LABORATORI NAZIONALI DI FRASCATI
SIS – Pubblicazioni

LNF–07/23 (IR)
November 5, 2007

ON POSSIBILITY OF SPIN MANIFESTATION IN CHANNELING RADIATION

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Abstract

The Dirac equation for both electron and positron moving under planar channeling condition was considered. The influence of projectile's spin on the bound energy levels of transverse motion was estimated within analytical approach. The estimates show that for ultra relativistic projectiles and for variety of crystals the presence of a spin results in a small splitting of a bound energy level. This effect, in principle, can be revealed by the use of precise techniques.

PACS.: 41.75.Ht, 61.80.Fe, 61.85.+p
Key words: electron/positron channeling, spin, Dirac equation

1 INTRODUCTION

As known the motion of spin particles is described by the Dirac equation[1]. A characteristic feature of the relativistic equation of motion is that the spin of the particle is built into the theory from the beginning. This feature provides a useful measure of the applicability of a particular equation of motion to the description of a particular kind of particle. Hence, for the spin manifestation at electron/positron channeling, the Dirac equation has to be solved under the channeling conditions for the projectiles.

Generally, due to negligible small spin-related terms in the Dirac equation, solving the motion equation for specified particles channeled in crystals, does not need to take into account the spin influence to “projectile-crystal” interaction[2].

Here we consider the planar channeling, when the particles move at small angle to the crystal plane. It is well known that transverse-plane motion of a channeled particle is characterized by a set of bound energy levels; the radiation spectrum is defined by the transitions between various levels. Solving the Dirac equation without simplifications at the beginning enables to obtain the bound energy levels in presence of a particle’s spin.

In[3] the Dirac equation for ultra relativistic particles, which move in one-dimensional electric field under small angle to a plane orthogonal to the field, is solved. It is shown that one needs to solve the motion equations for two possible projections of a particle’s spin onto the electric field direction, i.e.

$$\frac{\partial^2 f_1(x)}{\partial x^2} + \frac{1}{2E_{\|}} \frac{\partial V(x)}{\partial x} \frac{\partial f_1(x)}{\partial x} + \frac{2E_{\|}}{c^2 \hbar^2} \left(\varepsilon - V(x)\right) f_1(x) = 0 \qquad (1)$$

for a spin directed along the electric field, and

$$\frac{\partial^2 g(x)}{\partial x^2} + \frac{2E_{\|}}{c^2 \hbar^2} \left(\varepsilon - V(x)\right) g(x) = 0 \qquad (2)$$

for a spin - opposite the electric field. Here $E_{\|}$ is the longitudinal energy of a projectile, ε is the energy of a bound state of the transverse motion, $V(x)$ is the potential energy, c is the speed of light, and $\hbar$ is the Planck constant. The function $f_1(x)$ in (1) is the 1st component of the 4-component Dirac spinor; and the function $g(x)$ in (2) is related with a 1st Dirac spinor component by the expression

$$g(x) = \frac{1}{V(x) - \varepsilon} \frac{\partial f_1(x)}{\partial x} \,. \qquad (3)$$

Obviously, one can use these results to consider the spin manifestation for channeling phenomena. Below only under-barrier states are taken into account. In order to give some estimates we have used simple parabolic potential approximating real potential of a crystal plane.

2 POSITRON CHANNELING AT PARABOLIC POTENTIAL APPROACH

The parabolic potential is a good approximation for interaction potential of positrons with crystal field near the center of plane channel between crystal planes; the positron potential energy can be presented as $V(x) = \frac{4U_0}{d^2} x^2$, U_0 is the depth of a potential well, and d is the distance between crystal planes. Hence, Eqs.(1), (2) for a positron should be written in the following

$$\frac{\partial^2 f_1(x_1)}{\partial x_1^2} + \frac{4U_0}{E_{\|}} x_1 \frac{\partial f_1(x_1)}{\partial x_1} + \frac{2E_{\|} d^2}{c^2 \hbar^2} \left(\varepsilon - 4U_0 x_1^2\right) f_1(x_1) = 0 , \qquad (4)$$

$$\frac{\partial^2 g(x_1)}{\partial x_1^2} + \frac{2E_{\|} d^2}{c^2 \hbar^2} \left(\varepsilon - 4U_0 x_1^2\right) g(x_1) = 0 , \qquad (5)$$

where $x_1 = x/d$. These equations have the solutions, which are written in terms of Hermitian’s polynomials, and determine the spectrum of bound energy states

$$\varepsilon_n^{\uparrow} = c\hbar(2n+1)\left(\frac{2U_0}{E_{\|} d^2}\left(1 + \frac{c^2\hbar^2 U_0}{2E_{\|}^3 d^2}\right)\right)^{1/2} + \frac{c^2\hbar^2 U_0}{E_{\|}^2 d^2} \qquad (6)$$

for a spin along the electrical field of a plane, and

$$\varepsilon_n^{\downarrow} = c\hbar\left(\frac{2U_0}{E_{\|} d^2}\right)^{1/2} \left(2n+1\right) \qquad (7)$$

for a spin opposite directed; n = 0, ±1, ±2, ...

3 ELECTRON CHANNELING FOR A MODIFIED POSCHLE-TELLER POTENTIAL

When one deals with the channeled electrons, the modified Poschle-Teller potential is appropriate to describe interaction of a particle with a crystal field

$$V(x) = -U_0 \,\mathrm{ch}^{-2}(x/b) , \qquad (8)$$

where U_0 is the depth of a potential well and b is the tabular parameter for given crystal planes. In this case the crystal plane is located in the center of a channel considered. The bound states are formed for the transverse electron energies within the range $-U_0 < \varepsilon < 0$.

Substituting the potential (8) in Eqs.(1), (2), one can obtain the equations of motion:

$$\frac{\partial^2 f_1(x_1)}{\partial x_1^{\ 2}}+\frac{U_0}{E_\parallel}\frac{\mathrm{sh}(x_1)}{\mathrm{ch}^3(x_1)}\frac{\partial f_1(x_1)}{\partial x_1}+\frac{2E_\parallel b^2}{c^2\hbar^2}\left(\varepsilon+\frac{U_0}{\mathrm{ch}^2(x_1)}\right)f_1(x_1)=0\,, \tag{9}$$

$$\frac{\partial^2 g(x_1)}{\partial x_1^2}+\frac{2E_\parallel b^2}{c^2\hbar^2}\left(\varepsilon+\frac{U_0}{\mathrm{ch}^2(x_1)}\right)g(x_1)=0\,, \tag{10}$$

where $x_1 = x/b$. Eq.(10) has an analytical solution[4)] while analytical solution of Eq.(9) is unknown.

The 2nd term in (9) presents the influence of electron spin to energy levels and wave functions; estimating it[5)], one can conclude that this term is small in comparison with other terms. It should be underlined that this term is proportional to the strength of crystal electric field $\partial V(x_1)/\partial x_1$. Therefore, the influence of electron spin is greater near the center of a channel (for electrons it coincides with crystal plane); thus, we have to consider the behaviour of wave functions near the center of a channel.

In order to resolve a problem analytically, a crystal potential $V(x_1)$ can be expanded into the Taylor series near the channel center; we have used the expressions with the terms of the 2nd order including. In this area the crystal potential could be expanded into the Taylor expansion. The crystal potential $V(x_1)$ has been expanded to the second term. Furthermore, as the 2nd order term is small in comparison with the potential depth value, the 1st term in the expansion for $\partial V(x_1)/\partial x_1$ has been only taken into account. Substitution of these approximations gives us the equations

$$\frac{\partial^2 f_1(x_1)}{\partial x_1^{\ 2}}+\frac{U_0}{E_\parallel}x_1\frac{\partial f_1(x_1)}{\partial x_1}+\frac{2E_\parallel b^2}{c^2\hbar^2}\left(\varepsilon+U_0\left(1-x_1^2\right)\right)f_1(x_1)=0\,, \tag{11}$$

$$\frac{\partial^2 g(x_1)}{\partial x_1^2}+\frac{2E_\parallel b^2}{c^2\hbar^2}\left(\varepsilon+U_0\left(1-x_1^2\right)\right)g(x_1)=0 \tag{12}$$

The solutions of Eqs.(11) and (12) are similar to those for Eqs.(4) and (5), respectively. Hence, one can written the bound energy spectra replacing the corresponding terms of Eqs.(6) and (7). Thus, we obtain the following expressions

$$\varepsilon_n^{\uparrow}=c\hbar(2n+1)\left(\frac{U_0}{2E_\parallel b^2}\left(1+\frac{c^2\hbar^2U_0}{8E_\parallel^3b^2}\right)\right)^{1/2}+\frac{c^2\hbar^2U_0}{4E_\parallel^2b^2}-U_0 \tag{13}$$

for a spin along the electrical field of a crystal plane, and

$$\varepsilon_n^{\downarrow}=c\hbar\left(\frac{U_0}{2E_\parallel b^2}\right)^{1/2}\left(2n+1\right)-U_0 \tag{14}$$

for a spin directed opposite the field.

4 ESTIMATES AND DISCUSSIONS

As shown[3)], energy spectrum of ultra relativistic channeled particles in parabolic potential $V(x) = kx^2$, when its spin is neglected, defined by the expression

$$\varepsilon_n = c\hbar\left(\frac{k}{2E_{\parallel}}\right)^{1/2}(2n+1) \tag{15}$$

As in the case of positron channeling $k = 4U_0/d^2$, the expression for bound energies of a "spinless" positron is written as follows

$$\varepsilon_n = c\hbar\left(\frac{2U_0}{E_{\parallel}d^2}\right)^{1/2}(2n+1). \tag{16}$$

Let us estimate the influence of a particle's spin to its motion at planar channeling. One can compare the distance between spin-related components of (6) and (7) with the distance between neighbour energy levels (16).

From (16) valid for spinless particles the distance between two neighbour levels *n* and *n*+1 is constant for every *n* and defined as

$$\Delta_n = \varepsilon_{n+1} - \varepsilon_n = 2c\hbar\left(\frac{2U_0}{E_{\parallel}d^2}\right)^{1/2}. \tag{17}$$

Presence of a spin results in the level splitting with a gap

$$\delta_n = \varepsilon_n^{\uparrow} - \varepsilon_n^{\downarrow} = c\hbar(2n+1)\left(\frac{2U_0}{E_{\parallel}d^2}\right)^{1/2}\left(\left(\frac{c^2\hbar^2U_0}{2E_{\parallel}^3d^2}+1\right)^{1/2}-1\right)+\frac{c^2\hbar^2U_0}{E_{\parallel}^2d^2}. \tag{18}$$

To evaluate the influence of a projectile spin to observable effects, let calculate the quantity δ_n/Δ_n :

$$\frac{\delta_n}{\Delta_n} = \left(n+\frac{1}{2}\right)\left(\left(\frac{c^2\hbar^2U_0}{2E_{\parallel}^3d^2}+1\right)^{1/2}-1\right)+\frac{c\hbar}{4E_{\parallel}}\left(\frac{2U_0}{E_{\parallel}d^2}\right)^{1/2} \tag{19}$$

If the quantity (19) is equal or almost equal to 1, the spin of a particle should be taken into account at simulation. And vice versa, if this quantity is rather small, $\delta_n/\Delta_n << 1$, the influence of a spin is negligible.

For estimations the depth of a potential well for planar channeling can be taken as $U_0 \approx 20$ eV, and the distance between atomic planes as $d \approx 2\,\text{Å}$. Hence, the term $4U_0/d^2$ is about $2 \cdot 10^{17}$ eV/cm^2. It should be noticed that for ultra relativistic energies $E_{\|} > 50$ MeV the quantity $c^2\hbar^2 U_0/(2E_{\|}^3 d^2) << 1$. Therefore, the 1st term in (19) can be considered as negligible small.

Moreover, it should be underlined, that the 1st term in (19), which includes the dependence on the level number *n*, may be indeed considered as zero. Successfully, the energy splitting is defined by the 2nd term of Eq. (19), i.e., it is constant for all energy levels at the fixed projectile energy $E_{\|}$.

Then, we obtain from (19):

$$\frac{\delta_n}{\Delta_n} \approx \frac{c\hbar}{4E_{\|}}\left(\frac{2U_0}{E_{\|}d^2}\right)^{1/2} \tag{20}$$

Thus, for positrons we have $\delta_n/\Delta_n \approx 4.92 \cdot 10^{-11}$ at $E_{\|} = 1$ GeV; $1.4 \cdot 10^{-10}$ at $E_{\|} = 500$ MeV; and $4 \cdot 10^{-9}$ at $E_{\|} = 50$ MeV. All estimates show that influence of a positron's spin can be omitted for considered projectile energy range $E_{\|} > 50$ MeV. Of course, there is a special interest to evaluate (19) for the energies less than 50 MeV; however, for that interval the developed method for solving the Dirac equation becomes inapplicable.

Above one obtained that for very wide range of projectile energies $E_{\|}$ the influence of a spin can be omitted. But this effect may be observable for some very specific crystals. For positron energy 500 MeV and typical crystals we have

$$\delta_0/\Delta_0 \approx \frac{c\hbar}{4E_{\|}}\left(\frac{2U_0}{E_{\|}d^2}\right)^{1/2} \approx 10^{-10} \tag{21}$$

If one would like to observe the effect for such a positron under planar channeling conditions that may correspond to the value of $\delta_0/\Delta_0 \approx 0.05$, the coefficient $4U_0/d^2$ should be equal 10^{31} eV/cm^2. This quantity requires both very deep potential well $U_0 > 1$ MeV and very small distances between crystal planes $d < 1$ pm. These parameters are rather distant from the parameters of existing crystals.

Similar estimates can be given for a channeled electron, for which bound states are defined by the expressions (13), and (14). The energy spectrum of a "spinless" electron is defined as in (16):

$$\varepsilon_n = c\hbar\left(\frac{U_0}{2E_{\|}b^2}\right)^{1/2}(2n+1) - U_0 \tag{22}$$

For most of the crystals the parameter *b* is about the distance between crystal planes[6)]. It means that the estimates done for positrons are valid for electrons, too.

Finally, it is of theoretical interest to obtain the spin-related energy as a small correction to the energy of spinless particle. Above one obtained that the parameter $c^2\hbar^2 U_0 /(2E_{\|}^3 d^2) << 1$ in (6). Hence, one can use the Taylor expansion in the expression (6) for positron levels:

$$\varepsilon_n^{\uparrow} = c\hbar(2n+1)\left(\frac{2U_0}{E_{\|} d^2}\right)^{1/2} + \frac{c^2\hbar^2 U_0}{E_{\|}^2 d^2} + c\hbar(2n+1)\left(\frac{2U_0}{E_{\|} d^2}\right)^{1/2} \cdot \frac{c^2\hbar^2 U_0}{4E_{\|}^3 d^2} \qquad (23)$$

The first term in (23) is the energy levels of a spinless particle (16), and both the second and third terms are spin-related ones. The first term is proportional to $\hbar$, the second term is proportional to $\hbar^2$ and the third term - to $\hbar^3$. The second term does not depend on level's number n and related with a spin only. The third term depends on both the spin and energy level number. Hence, one can see obviously that spin-related energy is a small correction to the positron's energy obtained when the spin of a positron has been omitted.

5 CONCLUSIONS

It is known that the energy levels of a particle in parabolic potential are equidistant. Above it was obtained that influence of a projectile's spin results in splitting each energy level into two sublevels, which correspond to two possible spin projections onto the direction of electrical field. The distance between spin-related sublevels is almost the same for all levels.

One could observe two lines (with a small shift) in radiation spectra of particles, but the even modern experimental equipment does not able to separate these lines. Moreover, the broadening of levels due to various processes of scattering prevents observing of a "fine structure" of the levels near the peak of a potential well.

REFERENCES

(1) L.D. Landau, and E.M. Lifshits *Theoretical Physics. Quantum Electrodynamics*, chapters 3,4, Fizmatlit, Moscow, 2001 (in Russian).

(2) M.A. Kumakhov, *Radiation from Channeled Particles in Crystals*, Energoatomizdat, 1986 (in Russian).

(3) A. Babaev, *Proc. of SPIE*, **6634** (2007) 66340A.

(4) S. Flugge. *Practical Quantum Mechanics*, vol. 1., Springer-Verlag, Berlin, Heidelberg, New York, 1971.

(5) Haakon A. Olsen, and Yu. Kunashenko, *Phys. Rev*. **A 56** (1997) 321.

(6) V.A. Bazylev, and N.K. Zhevago, *Rad. Eff*. **54**(1) (1981) 41-50.