Laboratori Nazionali di Frascati

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C. Bernardini: NUMERICAL CALCULATIONS OF THE INFLUENCE OF THE RADIATION FLUCTUATIONS ON THE BEAMS INTENSITY FOR WEAK FOCUSING SYNCHROTRONS.

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C.BERNARDINI: Numerical calculations on the influence of the radiation fluctuations on the beam's intensity for weak focusing synchrotrons.

1.- In the following, some numerical calculations will be performed on:

a) the maximum amplitude of the synchrotron oscillations accepted by a given RF system.

b) the root mean square amplitude set up by radiation.

The basis for a) can be found for instance in Bohm, Foldy, Phys.Rev. 70,244,1946, or Persico, Suppl.al Nuovo Cimento,2,X,459,1955.

The basis for b) is the work by Kolomensky and Lebedev, CERN Symp.I.

In appendix a method is given for evaluating some Kolomensky-Lebedev's integrals.

The results are collected in fig.1 and, to avoid the reading of the paper, which could be some-what tedious, I will give soon the meaning of the graphs:

1) The full line represents the r.m.s. amplitude of synch. oscill. due to radiation fluctuations as a function of energy, both for the italian and Caltech machines since the difference in radf (a factor $\sqrt{\frac{375}{360}}$ ) does not matter with the used scale.

2) The datted (- - - -) lines represent the maximum amplitude of synchrotron oscillations for the Caltech synchrotron. There are the 50 and 150 kV (peak voltage of RF cavity) lines and also the end point of a similar curve for 200 kV (marked ⊙ at E = 1500 MeV).

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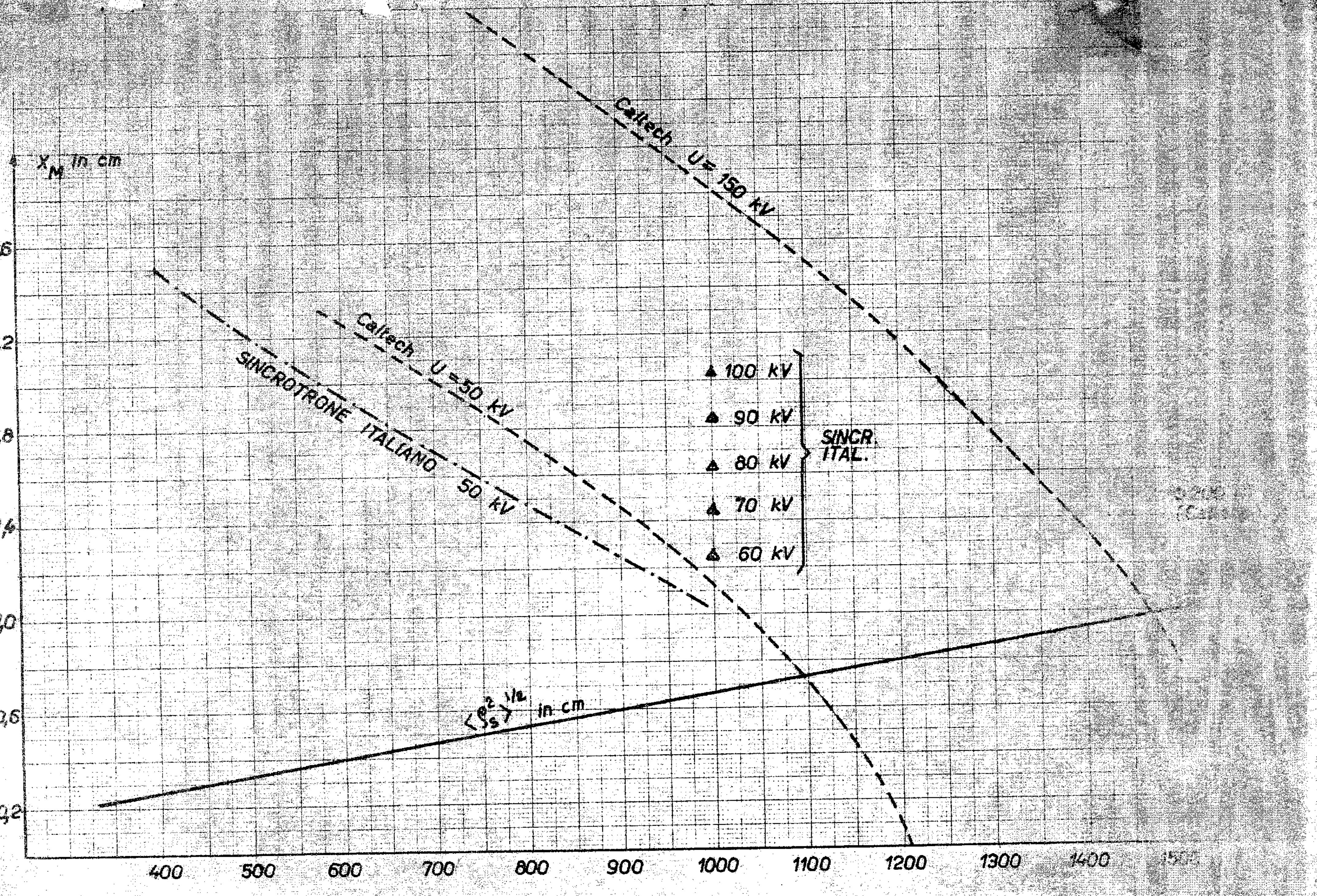
$X_M$ in cm
2,6
2,2
1,8
1,4
1,0
0,6
0,2
400
500
600
700
800
900
1000
1100
1200
1300
1400
1500
Caltech U = 150 kV
Caltech U = 50 kV
SINCROTRONE ITALIANO 50 kV
100 kV
90 kV
80 kV
70 kV
60 kV
SINCR. ITAL.
$\langle \rho_s^2 \rangle^{1/2}$ in cm

3) The datted (-.-.-.-) line represents the same as 2), for the italian machine in the 50 kV case. End points for 60 to 100 kV are marked $\Delta$ at the E = 1000 MeV line.

2.- Maximum amplitude of the synchrotron oscillations (radial).

Symbols:

R = radius of the principal orbit in a magnetic sector

k = RF harmonic number

n = field index

L = lenght of a straight section

$\Lambda = 1 + \frac{2L}{\pi R}$

U = peak RF voltage

E = energy

$\varphi_s$ = synchronous phase

$X_M$= maximum allowed oscillations amplitude

T = acceleration time lenght

t = time

w = t/T

$E_M$= maximum energy

$f(w) = \frac{E}{E_m}$ = magnetic cycle waveform

$u = 2\pi\Lambda R/c \cdot \frac{dE}{dt} + L$ , where:

L = radiation energy loss per turn

With this symbols, $X_M$ is given by the following formula:

$$X_M = C_o\left(2\cos\varphi_s + 2\varphi_s\sin\varphi_s - \pi\sin\varphi_s\right)^{1/2}$$

where

$$C_o = R\left(\frac{eU\Lambda}{\pi k(1-n)E}\right)^{1/2} \qquad \sin\varphi_s = \frac{u}{eU}$$

For practical purposes

$$u = 2.1\times10^{-2}\,\frac{\Lambda R E_M}{T}\,\frac{df}{dw} + \frac{88.5}{R}E_M^4 f^4(w)$$

...

with: R in meters
T " seconds
$E_M$ " BeV
u " keV

Caltech machine:

$f(w) = w$ $\quad R = 375$ cm $\quad n = 0.6$

$E_M = 1.5$ BeV $\quad \Lambda = 1.26$

$T = 250$ msec $\quad k = 4$

then

$$u = 0.57 + 120\, w^4 \text{ keV}$$

$$C_0 = 0.153 \frac{1}{\sqrt{w}} \sqrt{U} \text{ cm} \quad , \text{ U in kV}$$

Italian Machine:

$f(w) = \frac{1}{2}(1-\cos \pi w)$ $\quad R = 360$ cm $\quad n = 0.6$

$E_M = 1$ BeV $\quad \Lambda = 1.21$

$T = 1/40$ sec $\quad k = 4$

then

$$u = 3.5 \frac{df}{dw} + 24.6\, f^4(w) \text{ keV}$$

$$C_0 = 176.4\, (U/E)^{1/2} \text{ cm.} \quad , \text{ U in kV, E in MeV}$$

For not too high $\varphi_s$ the function

$$\Omega(\varphi_s) = (2\cos\varphi + 2\varphi_s \sin\varphi_s - \pi \sin\varphi_s)^{1/2}$$

is given with a very good approximation by

$$\Omega(\varphi_s) \approx -1.1\,\varphi_s + 1.4$$

The approximation is valid at least for $0 \leq \varphi_s \leq 0.6$

3.- R.M.S. amplitude of radiation-induced synchrotron oscillations.

As can be easily seen (s.appendix) the R.M.S. induced amplitude $\langle \rho_s^2 \rangle^{1/2}$ is, at the end of the acceleration cycle, a linear function of the energy

$$\langle \rho_s^2 \rangle^{1/2} \simeq \frac{55}{32\sqrt{3}} \cdot \frac{R\hbar}{mc(1-n)(3-4n)} \left(\frac{E_M}{mc^2}\right)^2 \cdot f^2(w)$$

...

That is, for the synchrotrons we are dealing with
$\langle \rho_s^2 \rangle^{1/2} \simeq 0.66\, E$ with $\langle \rho_s^2 \rangle^{1/2}$ in cm, E in BeV.

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Appendix I -

In Kolomenski and Lebedev theory one has to evaluate integrals like that appearing in the following formula

$$\langle x^2 \rangle = \frac{M^2}{f(w)} \int_0^w f^6(w')\, dw' \exp\left[-N^2 \int_{w'}^{w} f^3(w'')\, dw''\right]$$

This can be approximatily done as follows:
put $s = \int_0^w f^3(w')\, dw'$ and $\psi(s) = f^3(w)$, then

$$\langle x^2 \rangle = \frac{M^2}{f(w)} \int_0^s \psi(s')\, e^{-N^2(s-s')} ds' = \frac{M^2}{f(w)} \int_0^s \psi(s-s'')\, e^{-N^2 s''} ds''$$

if $N^2 s \gg 1$, then

$$\int_0^s \psi(s-s'')\, e^{-N^2 s''} ds'' \approx \int_0^s \left(\psi(s) - s'' \frac{d\psi}{ds}\right) e^{-N^2 s''} ds'' \approx$$

$$\approx \frac{\psi(s)}{N^2} + \dots = \frac{f^3(w)}{N^2}$$

That is $\langle x^2 \rangle \approx \frac{M^2}{N^2} \cdot f^2(w)$

In the case of the synchr. oscill.

$$N^2 = \frac{2}{3} \, \frac{3-4n}{1-n} \cdot \frac{T}{\Lambda E_M} \, \frac{e^2 C}{R^2} \left(\frac{E_M}{mc^2}\right)^4$$

and, for f(w) = w

$$N^2 s = \frac{1}{6} \, \frac{3-4n}{1-n} \, \frac{T}{\Lambda E_M} \, \frac{e^2 C}{R^2} \left(\frac{E}{mc^2}\right)^4$$

As can be seen the approximation starts to be good, for our cases, at about E = 500 MeV.