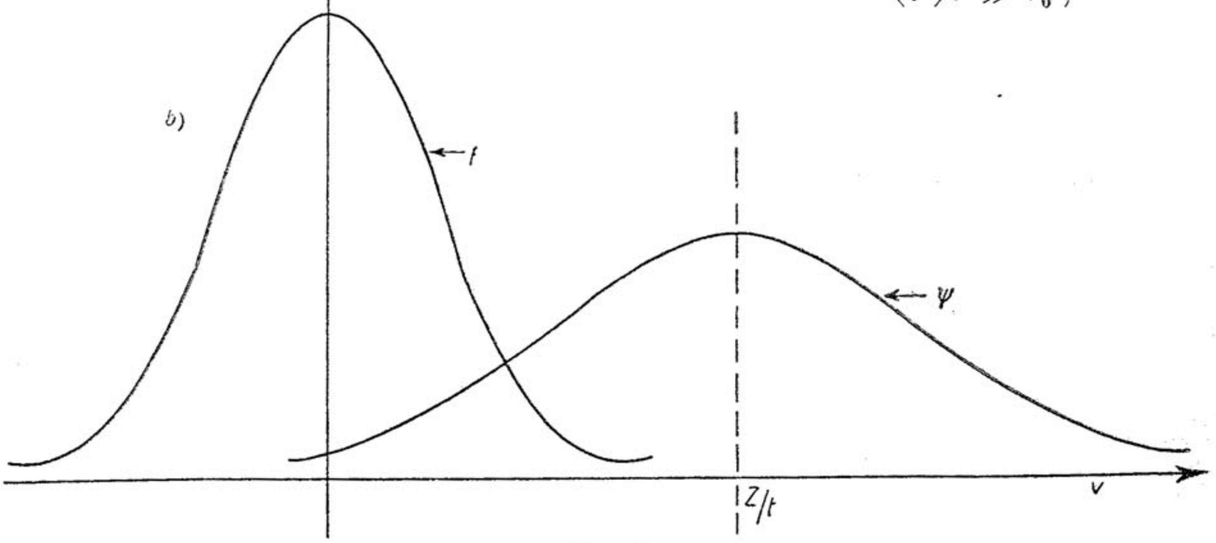


Fig. 2.

we can substitute for ψ a δ function:

$$\psi(x, z) \simeq u(x)\delta(z - vt) .$$

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Here u takes into account the x dependence. Thus, for this case

$$I(x, z, t) \simeq \frac{1}{|t|}\, u(x)\left(\frac{M}{2\pi kT}\right)^{\frac{1}{2}} \exp\left[-\frac{Mz^2}{2kT}\cdot\frac{1}{t^2}\right] \simeq \frac{1}{|t|}\, u(x)\left(\frac{M}{2\pi kT}\right)^{\frac{1}{2}}.$$

The last approximation comes out from the fact that we are mainly interested in z values less than r_0.

On the other hand, if

$$\langle v^2\rangle t^2 \ll r_0^2\,,$$

and if $z \neq 0$, the $\psi(z - vt)$ function, as a function of v, displaces its peak toward infinity as $t \to 0$, so that it does not overlap appreciably with the velocity distribution, (Fig. 2b).

There is an intermediate region in which

$$z^2 < t^2\langle v^2\rangle < r_0^2\,,$$

where things are somewhat more complicated. This region is, however, of no interest for the final result. One can check all the qualitative considerations just made by calculating what happens for a gaussian shaped ψ. Eventually one obtains that:

$$\left.\begin{aligned} I(x, z, t) &\simeq 0\,, && \langle v^2\rangle\, t^2 < r_0^2 \\ &\simeq \frac{1}{|t|}\, u(x)\left(\frac{M}{2\pi kT}\right)^{\frac{1}{2}}, && \langle v^2\rangle\, t^2 > r_0^2 \end{aligned}\right\}\ |z| < r_0\,.$$

Using this result to calculate

$$\langle n_+\rangle = \int_{-\infty}^{+\infty} n_+\,\mathrm{d}v\,,$$

one obtains

$$\langle n_+\rangle = c\sigma_i N \int_0^t N_e(t')\, I(x, z, t - t')\,\mathrm{d}t' \simeq c\sigma_i N u(x)\left(\frac{M}{2\pi kT}\right)^{\frac{1}{2}} \int_{r_0/\sqrt{\langle v^2\rangle}}^{t} N_e(t - t')\,\frac{\mathrm{d}t'}{t'}\ ;$$

neglecting the time dependence of N_e:

$$\langle n_+\rangle \simeq \frac{1}{2}\, c\sigma_i N u(x)\left(\frac{M}{2\pi kT}\right)^{\frac{1}{2}} N_e \ln\left(\frac{\langle v^2\rangle\, t^2}{r_0^2}\right) = \frac{1}{2}\, c\sigma_i N u(x)\left(\frac{M}{2\pi kT}\right)^{\frac{1}{2}} N_e \ln\left(\frac{2kTt^2}{Mr_0^2}\right).$$

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The log term is of no importance.

$$N_+ \simeq r_0 \int \langle n_+ \rangle \,\mathrm{d}x = \frac{1}{2} c\sigma_i N r_0 \left(\frac{M}{2\pi kT}\right)^{\frac{1}{2}} N_e \ln\left(\frac{2kTt^2}{Mr_0^2}\right).$$

This calculation shows that the qualitative arguments of Sect. **3** can be followed with some confidence.

APPENDIX B

Ion trapping.

The ion trapping mechanism can be described by means of matrix technique as follows: the ions in a given azimuthal position are periodically inside a bunch of electrons for a time $R\Delta\Phi/c$, outside of the bunches for a time $(R/c)((\pi/2)-\Delta\Phi)$.

When inside, the ions equations of motion are of the form

$$\ddot{z} + \omega_e^2 z = 0;$$

when outside, however,

$$\ddot{z} - \omega_+^2 z = 0,$$

where

$$\omega_e^2 = \frac{1}{2}\,\frac{\zeta N_e e^2(1-\varepsilon)}{R\Delta\Phi M r_0^2}$$

$$\omega_+^2 = \frac{1}{2}\,\frac{\zeta N_e e^2 \varepsilon}{R\Delta\Phi\, M r_0^2}.$$

This is a time alternating focusing defocusing structure, whose matrix trace is given by

$$\frac{1}{2}\,\mathrm{Tr}\,(\text{transf. matrix}) =$$

$$= \cosh \gamma_1\sqrt{\varepsilon}\cos\gamma_2\sqrt{1-\varepsilon} - \frac{1}{2}\,\frac{1-2\varepsilon}{\sqrt{\varepsilon}\sqrt{1-\varepsilon}}\cdot\sinh\gamma_1\sqrt{\varepsilon}\sin\gamma_2\sqrt{1-\varepsilon}\,.$$

where we put for brevity

$$\begin{pmatrix}\gamma_1\\ \gamma_2\end{pmatrix} = \left(\frac{\zeta N_e R e^2}{2Mc^2 r_0^2 \Delta\Phi}\right)^{\frac{1}{2}} \begin{pmatrix}(\pi/2)-\Delta\Phi\\ \Delta\Phi\end{pmatrix} = \gamma \begin{pmatrix}(\pi/2)-\Delta\Phi\\ \Delta\Phi\end{pmatrix}.$$

When $\gamma \ll 1$ the analysis of the trace is quite easy:

$$\tfrac{1}{2}\,\mathrm{Tr}\,(\text{transf. matrix}) \simeq 1 - \tfrac{1}{2}\,\gamma_2(\gamma_1+\gamma_2) + \tfrac{1}{2}\,\varepsilon(\gamma_1+\gamma_2)^2\,,$$

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and there follows that trapping is possible until ε reaches the value

$$\varepsilon = \frac{2\Delta\Phi}{\pi}\,;$$

the assumption of this ε value as equilibrium value would lead to a pressure independent shift of betatron frequencies. But the just described trapping mechanism must be interpreted with some care: in fact we claim that the situation we shew in Sect. **3**, based on thermal motion considerations for free ions, is more likely to be true for the following reasons. Let us suppose an ion oscillates along the z axis (i.e. it is trapped). If it leaves the beam axis with a velocity $\langle v\rangle$ (thermal average), the amplitude of its oscillations will be $\langle v\rangle/\Omega$ where Ω is the characteristic frequency of the composite structure we deal with in this appendix. Trapping is effective only if

$$\frac{\langle v\rangle}{\Omega} < r_0\,,$$

because if the ion goes outside the electron beam boundary repulsive space charge forces prevail and make it reach the doughnut's walls. Now Ω is given, in the $\gamma \ll 1$ approximation, by $\Omega \simeq \sqrt{8\pi}\gamma(\Delta\Phi - (\pi/2)\varepsilon)^{\frac{1}{2}}(c/R)$ and

$$\frac{\langle v\rangle}{\Omega} \simeq \frac{R}{\sqrt{8\pi}\gamma}\,\frac{1}{(\Delta\Phi-(\pi/2)\varepsilon)^{\frac{1}{2}}}\,\frac{\langle v\rangle}{c} \simeq 4\cdot 10^{-3}\,r_0\left[\frac{R\,\Delta\Phi}{\zeta(N_e/10^{10})(\Delta\Phi-(\pi/2)\varepsilon)}\right]^{\frac{1}{2}},$$

and this shows that trapping will be effective in fastening the ion cumulation but, as soon as $\Omega \to 0$, the trapping mechanism becomes less efficient and the pure thermal motion considerations give a good approximation.

We remark that because of this phenomenon the pressure dependence of ε could be far from linear.

Appendix C

The ion distribution. General case.

To show the possibility of appearance of log terms in the electric field formula, we develop here the potential due to an ion distribution of density $n_+(r,\varphi)$. The potential $V(r,\varphi)$ is given by

$$V(r,\varphi) = \text{const} - 2\zeta e\int_0^{2\pi}\mathrm{d}\bar\varphi\int_0^{\infty}\bar r\,\mathrm{d}\bar r\, n_+(\bar r,\bar\varphi)\ln\sqrt{r^2+\bar r^2-2r\bar r\cos(\varphi-\bar\varphi)} =$$

$$= \text{const} - 2\zeta e K(r,\varphi)\,,$$

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Expanding in Fourier series

$$n_+(r, \varphi) = \sum_{-\infty}^{+\infty} q_k(r) \exp[ik\varphi] \qquad (q_k = q^*_{-k})$$

and

$$K(r, \varphi) = \pi \ln r \int_0^r \bar{r} q_0(\bar{r})\, \mathrm{d}\bar{r} + \pi \int_r^\infty \bar{r} \ln \bar{r} q_0(\bar{r})\, \mathrm{d}\bar{r} -$$

$$- \frac{\pi}{2} \sum_{-\infty}^{+\infty}{}_{k\neq 0} \frac{\exp[ik\varphi]}{|k|} \left\{ \frac{1}{r^{|k|}} \int_0^r \bar{r}^{|k|+1} q_k(\bar{r})\, \mathrm{d}\bar{r} + \int_r^\infty \frac{1}{\bar{r}^{|k|-1}} q_k(\bar{r})\, \mathrm{d}\bar{r} \right\}.$$

Logarithmic terms can appear in the last term of the expansion, whenever

$$q_k(r) \sim r^{|k|-2} \qquad (\text{for not too large } r;\ k \neq 0)$$

The case we studied in Sect. 4 is that of $q_{\pm 2} = \text{const}$ for $r < r_0$. Attention must be given to the condition $n_+(r, \varphi) > 0$; in the model of Sect. 4 this condition requires, for instance, $e_0 > -\frac{1}{2}$.

RIASSUNTO

Si studiano gli effetti di carica spaziale (tenendo conto della presenza di ioni positivi) per stabilire se possono dar luogo a serie limitazioni per l'intensità del fascio di un elettrosincrotrone a focalizzazione debole. Si mostra che sono possibili effetti lineari di risonanza ed effetti non lineari di sdoppiamento del fascio. Si valuta inoltre il numero di ioni positivi prodotti nella camera a vuoto.

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