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G. Salvini: THE ITALIAN DESIGN OF A 1000 MeV ELECTRON SYNCHROTRON: A COMPARISON BETWEEN THE STRONG AND THE WEAK FOCUSING.

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The Italian Design of a 1000 MeV Electronsynchrotron: a Comparison between the Strong and the Weak Focusing.

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1. – The Choice of the Injector.

Once the final energy is established, one of the most important decisions which has to be taken in the design of an electrosynchrotron is the choice of the injector; in fact not only the entire design will heavily depend upon the quality of the injector, but also some basic decisions, for instance the choice between weak and strong focusing, will depend on the injector.

This sentence may seem too extreme to him who considers the problem of the design of an electronsynchrotron with the same spirit of the old days (1946), when the injector was an almost isotropic furnace of low energy electrons [1], but the opinion will change when one considers the high energy injectors of today, which pour in the magnet a well-collimated beam of relativistic electrons through an inflector of high precision in the large space of one of the straight sections.

Let's try to demonstrate this statement considering three of the main characteristics of an injector (we will limit us to the electronsynchrotrons, but of course, some of the considerations apply to the machines for protons too). These characteristics are: the energy, the momentum spread and the geometrical dimensions of the injected beam.

Let's start with the energy of the injected beam: a high energy is generally to be preferred. With an injector of high energy we will inject at a higher induction field of the magnet. This will lessen the fondamental difficulty of controlling and shaping the induction field near the injection, because of the reduced percent effect of the remanence and the eddy currents at injection.

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(1) CORNELL UNIVERSITY: *Synchrotron progress Report* (1949).

As we know, the injection at low fields—for instance 20 gauss—is a much greater difficulty in the electronsynchrotrons than in the protonsynchrotrons, where the injection field easily is 5-10 times higher due to the heavier mass of the proton.

Another advantage of the high injection energy is the reduced effect of the scattering in the residual gas inside the donut: the growth of the betatron oscillations induced by this scattering will be smaller than at low energies, with the behavior that we will consider in the following.

All these arguments indicate that the dimensions of the gap of the magnet may be smaller when the injection energy is higher. This assumption, which we will try to define more quantitatively later, is certainly right.

Another important advantage of the injection at high energy is the fact that the construction of the radiofrequency system may become easier: in fact, if we inject electrons of an energy of a few MeV ($2\div4$ MeV total energy) the velocity of the electrons differs from the velocity of the light by $3\div1$ percent and therefore we may not even need the frequency modulation: the instantaneous equilibrium orbit will remain inside the donut also if a constant value of the radiofrequency is assumed. For instance we remember that the 550 MeV electronsynchrotron of Cal. Tech., which has a very large gap, accelerates electrons injected from a pulse transformer of 1 million volt, and does not use frequency modulation. Electronsynchrotrons of a more economic structure could avoid frequency modulation only with electrons of a kinetic energy of at least 3 MeV.

The second quality of the injector that we are going to consider is the momentum spread of the electrons; a quantity which we may for instance define as the momentum interval inside which half the electrons emerging from the inflector are contained.

Let's indicate with Δp this quantity and with $\Delta p/p_0$ the percent spread around the nominal value p_0.

A magnet with weak focusing will generally [admit] only a small value of Δp, in the sense that all the electrons differing from p_0 for more than say $\pm 1\,\%$ will be lost against the walls. A strong focusing machine, on the contrary, has an higher momentum admittance, for instance by a factor 10. Therefore the choice of the type of focusing in the magnet may depend on the quantity $\Delta p/p_0$, characteristic of the injector.

The third important quality of the injector, is the geometrical dimension of the electron beam.

The dimensions of the beam (diameter and angular spread) are of great importance for the value of this efficiency: by improving the geometry of the injected beam, we may have the same final intensity of high energy electrons with a gap of the magnet which is considerably smaller.

2. – Types of Injectors

So, we begin to see that the ideal injector is a machine giving electrons of the highest energy (compatible with the inflecting system) and the minimum momentum spread, with a beam of a very high geometrical definition. Let's give a glance to what the market or the laboratory offer, keeping in mind that only after this inspection we will decide on the characteristics of the magnet.

In Table I we give a list of the injectors which may be taken in consideration

TABLE I.

Type of injector	Admittance	$\Delta p/p_0$	Max. Energy	Electrons/pulse
Van de Graaf . . .	$10^{-3} \div 10^{-4}$	$10^{-2} \div 10^{-3}$	~ 3 MeV	$10^{11} \div 10^{12}$
Cockcroft-Walton . .	$10^{-3} \div 10^{-4}$	$10^{-2} \div 10^{-3}$	~ 2 MeV	$(3 \div 5)\cdot 10^{12}$
Linear Accelerator (3)	$10^{-3} \div 10^{-4}$	$(3 \div 5)\cdot 10^{-2}$	~ 8 MeV	$10^{12} \div 10^{13}$
Pulse transformer . .	$10^{-3} \div 10^{-4}$	—	~ 1 MeV	$\sim 10^{12}$
Microtron	10^{-3} (?)	—	~ 6 MeV	$\sim 10^{11}$

for our purpose, with some of their characteristics. The numbers given in Table I may only constitute a rough indication of the possibility of these injectors, which should be discussed one by one.

With the quantity « admittance » we mean the value of the product: diameter of the beam times its angular spread at the exit from the inflector. This number does not give much more than an order of magnitude.

3. – Comparison between the Strong Focusing (Alternate Gradient) and the weak Focusing (Constant Gradient).

a) At this point, we have to go from the general to the specific: I do not think it is possible to give general rules for the choice of the injector, or for the choice of the magnet, and it is perhaps better for you if I present here the results we reached until now in the comparative study of two different designs, one of an e.s. of 1 000 MeV maximum energy, employing weak focusing the other of an e.s. of the same energy but with strong focusing.

These results are those obtained from the Italian group (Sezione Acceleratore) working at the construction of a nuclear machine.

The program of the Sezione Acceleratore is to build an electronsynchrotron of 1 000 MeV for the benefit of all the Italian Universities interested in nuclear physics and in quantum electrodynamics. The project is supported by the Istituto Nazionale di Fisica Nucleare. Therefore I am reporting here the results which were obtained with the contribution of all the members of our group.

Recent years are rather critical, for taking decisions on the design of an electronsynchroton of $0.8 \div 1.5$ GeV, and this mainly arises from the alternative in the choice between the strong focusing and the weak focusing solution for the magnet.

In order to make our decision as responsible as possible, we are calculating two designs: one electronsynchrotron with alternate gradient focusing (strong focusing) and one with constant gradient (weak focusing). This lecture is an invitation to you, and particularly to the French group who had to resolve a similar problem in the design of their protonsynchrotron of 2 GeV, to discuss the choice.

The two designs are not complete yet, and we will speak of them with this in mind. They have in common some basic dimensions which are the following:

the radius	$\sim$ 3.50 metres
the field at maximum . .	10 000 gauss
the injection energy . . .	2.5 MeV, total
the type of injector . . .	Cockcroft-Walton
the R.F. system	two resonant cavities.

In § 6 we give the exact data for the two designs; all the numerical calculations in the following were developed starting with the data of § 6.

The main problem is to obtain the intensity of the 1000 MeV electrons versus the cost, or better versus a certain nominal cost which takes into account also the time which the realization of the machine will require. Therefore we have to resolve the question of the dimensions of the gap in the two cases.

These dimensions depend as we already said on the choice of the injector, and we wish to justify our choice of a Cockcroft-Walton before all. The choice of the injector was made by prof. AGENO of the Istituto Superiore di Sanità, who will direct the construction with the help of the other members of his laboratory.

b) Looking again at Table I, we consider that the Cockcroft-Walton injector is more convenient thak the Van de Graaf and the pulse transformer. In fact, in comparison with the pulse transformer, the C.W. can give more intensity, for it has an higher capacitance (for instance $\sim$ 1000 cm against $\sim$ 100 cm), in pulsed operation. In comparison with the pulse transformer the C.W. can reach an higher energy, for the pulse transformer was never operated at more than 1 MeV, as far as we know, and a pulse transformer of two MeV or so would probably be a difficult and large machine: the injector of the Cal. Tech. electronsynchrotron is a pulse transformer and reaches 1 MeV kinetic energy.

On the contrary, as we see from Table I, the C.W. is as good as the two others in respect of the admittance, and the $\Delta p/p_0$ value.

And now we ask, why not a linear accelerator or a microtron? Linear accelerators are better for intensity and maximum energy, so that we could even hope to avoid frequency modulation. But linear accelerators (2) have a large $\Delta p/p_0$ value and this is certainly bad, at least with the weak focusing. Moreover, we have to rememebr that the use of an injection energy much higher than 2 MeV kinetic would make the problem of the inflector very hard: we should probably give up the usual electrostatic inflectors and study some new type employing magnetic fields. Another fact in favour of the C.W. is this: the Istituto di Sanità has a long experience with C.W. accelerators, while we never built in Italy a linear accelerator. Of course we could buy a l.a. in England (3), where they are commercially constructed, but we consider it to be of advantage to have a many years experience on the injector.

Going now to the microtron, we have to admit that we considered the microtron inconvenient for admittance and intensity, but we recently learnt that the swedish group (4) is planning to use the microtron as an injector in their strong focusing 1000 MeV electronsynchrotron, and we will try to know more on this machine.

With these points fixed, let's go back to the question of the dimensions of the gap, and therefore of the magnet, in the two cases of the strong focusing (alternate gradient, A.G.) and the weak focusing (constant gradient (C.G.).

c) Case of the alternate gradient (A.G.). The minimum gap dimensions for a 1000 MeV A.G. machine are, as it was already pointed out by doctor Bruck of Saclay in his lecture, hard to estimate we are mainly limited by pure mechanical precision and by the physical dimensions of the other pieces of the equipment: otherway the theoretical trend would be just to make n very large, and the gap as small as possible.

The question may be put in the following way. In order to have a linear induction field in the gap we have to make a gap with pole profiles which are in first approximation sections of an hyperbola; then the equipotentials have

(2) *Annual Review of Nuclear Science*, **1**, 199 e segg. (1952), M. Chodrow: *Rev Sci. Instr.*, **26**, 134 (1955).

(3) Two English factories are building l.a. of 3÷8 MeV. The following considerations particularly refer to these types.

(4) Private communication.

to obey a law of the type (see Fig. 1)

$$A(x_0 \mp x)z = \text{constant} \equiv K\,, \tag{1}$$

where z is the vertical coordinate, x is the coordinate along the axis.

The pole profiles of the magnet will be described by this curve, with the sign — and + alternately. The central orbit of the electrons is perpendicular the the plan of Fig. 1, in the point $X = Z = 0$.

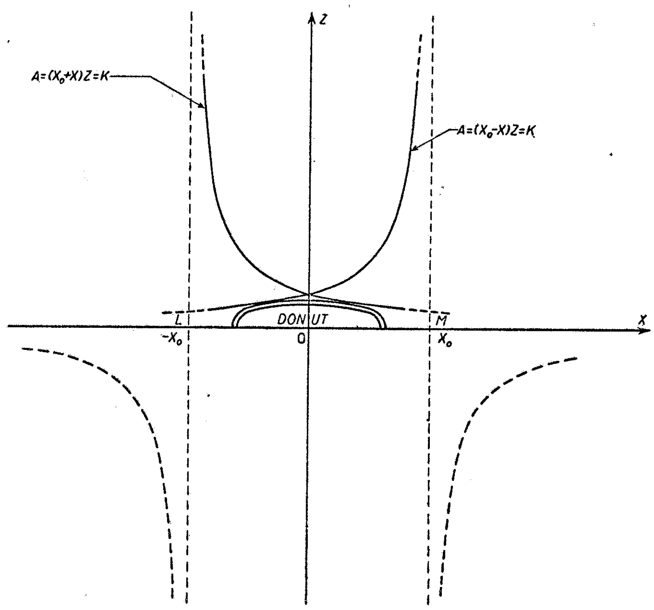

Fig. 1.

The value of the constant K determines the dimensions and the shape of the gap, and $\pm x$ is the distance of the vertical asymptots of the hyperbolae from the origin,

The component B_z of the magnetic induction is given by

$$B_z = \frac{\partial}{\partial z} A(x_0 \mp x)z = A(x_0 \mp x) \tag{2}$$

and this is evidently a field linear in respect of x.

On the other side, we can say that in an alternate gradient machine we need a field of the type

$$B_z = B_z^0\left(1 \mp \frac{nx}{R}\right), \tag{3}$$

where n is the field index, and R is the radius of the machine. By comparison between (2) and (3) we have:

$$B_z^0 = Ax_0, \quad n = \frac{R}{x^0}. \tag{4}$$

This means that if the radius R, which in both our projects is ~ 3.50 metres, is kept constant, the n value determines, at least to a certain extent, the dimensions of the donut. In fact in the conventional design of an alternative gradient magnet the useful region of the donut (see Fig. 1) will have a horizontal dimension not larger than the distance l between the asymptotes (length of the segment LM of Fig. 1) with the relation:

$$l = 2x_0 = \frac{2R}{n}.$$

In order to have a small gap, and therefore a small magnet, we have to increase for a given R the n value of the field.

Then we have to balance between two contrasting exigencies. On one side, if we want to take all the advantages of the strong focusing over the weak focusing solution we have to choose a high n value; on the other hand we cannot reduce the dimensions of the gap below certain values, which are imposed by other exigencies (dimension of the beam at the injection, dimension of the inflector, space for an useful spiralization etc.), and we cannot choose too high an n value, otherway the requirements on the mechanical precision of the parts of the magnet and on the precision in the measurement of the magnetic field become very hard or impossible to be fulfilled.

If we make reasonable assumptions of all these quantities, we conclude that in an A.G. e.s. with $R \cong 3.50$ metres n has to be chosen in the region of the tens (for instance $n = 15 \div 50$) rather than in the region of the hundreds. As you know, n is of the order of $200 \div 400$ in the case of the giant proton-synchrotrons with a radius of $50 \div 100$ metres.

In our design of the alternate gradient magnet we take $n \doteq 21$. The main reason for this choice is that the new e.s. of the University of Cornell which is now going in operation has this value, and so we will have an opportunity of comparing our theoretical previsions with the experimental results.

d) Case of a constant gradient machine. – The choice of n is in this case simpler, for n has to be in the limits $0 < n < 1$ and you cannot expect large advantages from a particular choice: the betatron oscillations will have frequencies proportional to $n^{\frac{1}{2}}$ (vertical oscillations) and to $(1-n)^{\frac{1}{2}}$ (horizontal oscillations) and you cannot hope to reduce one mode of oscillation if you don't enlarge the other.

The dimensions of the gap will then depend from the rather standard procedure of calculating the injection mechanism, the gas scattering, the radio frequency capture and so on.

4. – The Mechanism of Injection and Acceleration. Some General Remarks.

In the following part of this lecture we will compare the theoretical results of our two designs, the alternate gradient and the constant gradient. This comparison is not complete yet, and other calculations are in progress. The results come from the work of Professor PERSICO and his theoretical group (doctors CARLO BERNARDINI, SONA and TURRIN).

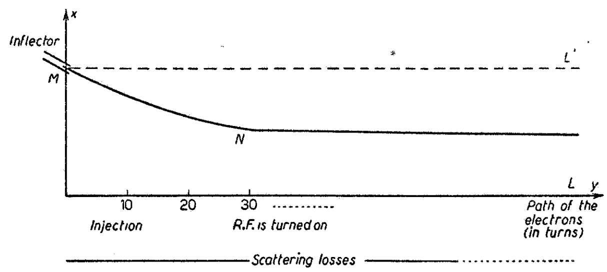

Fig. 2.

a) We will outline the general procedure, before giving the numerical results. – The electrons are injected in one of the four straight sections of the donut through the electrostatic inflector I (see Fig. 2). In Fig. 2 we represent the rectified path of the electrons, so that the abscissa gives the path of the electrons, and the distance between the two straight lines L and L' is the horizontal dimension of the donut. The sketch of Fig. 2, of course, is not at all in scale.

The beam emerging from the inflector will have certain geometrical dimensions (angle θ_0, width A) and a certain spread in momentum $\Delta p/p_0$. This is an electronsynchrotron, and we have to inject for many turns: in fact the period of revolution is of the order of 0.1 μs, which is a very short time, while an injection of $1\div2$ μs is what appears more convenient for us. There is therefore the problem of spiralizing the electrons inside the donut in such a way that we obtain the maximum efficiency.

The injection efficiency is something which we may define as the ratio between the number of electrons circulating inside the donut after the injection, until the R.F. is turned on, and the number of electrons which were injected. This efficiency (which we indicate with Q) is a function of many parameters:

$$Q = Q\left(\theta, \Delta p, A, \frac{\mathrm{d}B_z}{\mathrm{d}t}, n, \ldots\right),$$

which we have already defined.

If we disregard the possible inhomogeneities of the magnetic field, the simplest path of the electrons during the injection is a linear spiral in the case of the weak focusing (this path is represented with the line MN in Fig. 2), but it is a more complicated spiral curve in the alternate gradient case, due to the complicated nature of the instantaneous equilibrium orbits.

In the real case, the electrons will make betatron oscillations around the spiral orbit, with the behaviour which has been studied and exposed in his lecture by professor PERSICO.

Toward the end of the injection, at the proper moment, we will turn on the R.F. Of course, only a fraction of the electrons will be in the right phase relation with the R.F. system: we have therefore to define a F.R. capture efficiency, which we indicate by S. The function S is, directly or not, a function of the same variables than before, plus the rate of rise $\mathrm{d}V/\mathrm{d}t$ of the voltage of the R.F. at least:

$$S = S\left(\theta_0, \Delta p, A, \frac{\mathrm{d}B_z}{\mathrm{d}t}, n, \frac{\mathrm{d}V}{\mathrm{d}t}, \ldots\right).$$

The scattering of the electrons by the residual gas in the donut is important from the very beginning, up to an energy which more or less is three times the injection energy. This scattering will certainly produce a loss of electrons. We may indicate the efficiency for scattering in the gas with ϱ: this efficiency (probability that an electron survives the death for scattering from the residual gas on the walls of the donut) will be a function of many parameters: in particular it will depend on the mechanism and momentum p of injection, on the velocity of rise of the magnetic field, on the residual pressure P in the donut:

$$\varrho = \varrho\left(p, \frac{\mathrm{d}B_z}{\mathrm{d}t}, P, \ldots\right).$$

We remember that the value of $\mathrm{d}B_z/\mathrm{d}t$ at the injection is

$$\left(\frac{\mathrm{d}B_z}{\mathrm{d}t}\right)_{\text{in}} = \frac{n\sigma}{RT_0}(B_z)_{\text{in}},$$

where σ is the radial distance between two successive turns of our spiral MN, $B_{z\text{in}}$ is the field at injection, T_0 is the period of revolution of the electrons.

Q, S, ϱ, are the « efficiencies » that an electron has to pass before feeling reasonably safe. After these examinations it will not meet difficulties for quite a long time, until when the R.F. will switch from one cavity to the other (a problem which we did not completely resolve yet), and later when radiation losses and coherence phenomena become important.

With this history in mind, we will now sketch our results for our two designs, the one costant gradient, alternate gradient the other. We have to underline the preliminary character of these results.

The main characteristics of the two machines for which the numerical calculations were made are given in § 6.

5. – Results of our Numerical calculations for the Constant Gradient and for the Alternate Gradient Design.

a) Estimate of Q (5). – In the constant gradient case the efficiency Q was calculated with the following assumptions:

$$A = 0.8 \text{ cm}$$

$$\theta_0 = \pm 3 \cdot 10^{-3} \text{ rad}$$

$$\Delta p/p = \pm 10^{-2} \text{ (which means } \Delta p = 2 \cdot 10^{-2} p) .$$

In the calculation the inhomogeneities of the magnetic field were neglected. We indicate by I the electron current (number of electrons per second) from the injector, and we define the time of injection Δt:

$$\Delta t = \frac{T_0 a}{2\sigma} ,$$

where a is the horizontal width of the donut which may be considered at disposal for the injection; Δt is therefore the time which an electron spends to spiralize from the inflector to the center of the donut. T_0 and σ were already defined in § 4.

Then we have:

$$Q = \frac{\text{electrons iniected in the time } \Delta t \text{, and who survived}}{\text{electrons injected in the time } \Delta t} \cong 0.4 .$$

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(5) A. TURRIN: *Rapporto Teorico della Sezione acceleratore* n. 3.

As we see, the time Δt depends on σ and a. The best value of σ, considering the probability for the spiralizing electrons of missing the inflector, is of the order of a few millimetres. The efficiency goes down fast for $\Delta p/p > \pm 10^{-2}$.

In the alternate gradient case the problem of estimating Q is harder and not completely resolved. We will provisionally assume, with the same definitions as before and with the same numerical values of A, τ_0, $\Delta p/p$, that Q is still 0.4. But we have to consider that the time Δt is probably shorter, for a is reduced by a factor of the order of 2, (see § **6**) and therefore the number of electrons circulating in the donut after the injection is smaller than in the C.G. case.

One important remark is that in the A.G. case Q remains about the same also for $\Delta p/p$ as large as $\pm 3 \cdot 10^{-2}$. Some type of injector which may be inconvenient for the C.G. focusing (for instance the linear accelerator) could be successfully employed in the A.G. magnets.

b) Estimate of S. – Calculations were performed in the two cases (Professor PERSICO, Doctor BERNARDINI [6]), when the R.F. is turned on instantly (ideal case), and when the R.F. is turned on « slowly », for instance in 1-2 μ s. The two cases do not differ too much.

With the assumption $\sigma = 0.2$ cm we have:

Weak focusing: $S = 0.15$

Strong focusing: $S = 0.1$

S (weak)/S (strong) $= 1.5$.

c) Estimate of ϱ. – We give here our numerical results for the C.G. case, as they were obtained by Doctors SONA and C. BERNARDINI [7]:

$\sigma = 0.1$ mm	pressure $= 10^{-5}$ mm_{Hg}	$\varrho = 0.15$
$\sigma = 0.2$ mm	pressure $= 10^{-5}$ mm_{Hg}	$\varrho = 0.3$
$\sigma = 0.6$ mm	pressure $= 10^{-5}$ mm_{Hg}	$\varrho = 0.8$.

In the A.G. case the calculation is more difficult, for we cannot be sure that it only depends on the maximum amplitude of the betatron oscillations, and not on their particular shape. On the basis of some qualitative argument we consider it reasonable to assume the same values of ϱ than in the C.G. case. An argument in favour of the strong focusing in respect to ϱ is the fact that for

(6) E. PERSICO: *Rapporto Teorico della S. A.*, *n.* 4; E. PERSICO and C. BERNARDINI: *Rapporto Teorico della S. A.*, n. 12.

(7) C. BERNARDINI: *Rapporto Teorico della S. A.* n. 11; P. G. SONA: *Rapporto Teorico della S. A. P.T.*, V.

the same value of σ the value of dB_z/dt is higher in the A.G. case, and ϱ obviously increases with dB_z/dt. On the other side we should not forget that the increased value of dB_z/dt may constitute a difficulty in the A.G. case for the increased difficulty of controlling the shape of the magnetic field and the rate of change of the R.F..

d) Final comparison between C.G. and A.G. focusing. – At this point, a comparison of the efficiencies of the two designs may be made by estimating the quantity $Z = Q \times S \times \varrho$ in one case and the other. The ratio of the values of Z should give more or less the ratio of the final intensities (number of electrons of high energy per second) of the two machines, which we will indicate by T. More exactly, we have to consider the values of Δt in the two cases, and then the value of T is given by:

$$T = \frac{\text{C.G. intensity}}{\text{A.G. intensity}} = \frac{Z_{\text{C.G.}} \times (\Delta t)_{\text{C.G.}}}{Z_{\text{A.G.}} \times (\Delta t)_{\text{A.G.}}} \approx 3 .$$

Our results are summarized in Table II.

TABLE II. – *Comparison of the efficiency of our two designs.*

	Q	S	ϱ	$Z = Q \cdot S \cdot \varrho$	T
Weak focusing (C.G.) . .	0.4	0.15	0.3	0.018	3
Strong focusing (A.G.) . .	0.4	0.1	(0.3)	0.012	

If we strictly stick to these results, we should conclude that our weak focusing machine promises three times more electrons than the strong focusing. But considering the present uncertainties of our calculations, expecially in the A.G. case, and remembering that a factor 3 in the final intensity is not very significant in a nuclear machine, I prefer to say that the final intensities in the two cases are not significantly different.

6. – Some Other Numerical Results.

We will briefly report in the following some theoretical results which were of importance in the calculation of the quantities Z and T.

a) Betatron oscillations in weak focusing and strong focusing. – We will give here as an example the results in one particular case (8).

(8) A. TURRIN: *Rapporto Teorico della S. A.*, n. 14.

Let's define:

x, the horizontal distance of the electrons from the axis of the donut;

$dx/ds = \alpha$, the angle between the tangent of the trajectory of the electron and the axis of the donut (s is the coordinate along the axis of the donut);

M, the maximum value assumed by x (maximum displacement from the axis).

With these hypotheses we have, for an electron of initial conditions $x = 0$ and $dx/ds = \alpha$:

$$\text{constant gradient: } \left(\frac{M}{R}\right)_{x=0} = 1.67\alpha$$

$$\text{alternate gradient: } \left(\frac{M}{R}\right)_{x=0} = 1.3\alpha\,.$$

This is an indication that the amplitude of the betatron oscillations are not too different in the two cases, with a ratio $1.67/1.3 \approx 1.3$.

This result may surprise a little, but we have to consider the peculiar character of the strong focusing betatron oscillations, whose hyperbolic part strongly contributes to increase the maximum displacement M.

b) Momentum spread. – We give here the maximum displacement X_{max} from the axis for an electron injected along the axis ($x = 0$, $dx/ds = 0$) with a momentum $p = p_0 + \Delta p$, where p_0 is the momentum corresponding to the a is of the donut:

$$p_0 = eB_zR\,.$$

The ratio of the two values of X_{max} in the two cases, weak and strong focusing, is:

$$\frac{(X_{max})_{weak}}{(X_{max})_{strong}} \cong \frac{5R(\Delta p/p_0)}{0.5R(\Delta p/p_0)} = 10$$

for $\Delta p/p_0 = \pm 10^{-2}$.

From this ratio we may conclude that an injector with too large a $\Delta p/p_0$ value is more inconvenient to the weak focusing.

c) Amplitude of the synchrotron oscillations. – Doctor C. BERNARDINI calculated the ratio K between the synchrotron oscillation amplitudes in the C.G. and in the A.G. design, for electrons of the same energy and which were captured in the same phase in the two cases.

The results are:

$$K = \frac{X_{max}\ (\text{strong})}{X_{max}\ (\text{weak})} = 1.75 + 5.97\left(\frac{E_0}{E}\right)^2 + \ldots$$

E_0 is the rest energy of the electron, and E is the electron's total energy.

7. – The Basic Data of our Two Designs. Our R.F. System.

a) Constant gradient (weak focusing).

Injector:

Type	Cockcroft-Walton
Maximum voltage	$2\cdot10^6$ V
pulses/s	$\leqslant 50$/s
electrons/pulse	$\sim 5\cdot10^{12}$ (hope)
$\Delta p/p$	$10^{-2} \div 10^{-3}$

Pressurized with nitrogen and freon.

Magnet:

4 quadrants, with straigth sections of 100 cm;
$n = 0.6$
Excitation: resonant CL circuit with a bias of direct current;
Dimensions of the gap: 21×8 cm² ($\sim 16\times6$ useful);
injection field: 24.5 gauss;
maximum field: 10^4 gauss.

Radius of equilibrium orbit: 333 cm.
Maximum energy of the electrons: 1000 MeV.
R.F. system: two resonant cavities in 4th harmonic, with a final frequency of 48.1×10^6 Hz (see § 7-*c*).

b) Alternate gradient (strong focusing). – The data of our a.g. machine are, as we already said, very close to those of the e.s. which is under construction at the Cornell University. We are grateful to Prof. WILSON for his kind supply of data and informations.

Injector: the same than in the C.G. design.

Magnet:

4 quadrants, with straight Sections of 90 cm;
$n = 21$;
Excitation: resonant CL circuit with a bias of direct current;
Dimensions of the gap: $\sim 9\times4$ cm² (8×3 useful);
structure of the gradients: see Fig. 3;
injection field: 21.5 gauss;
maximum field: 10^1 gauss.

Radius of the equilibrium orbit: 381 cm.
Maximum energy: $1\,000 \div 1\,200$ MeV;
R.F. System: the same than in the C.G. design.

c) The R.F. system. – The radiofrequency has been studied by Professor QUERCIA, who is responsible of all the electronics of the Sezione Acceleratore, and by Ing. PUGLISI, who specialized on the problems of the Radio Frequency. What here follows is a summary from their reports.

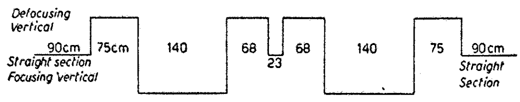

Fig. 3.

The frequency of revolution of the electrons, (which is of about 10 MHz at injection) increases of 2.1% as their energy increases from the injection energy (2.5 MeV total energy) to the final value of 1 GeV. In the present design the R.F. system employs two cylindrical resonant cavities which are located in two straigth sections. The first cavity drives the electrons from the energy of injection to about 10 MeV.

During this time the revolution frequency of the electrons increases by about 2.1%, and therefore the frequency of the electric field in the gap of the cavity has to increase by the same percent. The peak voltage of the cavity varies from about 3 to 7 kV.

The second cavity has to rise the energy of the electrons from ~ 10 MeV up to the final value. The peak voltage has to change during the process of acceleration from 7 kV to about 50 kV; this high value, as well known, is required to compensate for the radiation losses, which reach up to 27 kV at the maximum energy.

In Fig. 4 we give a block diagram of the apparati who feed and control the first cavity.

A pilot oscillator 4 guides the power amplifier 7 through the frequency multiplier 5 and the buffer amplifier 6. The power amplifier is directly connected to the cavity 19, which is the anodic resonant circuit of 7.

The frequency of the system, as we already said, has to change by $\sim 2.1\%$ and the rate of this change is a function of the energy of the electrons and therefore of the value $B_z(t)$ of the induction field at the equilibrium orbit. To control the frequency by the value $B_z(t)$, a coil is posed in the gap of the e.s.; the voltage induced in the coil is proportional to $\mathrm{d}B_z/\mathrm{d}t$; it is electronically integrated and this information goes to a function forming network 2 which controls the polarization current of an inductance with a ferroxcube nucleus. This inductance is a part of the circuit of the pilot oscillator 4, whose frequency is therefore controlled by the magnetic field in the gap and becomes a proper function of $B_z(t)$.

The precision required for this function is $\pm 1‰$.

The modulators 16 ad 20 perform the control in amplitude of the electric field of the cavity 19.

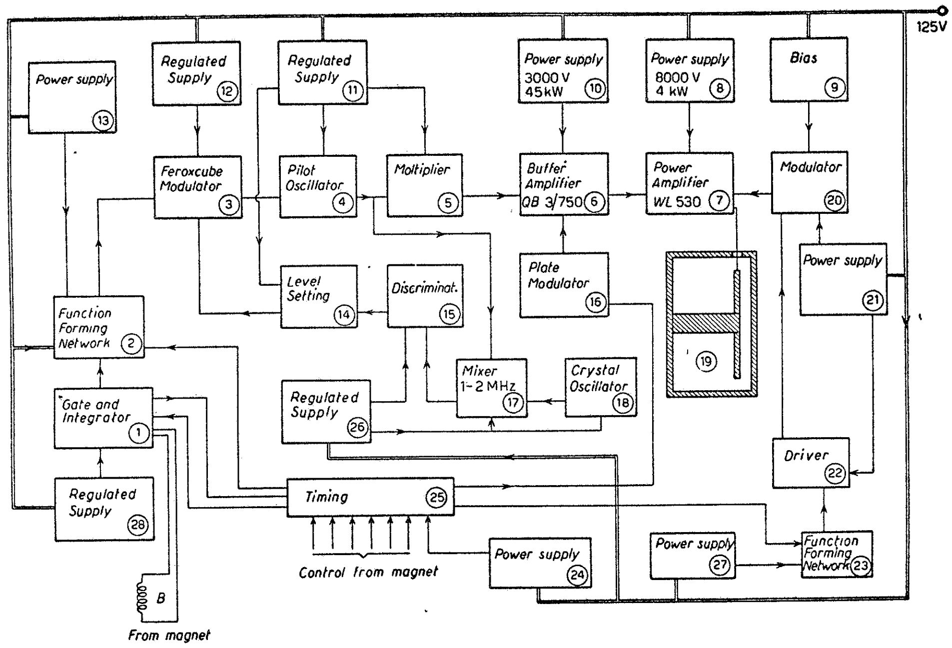

Fig. 4.

The R.F. system for the second cavity is not given in Fig. 3. This second cavity oscillates with a constant frequency, and the amplifiers feeding it are driven by the buffer amplifier 6.

8. – Conclusions.

From what we said we conclude that for an electronsynchrotron of about 1000 MeV a magnet employing strong focusing does not probably constitutes a definite advantage compared with the classical weak focusing design. On the contrary our numerical calculations for a comparison between the final intensities of two particular designs indicate the advantage of the Constant Gradient solution.

It certainly appears that in the C.G. case the problems connected with the injection of the electrons are more easy to resolve and understand, and probably we may have at disposal an injection time Δt larger than in the A.G. case.

To make our discussion more complete, we should compare the technical difficulties connected with the realization of the magnet in one case and the other: for instance the limits in the mechanical precision the tolerances on the magnetic properties of the iron etc. We will only observe that probably in our two designs these kind of difficulties are of the same order, may be with some advantage of the A.G. solution near the injection: in fact the distance Δn of the field value n_0 from the closest n values which lead to resonances is somewhat larger in the alternate gradient case.