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A B S T R A C T

It is shown that charge independence or charge symmetry give rise to following relations connecting the strengths of γ - transitions between corresponding states in mirror nuclei:a) the strengths of corresponding E1 transitions are always equal; b) the strengths of corresponding M1 transitions should not differ in general by more than a factor 1.5;c) for transitions with $|\Delta T| = 1$ the strengths of corresponding transitions of any multipolarity are equal.

The effects of the Coulomb and exchange corrections to the above rules are examined. The experimental data presently available are discussed.

γ TRANSITIONS BETWEEN CORRESPONDING STATES IN MIRROR NUCLEI

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1. Relations due to charge symmetry and charge indipendence.

The invariance with respect to charge symmetry has, for self conjugate nuclei, rather important consequences in connection with the strengths of γ transitions; the E1 selection rule is known since long time and has been subjected to many experimental tests; recently we have shown[1] that an M1 inhibition rule should also be true; such rule has already found useful application.[2][3] In particular the two rules are very powerful for determining the isotopic spin and the other characteristics of many nuclear levels in self conjugate nuclei.[2]

The purpose of the present note is to point out that charge symmetry and charge independence have also consequences for the transitions in mirror nuclei, and more generally in nuclei belonging to the same isotopic spin multiplet. The relations which we shall discuss may be considered the natural generalization of the rules holding for selfconjugate nuclei ($T_3=0$). In this section we shall consider an hypothetical situation in which the Coulomb and exchange interactions are absent; in sec. 2 we shall discuss the modifications which are implied by these neglected interactions; in sec. 3 we shall compare our results with the experimental data.

The matrix element for an electromagnetic transition between the levels a and b (with isotopic spin respectively T_a and T_b) in a nucleus with mass number A and with some value of T_3 (= (N-Z)/2) may be written in the form:

$$(1) \qquad M_{a,b}(T_3) = \langle a\, T_a \left| \sum_1^A \left(H_0^{(i)} + H_1^{(i)} \tau_3^{(i)} \right) \right| b\, T_b \rangle_{T_3}$$

where we have exhibited the fact that the interaction is the sum of a part proportional to $\tau_3^{(i)}$ and a part independent of $\tau_3^{(i)}$;

the index T_3 specifies the value of T_3 for the nucleus in question.

We are now interested in the relation between the values of $\mathcal{M}_{a\,b}(T_3)$ corresponding to the same levels and the same A but different values of T_3. The relation in question is provided by the Wigner-Eckart[4] formulas; for discussing the selfconjugate nuclei, use was made only of a particular case of such formulas; here we make a more complete use of them.

Writing, with an obvious notation:

$$\mathcal{M}_{a\,b}(T_3) = \mathcal{M}_0(T_3) + \mathcal{M}_1(T_3)$$

where we have explicitely separated the two parts of the matrix element indicated in (1), we note the following properties of $\mathcal{M}_0(T_3)$ and $\mathcal{M}_1(T_3)$:

a) $\mathcal{M}_0(T_3)$ is different from zero, only if $\Delta T = T_a - T_b = 0$; it is independent of T_3:

$$\mathcal{M}_0(T_3) = \mathcal{M}_0 \qquad (2)$$

b) $\mathcal{M}_1(T_3)$ is different from zero only if $\Delta T = 0$ or if $\Delta T = \pm 1$.

In the first case ($\Delta T = 0$):

$$\mathcal{M}_1(T_3) = \mathcal{M}_1 T_3 \qquad (3)$$

In the second case ($\Delta T = \pm 1$):

$$\mathcal{M}_1(T_3) = \sqrt{T_a^2 - T_3^2}\,\mathcal{M}_1 \qquad (T_a = T_b + 1) \qquad (4)$$

In the above formulas (2) (3) (4) $\mathcal{M}_0$ and $\mathcal{M}_1$ are quantities independent of T, though, of course, depending on the levels in question.

In the case of conjugate nuclei (nuclei with opposite sign of T_3, in particular mirror nuclei) we can therefore state:

1) the matrix elements for transitions with $|\Delta T| = 1$ of any multipolarity are the same.

2) The matrix elements for transitions with $\Delta T = 0$ in two conjugate nuclei are respectively $\mathcal{M}_0 + \mathcal{M}_1 T_3$ and $\mathcal{M}_0 - \mathcal{M}_1 T_3$. It follows that:

2a) The matrix elements in question are equal (aside from a sign)

for E1 transitions; there infact $\mathcal{M}_0 = 0$ (conservation of the total momentum),

2b) the matrix elements in question are nearly equal (aside from a sign) for M1 transitions: there infact, $|\mathcal{M}_0| \ll |\mathcal{M}_1|$, as it has been shown in ref(1).

The above inequality, due to an almost complete cancellation of the protonic, neutronic and orbital magnetic moments, has the following meaning(1): though individual deviations are possible, the average absolute value of $\mathcal{M}_0$ is expected to be 10^{-1} times smaller than that of $\mathcal{M}_1$. It follows that the ratio between the squares of the matrix elements for two corresponding (pure) M1 transitions should not deviate from unity, in the average, by more than 50%.

The above statements will be called, when necessary, rule 1, rule 2a, rule 2b.

It is appropriate at this point to remark that the interest of the above rules lies in the fact that they are model independent, being based only on the charge independence or charge summetry; they may therefore be used as a tool in identifying states in conjugate nuclei.

We should also add (with reference to the rule 2b) that in principle it is quite conceivable that $|\mathcal{M}_0| \ll |\mathcal{M}_1|$ (or viceversa) also in transitions different from E1 or M1; our only point is that it is difficult to prove this withouth making use particular models, that is using only arguments having the same degree of generality as those employed in the case of E1 and M1 transitions; perhaps in the case of ML transitions ($L > 1$), $\mathcal{M}_0$ can be small due to a partial cancellation of the protonic and neutronic magnetic moments(5); but for the EL transitions ($L > 1$) we have not found any similar argument.

It is reasonable, finally, to ask wich ones of the above results depend simply on charge symmetry and which need the charge independence. It is evident that all the results referring to $\Delta T = 0$ transitions can equally well be proved using only charge symmetry; on the other hand charge independence is necessary to prove the equality of the matrix elements in conjugate nuclei for transitions

with $|\Delta T| = 1$.(6)

2. Coulomb interactions.

We shall now briefly consider the role of the Coulomb interactions;(7) our object is to give an estimate of the average effect of the Coulomb interactions on the electromagnetic matrix elements; at the end we shall make some remarks concerning the individual deviations from the mentioned estimate of the average effect.

We write:

$$\mathcal{M}_{AB}(T_3) = \mathcal{M}_{ab}(T_3) + K_{ab}(T_3) \qquad (5)$$

where $\mathcal{M}_{ab}(T_3)$ is the matrix element of the electromagnetic interaction between two levels in the charge independent approximation, and $\mathcal{M}_{AB}(T_3)$ the corresponding matrix element when account is taken of the Coulomb interaction; the eq. (5) defines the Coulomb correction $K_{ab}(T_3)$ which we propose to calculate.

To show the necessity of such a calculation it is important to remark that on changing T_3 into $-T_3$, $K_{ab}(T_3)$ does not simply transform as $\mathcal{M}_{ab}(T_3)$; if this were the case, the Coulomb corrections would never affect the rules of the past section, quite independently of their magnitude. The fact that $K_{ab}(T_3)$ does not transform in general as $\mathcal{M}_{ab}(T_3)$, when T_3 is changed into $-T_3$, is physically clear and may be seen most simply on writting the first order term in a perturbation expansion of K:

$$K_{ab}(T_3) = \sum_n{}' \frac{C_{an}(T_3)\mathcal{M}_{nb}(T_3)}{E_a - E_n} + \sum_n{}' \frac{\mathcal{M}_{an}(T_3)C_{nb}(T_3)}{E_b - E_n} \qquad (6)$$

where $C_{an}(T_3)$ is the matrix element of the Coulomb interaction between the states a and n of the nucleus T_3 and the other symbols are obvious. It may be easily seen, writing the Coulomb interaction in the isotopic spin formalism, that $C_{an}(T_3)$ is the sum of two parts which transform with different signs when T_3 is changed into $-T_3$.

It is therefore necessary to give an estimate of K_{ab}, to be at least sure, that, apart from special cases, which will be illustrated later, the Coulomb corrections to the rules of the past section are not too large.

We might try to give such estimate starting from the espression (6) and making use of wave functions for the nuclear states obtained from some model, for instance the shell model. However once eccept, as we shall do, the use of a shell model for constructing the wave functions, the following approach is much more reasonable.

Suppose that we choose oscillator wave functions as our basis functions. This means that in our model the nuclear interaction between pairs of nucleons has been substituted with an average oscillator nuclear potential. It seems then also appropriate for our purposes, to substitute the sum of the Coulomb interaction between pairs of protons, by an average Coulomb potential in which each proton moves. This will be done in what follows.

Accordingly the Hamiltonian of our model is written:

$$(7) \qquad H = \frac{1}{2M} \sum_1^A \vec{p}_i^{\,2} + \frac{1}{2} \sum_1^A k \vec{r}_i^{\,2} + \sum_1^Z V_c(r_i)$$

where, for simplicity, we have distinguished between proton and neutron instead of using the isotopic spin formalism; the last term, summed over all protons (i = 1 ..... Z) represents the average Coulomb potential in which each one of them moves; the first two terms represent the (charge independent) Hamiltonian ($\mathcal{H}_o$) which one has in the absence of Coulomb forces.

The Coulomb potential V_c may be calculated as follows: consider for each state of $\mathcal{H}_o$, the charge density $\rho(r)$ in that state. The Coulomb potential $V_c(r)$ is then:

$$V_c(r) = e \int \frac{\rho(r')\, d\vec{r}'}{|\vec{r} - \vec{r}'|}$$

Here we shall make several approssimations: 1) we shall first neglect the dependence of V_c from the nuclear state, always taking for $\rho(r)$ its expression for the ground state; 2) we approximate the correct expression of $\rho(r)$ with:

$$(8) \qquad \rho(r) = Z e \Big/ \tfrac{4}{3} \pi R^3 \qquad r < R$$

$$\rho(r) = 0 \qquad r > R$$

In (8) the radius R is chosen so that the root mean square radius corresponding to the distribution (8) is equal to the root mean square radius corresponding to the correct expression of $\rho(r)$.

3) The potential which is calculated from (8) is, as well known:

$$V_c(r) = \frac{Z e^2}{2R}\left(3 - \frac{r^2}{R^2}\right) \qquad r < R \tag{9}$$

$$V_c(r) = \frac{Z e^2}{r} \qquad r > R \tag{10}$$

We assume that the expression (9) is valid for all r; this is an approximation similar to that of taking an oscillator nuclear potential for all r and not only for $r < R$.

It should be noticed that the approximations (1) and (3) can only increase the Coulomb effects; with respect to the approximation (2) it is a known fact that the shape of a distribution is not very important in low energy phenomena.

Inserting (9) in (7) and leaving out a constant the Hamiltonian may be written:

$$H = \frac{1}{2M}\sum_1^A \vec{p}_i^{\,2} + \frac{1}{2}\sum_1^Z k' \vec{r}_i^{\,2} + \frac{1}{2}\sum_{Z+1}^A k \vec{r}_i^{\,2} \tag{11}$$

where

$$k' = k - \frac{Z e^2}{R^3} \tag{12}$$

The only difference between protons and neutrons appears therefore in this model as a difference in the elastic constants.

In order to appreciate the value of this difference we must give a value for k; this can be done by fitting the root mean square radius a of the nucleus.

Experimentally it is [8]:

$$a \simeq r_0 A^{\frac{1}{3}}$$

with r_0 around $1.35\ 10^{-13}$ cm for the light nuclei, though not exactly constant. On the other hand a is related to k by:

$$a = \chi\left(\frac{\hbar^2}{M k}\right)^{\frac{1}{4}} \tag{13}$$

where χ is a factor which varies only slightly with the mass number; it has the values 1,25-1,5-1,71 respectively for He, O and Ca.

We approximate the above A dependence writing χ as $A^{1/7}$; one may check that in this way the above numbers are approximately reproduced. It follows ($a = \sqrt{\frac{3}{5}} R$):

$$k \cong \frac{2.8\, A^{\frac{4}{7}} \hbar^2}{M R^4} \tag{14}$$

The ratio $|k'-k|/k$ already gives an idea of the importance of the Coulomb corrections; such ratio is:

$$\xi \equiv \frac{|k'-k|}{k} \cong \frac{Z}{77\, A^{\frac{1}{2}}} \tag{15}$$

For instance we have $\xi \cong 6 \cdot 10^{-2}$ for Z = 10, $\xi \cong 10^{-1}$ for Z=20.

This shows that, in general, in the Z region in which we are interested, the Coulomb corrections to the matrix elements are expected to be relatively small; we now try to get an estimate of such corrections.

This is most easily done comparing directly the matrix elements $\mathcal{M}_{ab}(T_3)$ calculated putting $k' = k$ in (11) (that is disregarding the difference (12) between k' and k) with the corresponding matrix elements $\mathcal{M}_{AB}(T_3)$ calculated taking this difference into account; in other words, in the notation of (5), we directly calculate the quantity $\mathcal{M}_{AB}/\mathcal{M}_{ab}$ instead of calculating K_{ab}.

It is easy to show[10] that in our model, an estimate of the ratio $\mathcal{M}_{AB}/\mathcal{M}_{ab}$ may be obtained without specifyng the particular states between which the transition takes place; such estimate only depends on the multipolarity of the transition; the results are reported below for the various multipolarities.

We may generally decompose $\mathcal{M}_{AB}$ and $\mathcal{M}_{ab}$ in a proton part (suffix (p)) and in a neutron part (suffix (n)) as follows:

$$\mathcal{M}_{AB} = \mathcal{M}_{AB}^{(p)} + \mathcal{M}_{AB}^{(n)} \quad ; \quad \mathcal{M}_{ab} = \mathcal{M}_{ab}^{(p)} + \mathcal{M}_{ab}^{(n)}$$

It is then possible to show that it is generally:

$$\begin{aligned} \mathcal{M}_{AB}^{(p)} &= \lambda\, \mathcal{M}_{ab}^{(p)} \\ \mathcal{M}_{AB}^{(n)} &= \mathcal{M}_{ab}^{(n)} \end{aligned} \tag{16}$$

where the coefficient λ in (16) depends on the multipolarity L.

For magnetic transitions λ is generally given by:

$$\lambda = \left(k/k'\right)^{\frac{\ell-1}{4}} = (1+\xi)^{-\frac{\ell-1}{4}} \tag{17}$$

In particular no Coulomb corrections affect the M1 matrix elements in this model.

For electric transitions λ is given by:

$$\lambda = \left(k/k'\right)^{\frac{\ell}{4}} = (1+\xi)^{-\frac{\ell}{4}} \tag{18}$$

In particular for $L>1$ the neutron contribution to an EL transition may be generally neglected in comparison with the proton contribution, so that (18) gives the ratio between $\mathcal{M}_{AB}(EL)$ and $\mathcal{M}_{ab}(EL)$. The same argument does not apply to the E1 transitions because there it is essential to consider the neutron contribution which arises from the conservation of the total momentum.

In any case the formulas (16), (17), (18) provide us with a general estimate of the Coulomb corrections to the matrix elements; as anticipated such corrections turn out to be rather small in the Z region of interest.

To complete the discussion of the Coulomb corrections it is however essential as this point to note that so far we have only discussed the corrections to the matrix elements; in order to obtain the transition widths one has to multiply the square of the matrix element by some power of the energy difference of the two levels between which the transition takes place; this energy difference, which in general is different in the two conjugate nuclei which we are comparing, is in any case a known experimental quantity. We may therefore summarize the situation as follows: consider the widths for two corresponding transitions in conjugate nuclei and divide them by the appropriate power of the energy difference; the two squares of the matrix elements so obtained should satisfy the rules of the sec.1 with a precision expressed by the factor $(1+\xi)^{-\frac{\ell}{2}}$ for an EL transition and $(1+\xi)^{-\frac{\ell-1}{2}}$ for an ML transition.

It is important here to stress that, obviously, the above estimates represent only the average behaviour. It is quite possi-

ble that the matrix element for some particular transition in the hypothetical absence of the Coulomb interactions is small (say, of the order ξ) with respect to the average matrix element for the same kind of transition; this can be due to a particularly bad overlap of the two wave functions. In such case the relations of sec. 1 between the widths of the transition for the nucleus in question and for the conjugate nucleus may be lost, the transitions being dominated by the Coulomb interactions. The existence of the special cases mentioned above may be described using (5) and (6): if $\mathcal{M}_{ab}$ is small, K_{ab}, given by (6), can become very important or even dominating. It should be remarked, however, that the intensity of the transition in these special cases should be smaller, by a factor say ξ^2 , than the intensity of a normal transition of the same multipolarity; only for such weak transitions large deviations from the rules of sec. 1 should possibly be found.

To complete this section we have still to consider the possible effects due to the fact that the forces between the nucleons are transmitted through pions. We confine here to the case of E1 and M1 radiation. For the E1 radiation a correction in principle arises (11) due to the fact that the total momentum of the nucleus may undergo small fluctuations around its average value zero; this is because the quantity which is conserved is not the momentum of the (dressed) nucleons only, but the total momentum of the dressed nucleons plus mesonic field. This effect has been already considered for the selfconjugate nuclei. It is difficult to calculate; for an alpha particle to which the calculations of[11] refer, it turns out that the order of magnitude of the effect can be in some cases as large as the Coulomb one. The expression order of magnitude here, has however to be taken in an extremely loose sense on account of the very many approximations which are necessary in the computation. Moreover the effect should decrease rapidly with increasing A while the Coulomb corrections increase. There is no evidence from self conjugate nuclei for invoking this effect; so we may well neglect it here.

For the M1 radiation one may have exchange current effects. However, as already pointed out by Gell Mann and Telegdi the terms in the Hamiltonian responsible for them, transform as the H_1 part in (1) and should not, therefore, modify the present rules.

3. The experimental data

There are very few data, presently, to compare with above predictions, and the most part of them is affected by rather large errors.

Two kinds of measuments are of interest:

a) branching rations in γ-deexcitation of corresponding levels of conjugate nuclei.

b) The absolute values of the lifetimes of corresponding levels in conjugate nuclei.

We first discuss the class a) experiments; practically the only material to compare with our predictions is provided by some transitions in Al^{25} and Mg^{25}, investigated particularly by Gove and coll.(13)(14) The similarity of the branching ratios, in the cases in which they are known in both nuclei, has been already stressed by these investigators and is apparent from the fig. 1 below (where we have reported all the information existing on the levels (1) to (5) which is of interst for the following discussion); the results of the present paper show that such similarity, though expected, is by no means an obvious fact, and has to be discussed in each case.

Because the values of the individual intensities are not known, the only thing which we may compare in the two nuclei are the ratios of the intensities (for each nucleus) of competitive decay modes in corresponding transitions. For instance, considering the deexcitation of level (2) we compare in the two nuclei the ratio $\Gamma_{2,1}/\Gamma_{2,0}$ of the intensities to the first excited state (1) and to the ground state (0); such ratio we call ${}_2\eta_{1,0}$, where the left index refers to the deexciting state and the right indices refer, in the order specified, to the states which are formed. We introduce also the symbol ${}_2\eta^{c}_{1,0}$ to indicate again the ratio of the two widths $\Gamma_{2,1}$

and $\Gamma_{2,0}$, each being divided, however, by the appropriate power of the energy difference between the relevant levels, as explained in the past section:

$$ {}_2\eta^{c}_{1,0} = \frac{\Gamma_{2,1}}{(E_2 - E_1)^{l}} \Big/ \frac{\Gamma_{2,0}}{(E_2 - E_0)^{l'}} $$

where l and l' depend on the multipolarity of the emitted radiation.

The above notation, exemplified on a particular level, will be used generally in the following discussion where we shall compare the values of η and η^{c} , for those transitions in Al and Mg which are relevant to the rules discussed in sec. 1.

1) Deexcitation of level (2): from the diagrams 1 and 2 we see that all the transitions are M1 + E2; obviously $\Delta T = 0$. If we can disregard the E2 fraction (this assumption is perhaps unjustified; compare ref.(14)) the rule 2b should apply. We have ${}_2\eta_{1,0}(Al) = 1{,}38$ and ${}_2\eta_{1,0}(Mg) = 0{,}45$; therefore ${}_2\eta_{1,0}(Al) / {}_2\eta_{1,0}(Mg) = 1{,}45$. More meaningful is the ratio ${}_2\eta^{c}_{1,0}(Al) / {}_2\eta^{c}_{1,0}(Mg) = 0.70$

2)Deexcitation of the level (4): The similarity of the branching ratios in this case cannot be explained using our rules since the transition (4) $\longrightarrow$ (1) is pure E2. We may still compare the values of ${}_4\eta_{2,0}$ in Al and Mg, which refer to M1 + E2 transitions, but the fact that the E2 transition is so strong raises doubts about the smallnes of the amount of E2 radiation in the (4) $\longrightarrow$ (2), (4) $\longrightarrow$ (0) radiation.

3) Deexcitation of the level (5): the decay scheme is not exactly the same in Al and in Mg, as it appears from the fig. 1. However we may safely assume that in the Mg^{25} case, the (5) $\longrightarrow$ (2) radiation has been missed and compare the 16/84 ratio of Mg with a 13/87 ratio in Al. All the transitions are E1; they fall under the rule 2a. We have ${}_5\eta_{1,0}(Al) / {}_5\eta_{1,0}(Mg) = 0.79$; for the ratio between the η^{c}'s we have: ${}_5\eta^{c}_{1,0}(Al) / {}_5\eta^{c}_{1,0}(Mg) = 0.76$.

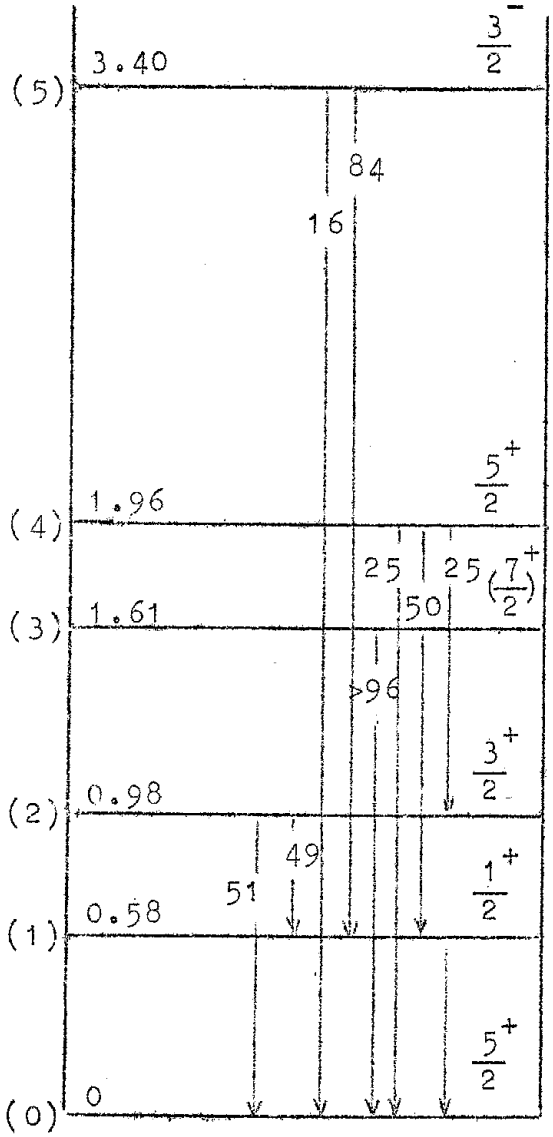
Summarizing the situation we may say that the agreement between the predictions and the experimental data is fair, though not strikingly good; however the experimental branching ratios are probably affected by rather large errors: in the case of the deexcitation

of the level (5) we have assumed in the discussion above that the (5) $\rightarrow$ (2) radiation has been missed in the Mg decay; this implies errors of the order 20%; in the case of the deexcitation of the level (2) it is sufficient to say that Maeder and Stahelin[15] give for $\gamma_{1,0}^{(Mg)}$ the value 0,44 $\pm$ 0,07 instead of the value 0.95 quoted above.

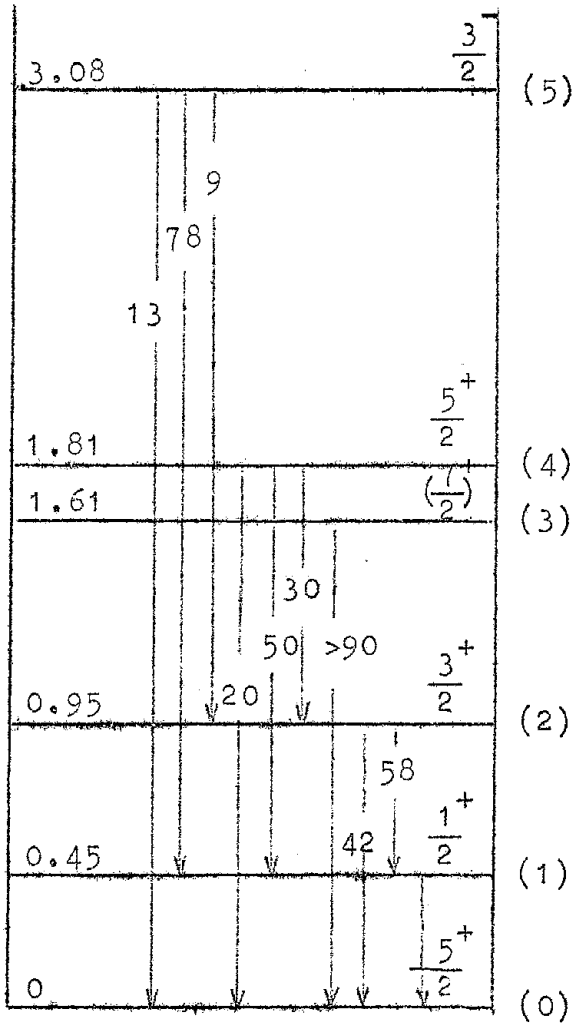
In view of this it is better to wait for more precise data to continue this discussion; we note only that in any case one would like to have the branching ratios established also in other pairs of mirror nuclei possibly with a lower Z and with small E2-M1 admixtures; from the 1955 compilation of Ajzenberg and Lauritsen[16] we learn that the only other reported case in which a branching ratio in corresponding transitions in mirror nuclei is known is one in N^{13}, C^{13}; the radiations involved are here M1 and E1.No disagreement with the results of the present paper exists, but again the data are not sufficient to say more.

Finally only in one case the value of the lifetimes of corresponding excited states in mirror nuclei are known[17]: those of the Be^7 and Li^7 first excited states which go into the ground state through an M1 transition; they are respectively 2.7 10^{-13} sec. with an error of $\pm$ 50% and 7.7 10^{-14} sec. with an error of $\pm$ 20%. To compare the two values we multiply the latter by $(4,77/4,3)^3$, the cube of the ratio of the energies involved; we therefore have to compare 2.7 . 10^{-13} with 1.05 10^{-13}. Within the errors no discrepancy with the rule 2b exists.

We end these considerations by stating that it would be interesting to collect more precise data of the kind discussed; we notice that once the rules have been cheked accurately in some cases, they can become an useful tool several circumstances.



Mg^{25}



Al^{25}

FIG. 1

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(1) G. Morpurgo, Phys. Rev. 110, 721 (1958); this paper contains also references to work on the E1 rule; for a complete survey of the E 1 rule compare also W.K. MacDonald. Isotopic spin selection rules (to be published in Nuclear Spectroscpy edited by F. Ajzenberg-Selove). I thanck Dr.MacDonald for a proprint.

(2) E.K. Warburton: Inhibition of M1 radiation and the identification of T = 1 states in light, selfconjugate nuclei. NYO-8138-I thank Dr. Warburton for having sent this paper before publication.

(3) G.M. Temmer and N.P. Heydenburg. Phys. Rev. 111, 1303 (1958).

(4) Compare, e.g.: E. Wigner, Gruppentheorie (Edward Brothers, inc. 1944)

(5) E.K. Warburton. Phys. Rev. Letters 1,68 (1958)

(6) The rules 1,2a, 2b may be easily proved as follows without using the isotopic spin formalism: in the nucleus Z,N the matrix element $M(Z,N)$ for the transition a → b may be written: $\langle a(Z,N)|\sum_1^Z u_i + \sum_{Z+1}^A v_i|b(Z,N)\rangle$; where the first Z coordinates are those of the protons, the last N those of the neutrons and u_i, v_i are the operators inducing the transition. The above expression may be identically rewritten as the difference of two terms, say A and B where A = $\langle a(Z,N)|\sum_1^A (u_i + v_i)|b(Z,N)\rangle$ and B = $\langle a(Z,N)|\sum_{Z+1}^A u_i + \sum_1^Z v_i|b(Z,N)\rangle$. Now, if the charge symmetry holds, the wave functions of the states a,b of the mirror nucleus N,Z are obtained from the wave functions of the nucleus Z,N simply calling protons the particles with i = Z + 1 ..... A and neutrons the particles with i=1 ..... Z; therefore the matrix element B is, apart from a phase factor, the matrix element for the transition a → b in the nucleus N,Z. This proves the rules if the matrix element A vanishes or (case 2b) nearly vanishes. It is clear that this always happens for E1 and M1 transitions because then the operator $\sum_1^A (u_i + v_i)$ vanishes or nearly vanishes; moreover this also happens for any multipole if the functions a and b belong to different representations of the group of the neutron-proton permutations because the operator $\sum_1^A (u_i + v_i)$ is completely symmetrical; this last assertion is equivalent to say that a and b have a different value of T; it has a meaning only if the forces are charge independent.

(7) The problem here is rather different from the corresponding one for self conjugate nuclei treated by L. Radicati (Proc. Phys. Soc. (London) A66,139, (1953)) and W. MacDonald (Phys. Rev. 100, 51 (1955)); there, for instance, only a mixing with states with different T is important; this is not the case here.

(8) R. Hofstadter. Annual Review of Nuclear Science, 7, 311 (1957).

(9) Compare ref. (8) and also E. Feenberg. Shell theory of the nucleus (Princeton 1955).

(10) To show this we compare the wave function corresponding to some given nuclear state when one takes into account or disregards the Coulomb interaction. We call these (normalized) wave functions respectively Φ_A and Φ_a ; both Φ_A and Ψ_a are in general constructed through products of hermite and spin functions; the only difference between the two is that in one case (Φ_a) the "unit of lenght" for the protons is $\left(\hbar^2/kM\right)^{\frac{1}{4}}$ in the other (Φ_A) it is $\left(\hbar^2/k'M\right)^{\frac{1}{4}}$; in other words the normalized Φ_a is obtained from the normalized Φ_A simply replacing everywhere $\left(\hbar^2/k'M\right)^{\frac{1}{4}}$ with $\left(\hbar^2/kM\right)^{\frac{1}{4}}$; from this properties the simple proportionality relations (16) follow easily.

(11) G. Morpurgo, Nuovo Cimento 12, 60 (1954)

(12) M. Gell Mann and V. Telegdi, Phys. Rev. 91, 169 (1953)

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Figure caption

Fig. 1 - The levels of interest in Mg^{25} and Al^{25}.