LABORATORI NAZIONALI DI FRASCATI
SIS-Pubblicazioni

LNF-07/25(IR)
November 16, 2007

EXPLORER AND NAUTILUS CORRELATION FOR DAMPED SINUSOID SIGNALS

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Abstract

The correlation coefficient between the data of the cryogenic resonant detectors EXPLORER and NAUTILUS, for the case of damped sinusoid gravitational wave signals, has been studied. It is found that it is possible to obtain a 5 σ result for gravitational waves with amplitude of the order of $h \sim 10^{-20} - 10^{-19}$.

PACS: 04.80; 04.30

1 Introduction

The Rome group has operated the resonant gravitational wave (GW) detectors EXPLORER and NAUTILUS since 1994, applying correlation techniques between the two detectors such as the search for coincidences between events obtained by thresholding the measured data. The data are processed independently for each detector by applying optimum filters [1] matched to gravitational wave signals with a delta-like shape, principally aiming to detect events as supernova explosion. In a larger astrophysical scenarios as merger of two massive objects, or strong instabilities in rapidly rotating stars, energy can emitted in the form of a damped sinusoidal signal. The soft γ-ray repeaters (SGR) are considered typical sources of this kind. In a recent study ([2] and references therein), taking in the account the increased bandwidth [4,5], the capability of the resonant detectors to detect also signals with different wave shapes, is been exploited. In particular, the response to a damped sinusoid, typical signal emitted from stellar quasi-normal mode (QNM), is been analyzed on the development of the matched filter.

The purpose of this note is to study how to extend the correlation procedure to all data with no applied threshold and to consider GW signals with a duration longer than that generally adopted for delta-like pulses (order of one millisecond). In the second section, corresponding to a typical real experimental status of the detectors, the correlation coefficient, that shows a robust gaussian distribution, is evaluated. In the third section, following some previous considerations on the adopted signal, a software calibration will be performed, considering a large spectrum of physical hypotheses. Several amplitude values of the incident wave and also multi-wave packets with smaller amplitude will be considered. The last case might result especially interesting comparing the cross-correlation technique and the coincident events search which requires, in order to be efficient, an high SNR for the single signal of the packet on the each detector.

2 Experimental background

In order to study the correlation coefficient between the data of EXPLORER and NAUTILUS an entire day with typical experimental conditions is chosen.

In the Table 1 the main characteristics, referred to that day, are reported for each detector. The acquired data with the sampling time of 3.2 ms are processed with a filter matched to a delta-signal. The sequences of the data $x(t)$ and $y(t)$ are then divided in 27000 sub-periods, each one with the duration of 3.2 seconds[1].

[1]This choice is done by taking into account both typical astrophysical hypotheses and the peculiar instrumental conditions.

Table 1: Main characteristics of EXPLORER and NAUTILUS, referring to the chosen day used for the background measurements.

detector	frequencies Hz		aliased freq. Hz		bandwidth Hz	S_n $\frac{1}{Hz}$
EXPLORER	904.7	927.5	74.82	97.53	8.7	$(3 \cdot 10^{-21})^2$
NAUTILUS	926.3	941.8	74.85	89.89	9.6	$(3 \cdot 10^{-21})^2$

The correlation coefficient between two quantities $x(t)$ and $y(t)$ is calculated according to the following equation, where $\bar{x}$ is the average value of x, $\bar{y}$ that of y and $E[...]$ is the $expectation$.

$$r = \frac{E[(x-\bar{x})(y-\bar{y})]}{\sqrt{E[(x-\bar{x})]^2 E[(y-\bar{y})^2]}} \tag{1}$$

For uncorrelated x and y we have $E[r] = 0$ and the standard deviation is

$$\sigma = \sqrt{\frac{1}{N}} \tag{2}$$

where N is the number of independent samplings in each interval of 3.2 s. In the present case, the number N is less than 1000, because of the bandwidth of the detectors, so that we expect a standard deviation greater than $\sigma = \sqrt{\frac{1}{N}}$ by a certain factor which will be determined experimentally.

The result for the 27000 determinations is shown in fig.1. The correlation coefficient values are distributed according to the Gaussian law. It results $\sigma_{exp} = 1.8\sqrt{\frac{1}{N}}$. This means, roughly, that the samplings can be considered independent if taken in groups of $1.8^2 \sim 3$.

3 Calibration and simulation

In order to calibrate the correlation procedure in the hypothesis of an incoming damped sinusoid signal, we employ the study by Pai *et al.* [2], where a GW decaying wave-packet at the frequency ω_g, with duration τ_g is considered.

$$h(t) = h_o e^{-\frac{t-t_o}{\tau_g}} cos(\omega_g(t-t_o)) \quad for \ \ t > t_o \tag{3}$$

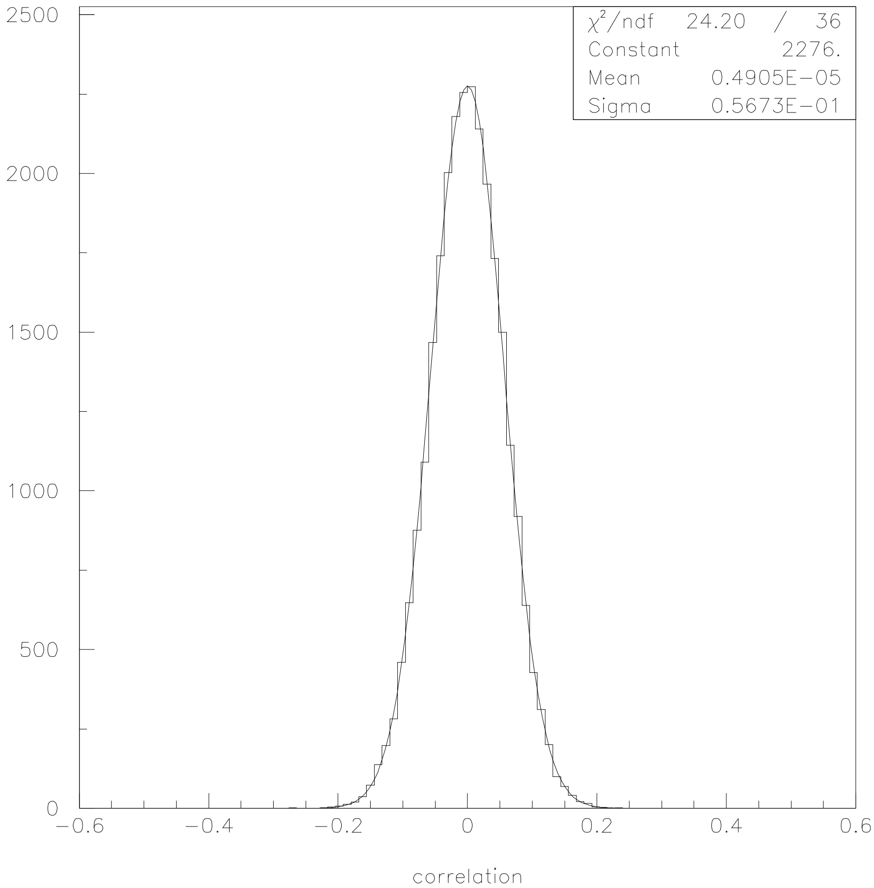

Figure 1: Correlation coefficient calculated over 3.2 s. Distribution of 27000 values during one entire day. The measured rms is 0.057, 1.8 times greater than the value $\sqrt{\frac{1}{1000}} = 0.0316$ as expected if the samplings were independent (see text). The continuos line is the gaussian fit.

3.1 The single detector

The response has been evaluated using a filter matched to the above waveform. It is found [2]:

$$g(t) = \frac{-h_o^2 L^2 \omega_g^2}{16 S_n} \Re[\tau_- \; e^{-t/\tau_-} P_1 + \tau_+ \; e^{-t/\tau_+} P_2 + \tau_g \; e^{-t/\tau_g} P_3] \tag{4}$$

where $\omega_\pm$ and $\tau_\pm$ are the two resonant modes and relative decay times after applying the filter, $P_{1,2,3}$ are complex functions of $\omega_\pm, \omega_g, \tau_\pm, \tau_g$ and time, the impulse amplitude h_0, the effective length, $L = 4L_{bar}/\pi^2$, and S_n the noise spectral density. We define the function $g'(t)$ as

$$g(t) = -\frac{h_0^2 L^2 \omega_o^2}{16 S_n} g'(t) \tag{5}$$

Although the experimental data have been obtained by applying a filter matched to a delta-signal, the function defined by Eq. 4 shall be used, considering it maintains a good validity for signal durations $\tau \leq 100\ ms$, as can be seen from the ref. [3] (see fig.3 therein), in the frequency range 900-950 Hz.

Adding the function $g(t)$ to the filtered data $x(t)$ and $y(t)$ that are given in $\sqrt{kelvin}$ units, it is reasonable to rewrite it in the terms:

$$g(t) = \sqrt{T_{eff}} \frac{h_o^2}{h_\delta^2} \frac{g'(t)}{g'_\delta} \tag{6}$$

where $T_{eff} = \overline{x(t)^2}$ is the effective noise temperature, h_δ is the minimum amplitude of a detectable delta-signal, and the quantity g'_δ is calculated with the parameters of the relative detector according to the Table 1, at time $t = 0$ and with $\tau_g = 1\ ms$.

3.2 Correlated response

Before performing the injections of signals on the real background, it is interesting to evaluate the correlation coefficient just considering the damped sinusoid signals.

In order to calibrate the correlation procedure the dependency of the correlation coefficient on both the frequency ω_g and the duration τ_g of the incoming g.w. signal is studied. The calculation of the correlation coefficient, in absence of background, gives the result shown in fig.2.

We remark that the correlation coefficient tends to vanish for frequency above 920 Hz. This depends on the particular adopted aliasing. From the Table 1, we see that the two lowest aliased resonance frequencies of EXPLORER and NAUTILUS overlap at 74.8 Hz. An incoming g.w. with $\omega_g \sim 915\ Hz$ and $\tau_g \leq 100\ ms$, small enough that both detectors be simultaneously exited in their lowest modes, could produce a reasonable

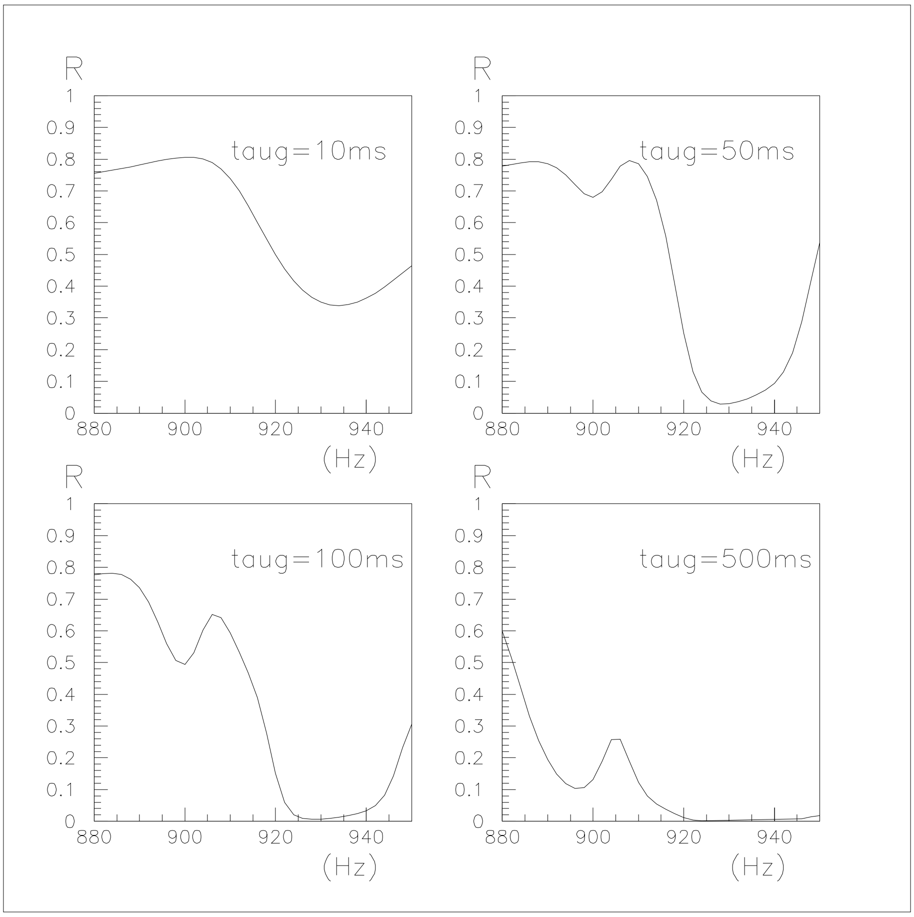

Figure 2: Correlation coefficient versus the frequency of the applied signals, with various signal durations.

large correlation coefficient. Instead a g.w. with $\omega_g \sim 927\ Hz$ would not produce in the aliased data a reasonable large correlation coefficient.

3.3 Software calibration

In order to evaluate the efficiency of the procedure for the typical experimental status, by using the Eq.6 a damped sinusoid signal is added to the background with characteristics given in the Table 1.

Fifty periods of length 3.2 s are extracted from the archive of the filtered data of each detector and several signals at frequency $f_g = 915\ Hz$ and of various duration τ_g and amplitude h_o are added to these data, examining also the case of signals with several pulses in the same 3.2 period.

The correlation coefficient is calculated for each one of the fifty periods as follows:
a)one signal with amplitude $h_o = 8 \cdot 10^{-20}$.
b)ten signals with amplitude $h_o = 3.5 \cdot 10^{-20}$.
c)three signals with amplitude $h_o = 3.5 \cdot 10^{-20}$.

The values obtained by averaging the fifty correlation coefficients are reported in fig.3, with respect to the duration of the applied signals. It is evident the systematical effect of the characteristic waves on the correlation parameter[2].

In the fig.4, the relative response at $t = 0$ (i.e. at the instant the response gets its maximum value) on the single detector (averaged on the fifty values of EXPLORER and the fifty of NAUTILUS) in terms of SNR referred to the value $h_\delta = 4 \cdot 10^{-19}$ is reported.

By inspecting figs. 3 and 4 it is relevant to note that it is possible to obtain significant values of the correlation coefficient also in the case with small SNR for the single sequence of the filtered data. For instance, restricting to the case of a signal with ten pulses, each one with $h_o = 3.5 \cdot 10^{-20}$ and duration $\tau \leq 50\ ms$, we obtain a correlation coefficient as high as $R = 0.26$, corresponding (see fig. 1) to over five standard deviations, while the response of the single detector has $SNR \leq 2$.

So, in the hypothesis of an emission of several small pulses of a few tens of milliseconds each one, the search by correlation could be more efficient with respect to the coincidence analysis that employs a threshold for $SNR \geq 4$ on the filtered data of the single detector.

[2]The possible difference of phase between the two detectors is not considered in this study, that is mainly intended as an evaluation of the global effect of an emission of GW with a certain duration. Taking into account the exact time difference implies the perfect knowledge of the time structure of the real event, of which Eq.3 is an ideal approximation.

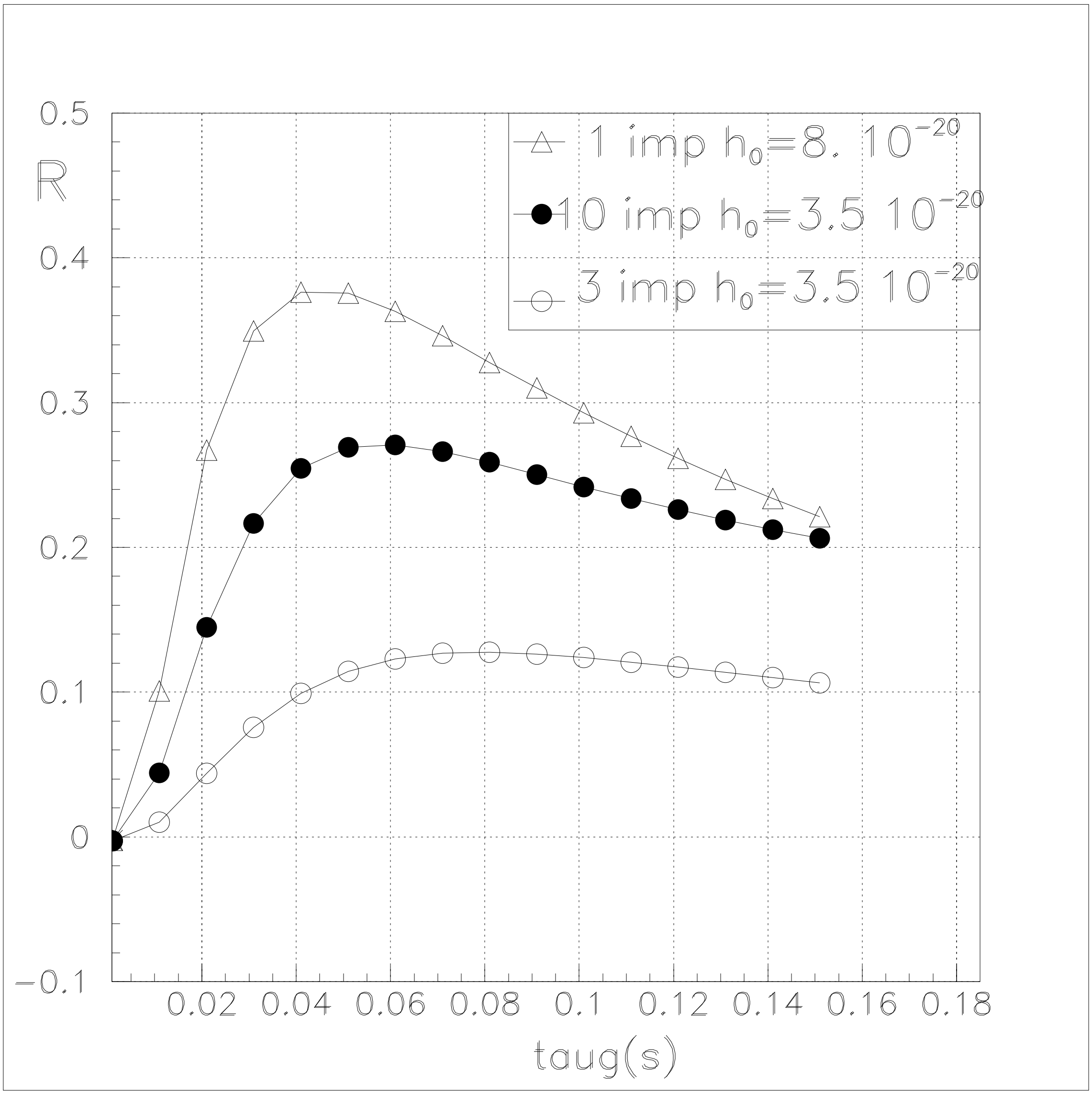

Figure 3: Correlation coefficient for the simulated signals of various duration τ_g, amplitude h_o and frequency 915 Hz, referred to the experimental background as reported in the Table 1.

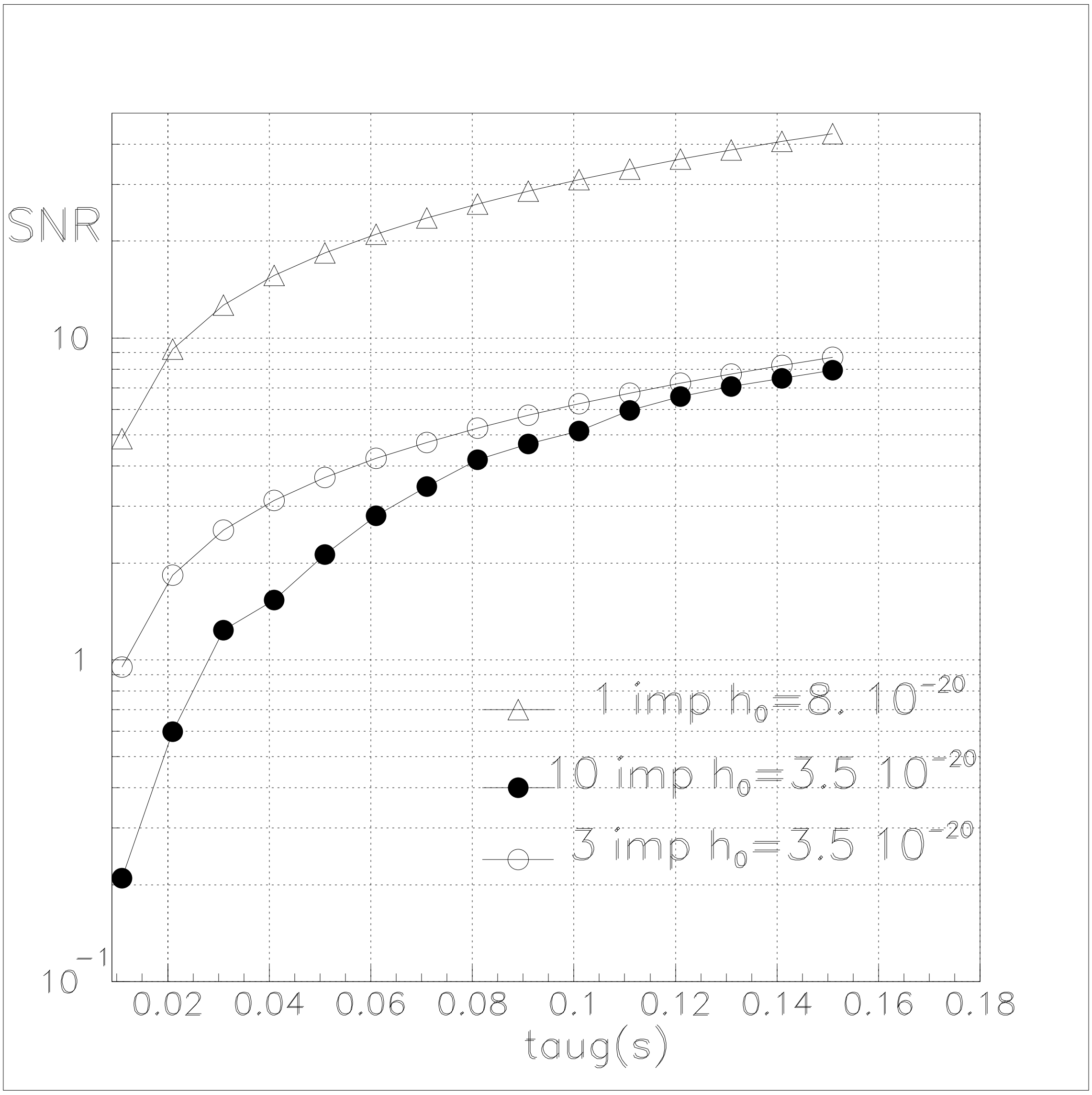

Figure 4: SNR for the single detector plus simulated signals of various duration τ_g, amplitude h_o and frequency 915 Hz, referred to the value $h_\delta = 4 \cdot 10^{-19}$.

References

[1] P.Astone *et al.* , Il Nuovo Cimento **20**, 9 (1997).

[2] A Pai *et al.* , Class. Quantum Grav. **24**, 1457-1477 (2007).

[3] S. D'Antonio *et al* J. Phys.: Conf. Ser.**32,** 192-197 (2006).

[4] P.Astone *et al.* , Phys. Rev. Lett. **91**, 111101 (2003).

[5] P Astone *et al.* , Class. Quantum Grav. **23** , S169-S178 (2006).